BCS pairing with time-dependent interaction

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Issues:

- coherent BCS dynamics
- How does BCS settle down?
- Manifestations of pairing in a cold trapped gas

collaboration:

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BCS dynamics:

$$H = \sum_p \xi_p a_p^\dagger a_p - \frac{\lambda}{2} \sum_{p_1+p_2=p_3+p_4} a_1^\dagger a_2 a_3^\dagger a_4$$

λ ← time-dependent coupli[ng]

Mean field theory $\qquad \xi_p = \frac{p^2}{2m} - E_F$

$$H = \sum_p \xi_p a_p^\dagger a_p + \Delta(t)\, a_p^\dagger a_{-p}^\dagger + \bar{\Delta}(t)\, a_{-p} a_p$$

Bogoliubov

A $\quad \Delta(t) = \lambda(t) \langle a_{p\uparrow}(t)\, a_{-p\downarrow}(t) \rangle$

B $\quad i\dot{a}_p = [a_p, H] = \xi_p a_p + \Delta\, a_{-p}^\dagger$

$\quad\;\; i\dot{a}_{-p}^\dagger = [a_{-p}^\dagger, H] = \xi_p a_{-p}^\dagger + \bar{\Delta}\, a_p$

Condensate dynamics $\Delta(t)$
coupled with

Quasiparticle dynamics $a_p(t)$
(condition required: $\Delta \gtrsim \frac{1}{\tau_E}$,
true not too close to T_c)

Unlike bosons, where qp's are slaved
to condensate

Magnetically trapped fermions

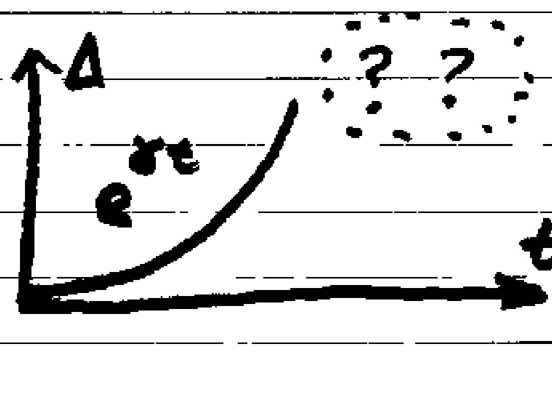
New possibility: time-varying coupling (Feshbach resonances)

Problem: Given $\lambda(t)$, e.g. step-like find $\Delta(t)$

Instability of a Fermi sea ($\Delta=0$) in the presence of pairing inter.

Linearized Eqs A, B:

$\Delta \propto e^{\gamma t}$, $a_p \propto e^{-i\xi_p t}$



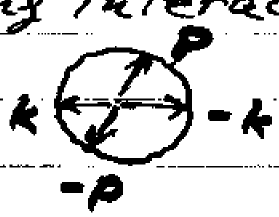
$1 = \lambda \nu \,\mathrm{Re} \int \frac{\tanh \frac{1}{2}\beta E}{E + i\gamma}\, dE \Rightarrow \gamma \approx \Delta_{equil}$

- BCS state emerges over time $\sim \frac{\hbar}{\Delta}$
- What happens after that?

Pseudo spin representation

$$H = \sum_p \xi_p a_p^+ a_p - \lambda \sum_{pk} a_k^+ a_{-k}^+ a_p a_{-p}$$

Pairing interaction



Spin operators

$$\sigma_p^- = a_p a_{-p}, \quad \sigma_p^+ = a_p^+ a_{-p}^+$$

Anderson, Richardson

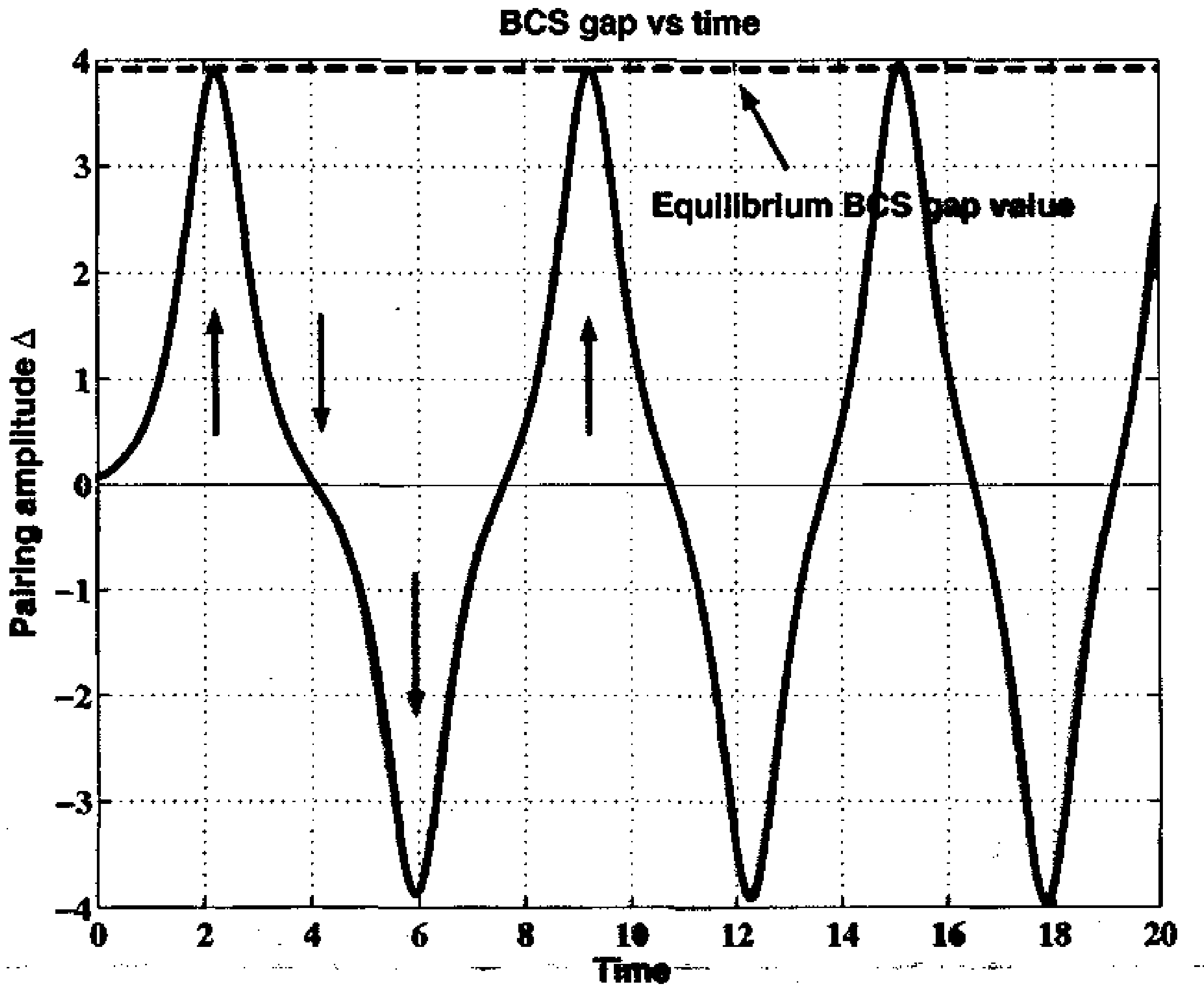
$$H = -\sum_p \xi \sigma_p^z - \lambda(t) \sum_{pp'} \sigma_p^+ \sigma_{p'}^-$$

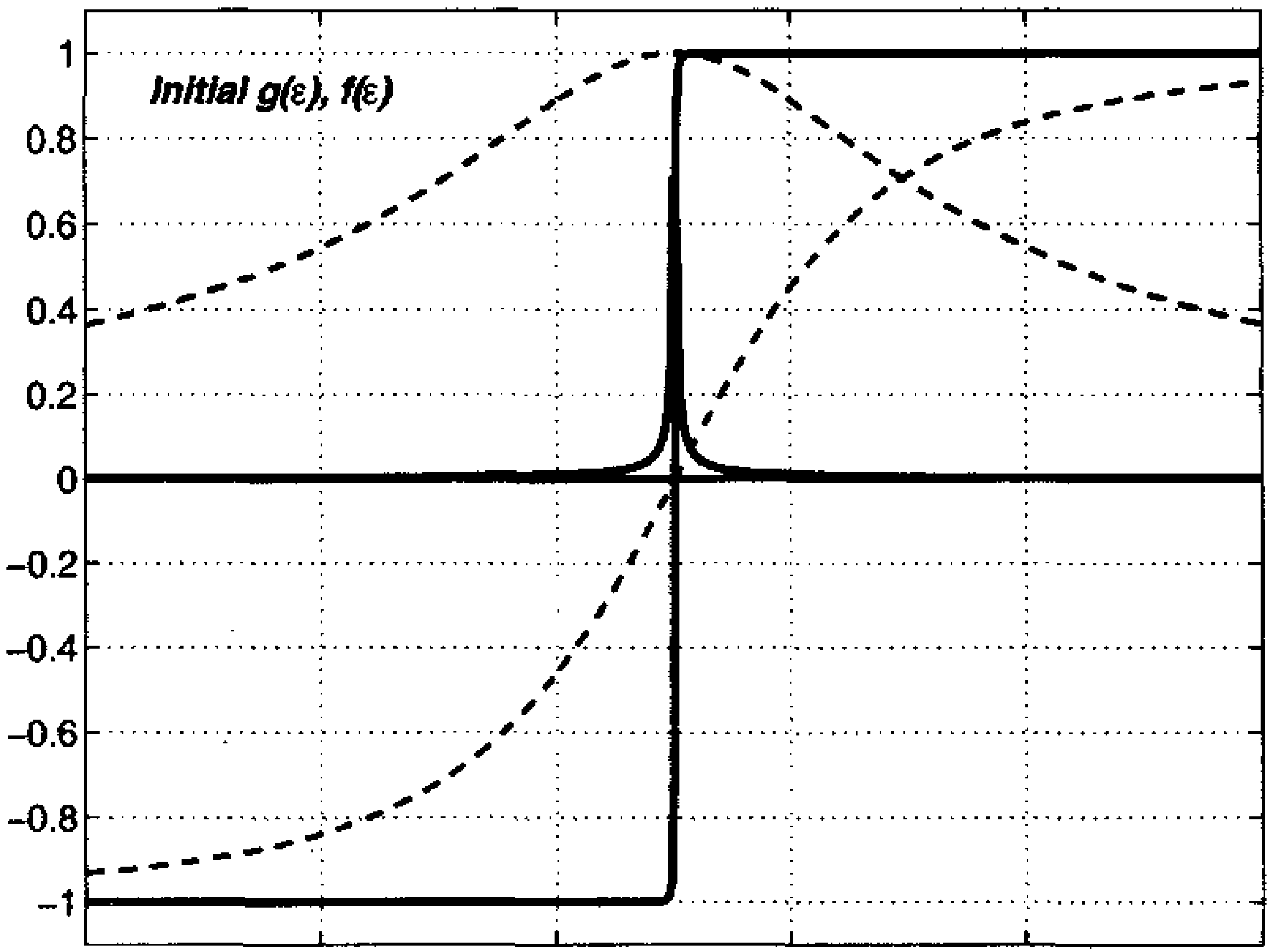
$\downarrow$ Mean field theory

Each spin is driven by an effective field $\vec{B}_p = \left(\frac{\lambda}{2}\langle\sigma^x\rangle, \frac{\lambda}{2}\langle\sigma^y\rangle, \xi_p\right)$

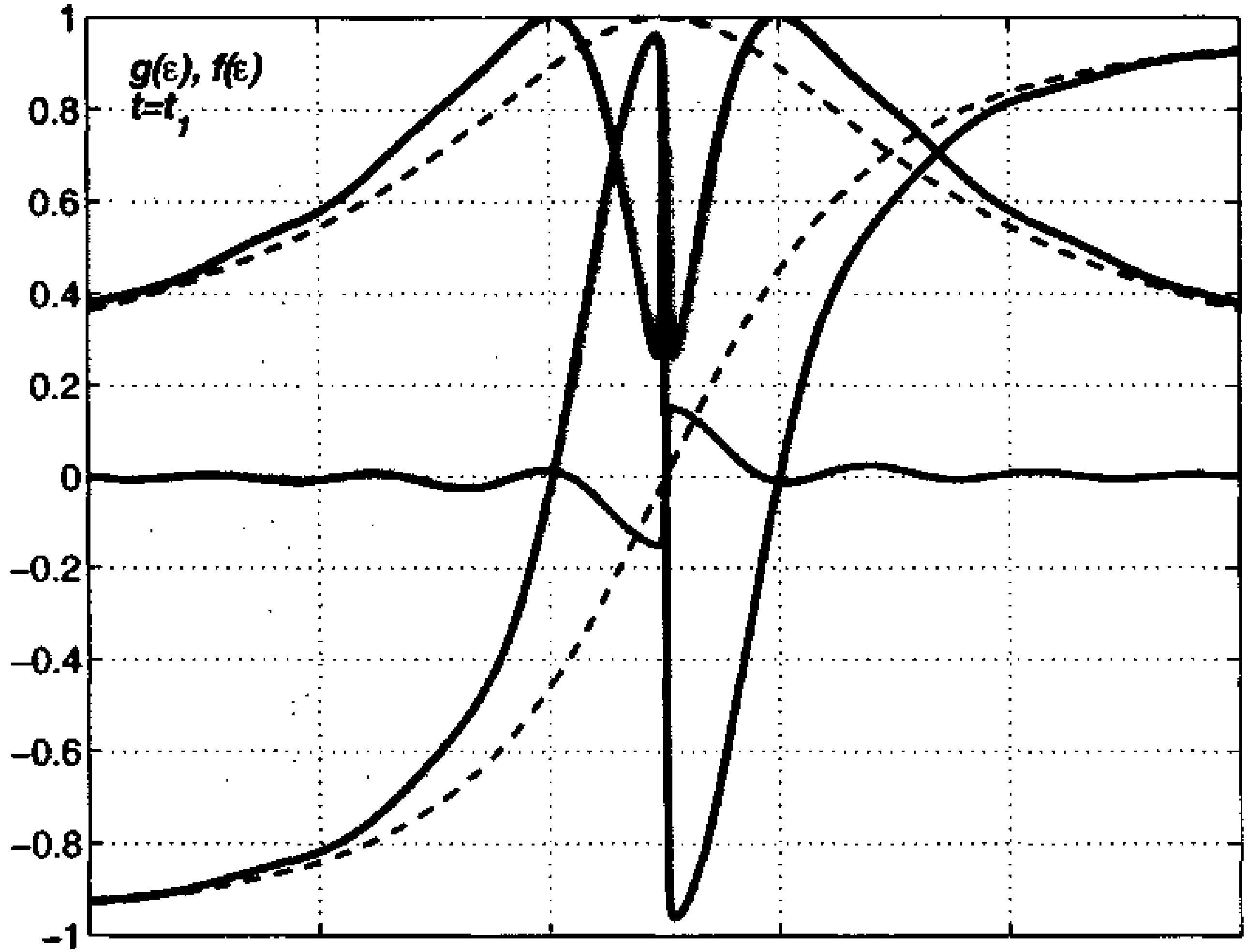
the same for all spins

$$\frac{d}{dt}\vec{\sigma}_p = \vec{B}_p \times \vec{\sigma}_p$$ Block precession


BCS gap vs time
Equilibrium BCS gap value
Pairing amplitude Δ
Time
4
3
2
1
0
−1
−2
−3
−4
0
2
4
6
8
10
12
14
16
18
20



Initial g(ε), f(ε)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1


g(ε), f(ε)
t=t1
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1

Features:

- Ground state identical to BCS

$\Delta = \frac{1}{2}\langle \sigma_x \rangle$, $\langle \vec{\sigma}_p \rangle = \frac{\vec{B}_p}{|B_p|} = (f, 0, g)$

$f = \frac{\Delta}{(\xi^2+\Delta^2)^{1/2}}$, $g = \frac{\xi}{(\xi^2+\Delta^2)^{1/2}}$

- Finite temperature behaviour same as BCS

- Collective modes, $\omega = \Delta, \ldots$

- Integrability:

$$L_p = -\sigma_p^z - \lambda \sum_{p'} \frac{\sigma_p \cdot \sigma_{p'}}{\xi - \xi'}$$

Sierra, Gaudin

commute with Hamiltonian

Manifestations in experiment

- Turn on λ, look for ringing
- $\omega \sim \Delta_{equil}$, i.e. strongly depends on λ
- For $\Delta \sim E_F$, strong coupling to the density mode.

  but different from hydro. modes

  $\omega_{BCS} \gg \omega_{hydro}$

- Damping mechanisms:

1. Energy relaxation via elastic collisions, $\tau_E^{-1} \sim E_F \left(\frac{T}{E_F}\right)^2$
2. "Inhomogeneous averaging"
   ω, Δ depend on initial conditions
3. States w. particle-hole imbalance compete w. BCS