Dynamics of two level system (qubit) interacting with random and deterministic classical field.

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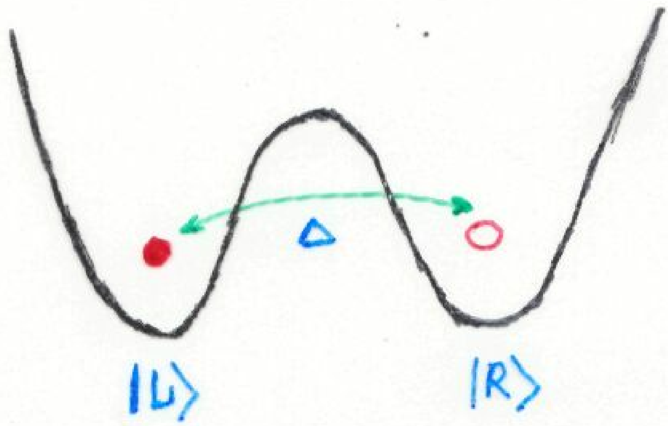
Definition of the model.

Quantum bit interacting with random classical field.

$$\hat{H} = \hat{H}_{qb} + \hat{q}\hat{E} + \hat{H}_{env}$$



$$\hat{H}(t) = -\frac{1}{2}\hbar\Delta\,\hat{\sigma}_x + \frac{q_0}{2}\phi(t)\hat{\sigma}_z\ ,$$

$\hat{\sigma}_x$, $\hat{\sigma}_z$ - the Pauli matrixes,

Δ - the tunneling matrix element between wells,

$\phi(t)$ - random classical field with Gaussian statistic:

$$\langle\phi(t_1)\phi(t_2)\rangle = \langle\phi^2\rangle\,\delta(t_1 - t_2)$$

The quantities of our interest.

Consider the states of the particle in two-well potential:

$|L\rangle$ -localized in the left well,

$|R\rangle$ -localized in the right well.

$|S\rangle = a|L\rangle + b|R\rangle$ -delocalized state.

We will be interested in quantum-mechanical probability for the particle to go from initial state $|S\rangle$ to either of the wells at time t averaged over random field $\phi(t)$:

$$\langle\, P_{S\to L(R)}[\phi(t)]\,\rangle_\phi = ?$$

$$\langle\, P^n_{S\to L(R)}[\phi(t)]\,\rangle_\phi =$$

$$= \langle P_{S\to L(R)}[\phi(t)] \times \ldots \times P_{S\to L(R)}[\phi(t)]\rangle = ?$$

The probability $P_{S\to L(R)}[\phi(t)]$ is *a random quantity* random field $\phi(t)$.

The calculation scheme.

Let $q(t) = \pm 1$ is the trajectory of the particle.

$$\langle P_{L\to L}(t)\rangle_\phi =$$

$$\int\int Dq_1(t) \quad Dq_2(t)\; A[q_1]A[q_2]^* F[q_1, q_2]$$

where $F[q_1, q_2]$ the influence functional:

$$\begin{aligned} F[q_1, q_2] &= \left\langle \exp\left[-\frac{i}{\hbar}\int_0^t \phi(\tau)(q_1(\tau) - q_2(\tau))d\tau\right]\right\rangle_\phi \\ &= \exp\left[-\Gamma\int_0^t (q_1(\tau) - q_2(\tau))^2 d\tau\right] \end{aligned}$$

$$\Gamma = \frac{q_0^2\langle\phi^2\rangle}{2\hbar^2}$$

$A[q_1]$ is the one transition amplitude:

$$A[q] = i\frac{\Delta}{2}.$$

The calculation scheme.

There are only four different states of the pair of trajectories $[q_1(t), q_2(t)]$:

$$[L, L], \quad [L, R], \quad [R, L], \quad [R, R].$$

The state of the pair of trajectories can be described by four-dimensional state vector.

Let us introduce the jump matrix to go from one state $\vec{E}_n$ to another $\vec{E}_{n+1}$:

$$\mathbf{\Lambda} = i\frac{\Delta}{2}\begin{bmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Next let us introduce 4×4 matrix for influence functional:

$$\mathbf{U}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\Gamma t} & 0 & 0 \\ 0 & 0 & e^{-\Gamma t} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The calculation scheme.

$$\langle P_{L\to L}(t)\rangle = \vec{E}_L^t \times \left[\sum_{n=0}^{\infty}\int_0^t dt_n \int_0^{t_n} dt_{n-1} \dots \int_0^{t_2} dt_1 \right.$$

$$\left. \mathbf{U}(t-t_n)\mathbf{\Lambda}\mathbf{U}(t_n - t_{n-1})\mathbf{\Lambda}\dots\mathbf{\Lambda}\mathbf{U}(t)\right] \times \vec{E}_L$$

Making the Laplace transform:

$$\langle P_{L\to L}(\lambda)\rangle = \int_0^{\infty} \langle P_{L\to L}(t)\rangle\, e^{-\lambda t}\, dt$$

$$\langle P_{L\to L}(\lambda)\rangle = \vec{E}_L^t \times [\mathbf{U}^{-1}(\lambda) - \mathbf{\Lambda}]^{-1} \times \vec{E}_L$$

then

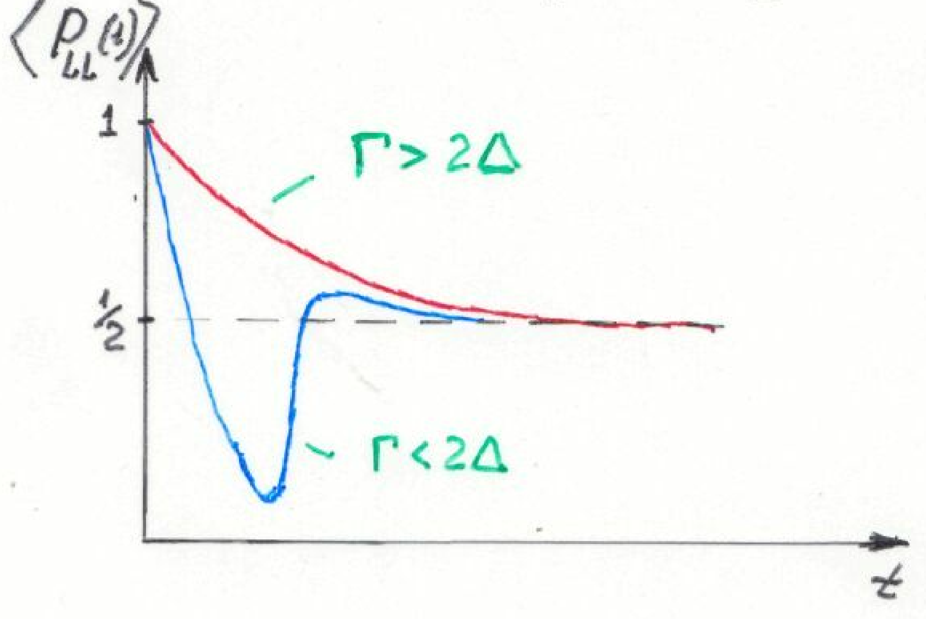
$$\langle P_{L\to L}(t)\rangle = \frac{1}{2\pi i}\int_C \vec{E}_L^t \times [\mathbf{U}^{-1}(\lambda) - \mathbf{\Lambda}]^{-1} \times \vec{E}_L e^{\lambda t} d\lambda$$

The results. $\langle P_{L\to L}(t)\rangle$.

$$\langle P_{L\to L}(t)\rangle = \frac{1}{2}$$
$$+ \frac{\exp\left\{\frac{t}{2}(-\Gamma + \sqrt{\Gamma^2 - 4\Delta^2})\right\}(\Gamma + \sqrt{\Gamma^2 - 4\Delta^2})}{4\sqrt{\Gamma^2 - 4\Delta^2}}$$
$$+ \frac{\exp\left\{-\frac{t}{2}(\Gamma + \sqrt{\Gamma^2 - 4\Delta^2})\right\}(-\Gamma + \sqrt{\Gamma^2 - 4\Delta^2})}{4\sqrt{\Gamma^2 - 4\Delta^2}}$$

- $\langle P_{L\to L}(t = \infty)\rangle = \frac{1}{2}$.
- $\Gamma > 2\Delta$ - the exponential relaxation regime.
- $\Gamma < 2\Delta$ - damped oscillation regime.
- in regime $\Gamma \gg \Delta$ two relaxation times appear:

$$\tau_1 = \frac{1}{\Gamma}, \quad \tau_2 = \frac{\Gamma}{\Delta^2}$$



The off-diagonal elements of density matrix.

$$\langle \Psi_L^*(t)\Psi_R(t)\rangle =$$

$$\frac{i\Delta}{2\sqrt{\Gamma^2-4\Delta^2}}\exp\left\{-\frac{t}{2}(\Gamma+\sqrt{\Gamma^2-4\Delta^2})\right\}$$
$$-\frac{i\Delta}{2\sqrt{\Gamma^2-4\Delta^2}}\exp\left\{\frac{t}{2}(-\Gamma+\sqrt{\Gamma^2-4\Delta^2})\right\}$$

- Density matrix vanishes at big times

$$\langle \Psi_L^*(t=\infty)\Psi_R(t=\infty)\rangle = 0$$

- In the limit $\Gamma \gg \Delta$ the relaxation time is equal to

$$\tau_1 = \frac{1}{\Gamma}.$$

The higher moments of $P_{L\to L}(t)$

$$\langle P^n_{L\to L}((t=\infty))\rangle = \frac{1}{n+1}$$

This result can be interpreted in terms of distribution function for $P_{L\to L}(t=\infty)$:

$$\langle P^n_{L\to L}\rangle = \int_0^1 (P_{L\to L})^n \, \mathcal{P}(P_{L\to L})\, dP_{L\to L} = \frac{1}{n+1}$$

- The random quantities $P_{L\to L}(\infty)$ has an uniform distribution:

$$\mathcal{P}(P_{L\to L}(\infty)) = 1$$

- $\mathcal{P}(P_{L\to L}(\infty))$ does not depend on initial state of the particle:

$$\langle P^n_{S\to L}(t=\infty)\rangle = \frac{1}{n+1}$$

$$|S\rangle = a|L\rangle + b|R\rangle \quad |a|^2 + |b|^2 = 1$$

Sensitivity to initial state of the particle.

- Sensitivity of the final state to initial can be determined by the correlator:

$$\langle (P_{S\to L}(\infty) - P_{S'\to L}(\infty))^2 \rangle = \frac{|ab' - a'b|^2}{3}$$

$$|S\rangle = a|L\rangle + b|R\rangle \quad |S'\rangle = a'|L\rangle + b'|R\rangle$$

- Sensitivity to variation of external field $\phi(t)$:

$$\phi(t) = \phi_{ran}(t) + \delta\phi(t)$$

$$|S'\rangle = \exp\left[\frac{i}{\hbar}\int d\tau \delta\phi(\tau)\,\hat{\sigma}_z\right][a|L\rangle + b|R\rangle]$$

$$|S'\rangle = ae^{-i\Phi}|L\rangle + b\,e^{i\Phi}|R\rangle$$

$$\Phi = \frac{1}{\hbar}\int \delta\phi(\tau)\,d\tau$$

$$\langle (P_{S\to L}\{\phi(t)\} - P_{S\to L}\{\phi(t) + \delta\phi(t)\})^2 \rangle =$$

$$= \langle P_{S\to L} P_{S\to R} \rangle \frac{4\sin^2(\Phi)}{3} = \frac{2}{9}\sin^2(\Phi)$$