Quantum Coherence in Josephson Qubits

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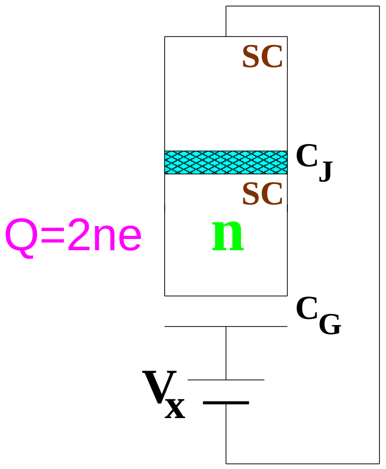
- **Josephson qubits and sources of dissipation**
- **Models of environment & analytical methods**
- **Noise spectra: Ohmic environment, 1/f noise**
- **Higher-order contributions, nonlinear coupling**
- **Energy renormalization and the Berry phase**

Josephson Charge Qubits

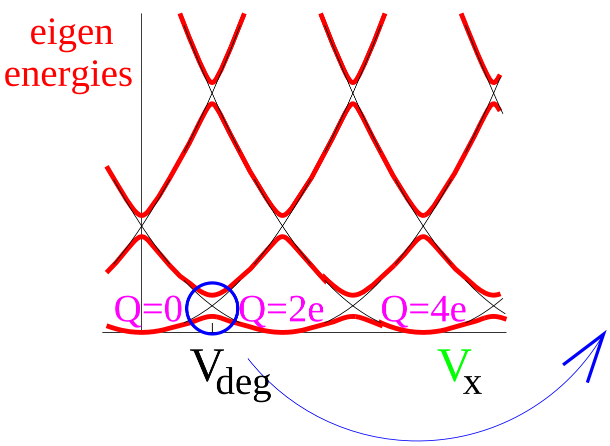


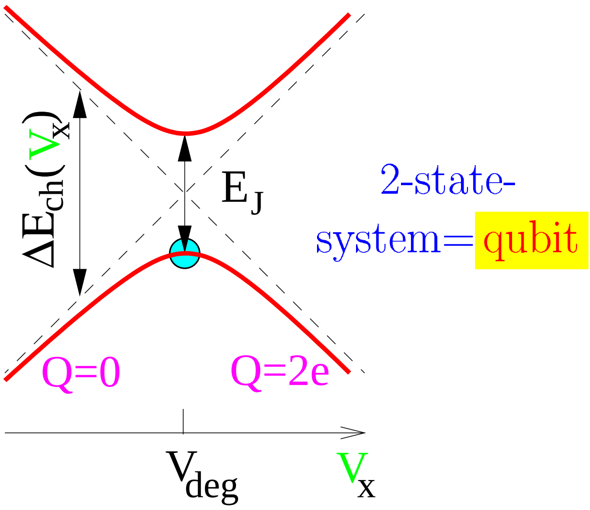
- charging energy (Cooper-pairs) $\quad Q_{\mathrm{g}} = C_{\mathrm{g}} V_{\mathrm{x}}$

$$E_{\mathrm{ch}}(n, V_{\mathrm{x}}) = \frac{(2ne - Q_{\mathrm{g}})^2}{2(C_{\mathrm{g}} + C_{\mathrm{J}})}$$

- Josephson coupling

$$E_{\mathrm{J}} \cos\varphi \quad \rightarrow \quad \frac{E_{\mathrm{J}}}{2} \, |n\rangle \langle n \pm 1|$$

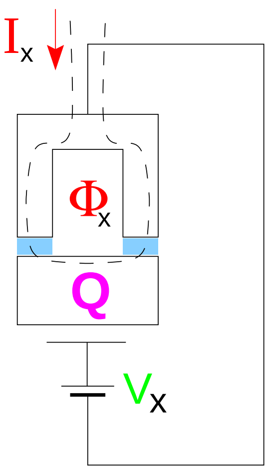




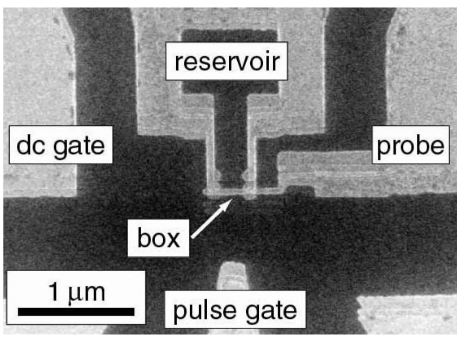
$$\mathcal{H} = \quad \Delta E_{\mathrm{ch}}(V_{\mathrm{x}}) \, \hat{\sigma}_z + E_{\mathrm{J}} \, \hat{\sigma}_x$$

Flux-controlled qubit

controlled Josephson coupling

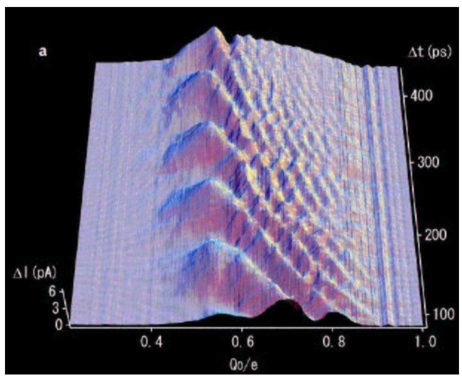


$$\mathcal{H} = \quad \Delta E_{\mathrm{ch}}(V_{\mathrm{x}}) \, \hat{\sigma}_z + E_{\mathrm{J}}(\Phi_{\mathrm{x}}) \, \hat{\sigma}_x$$



Coherent oscillations

(Nakamura et al. '99)

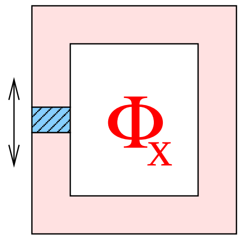


Josephson Flux / Phase Qubits

$E_J \gg e^2/2C$

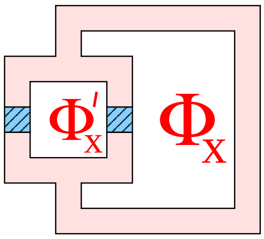
- SQUID type devices

$$2\pi \frac{\Phi}{\Phi_0} = \phi$$

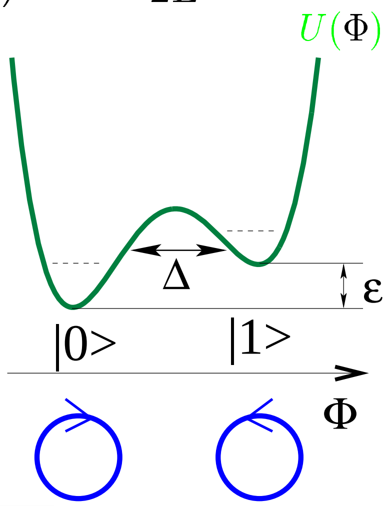


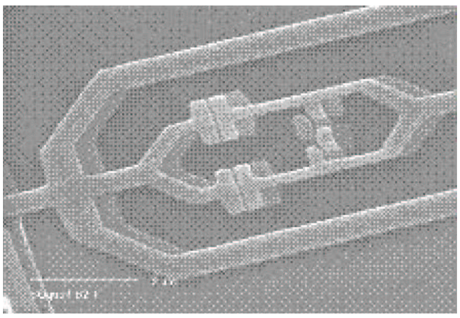
$$\mathcal{H} = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L}$$

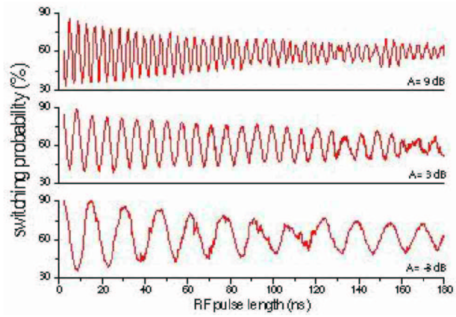
$$\mathcal{H} \approx \varepsilon(\Phi_x)\, \hat{\sigma}_z + \Delta\, \hat{\sigma}_x$$



$\Delta = \Delta(\Phi'_x)$ - controlled

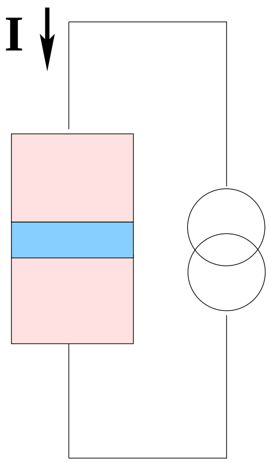


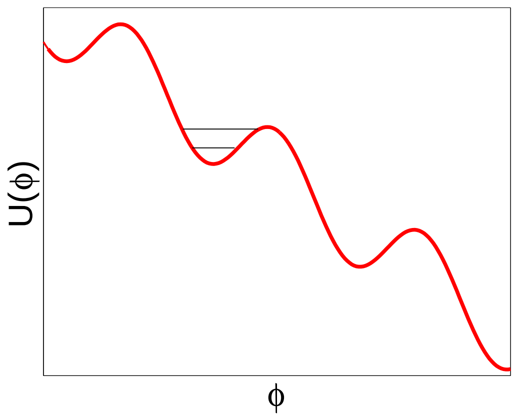




Chiorescu et al. '02

- Current biased Josephson junctions



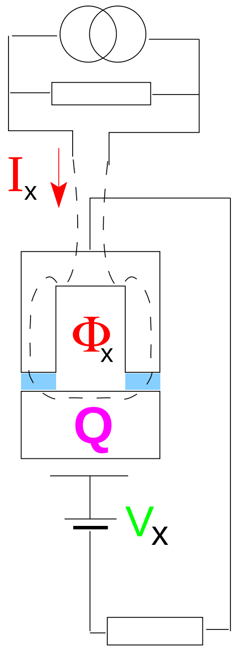


Martinis et al. '02
Han et al. '02

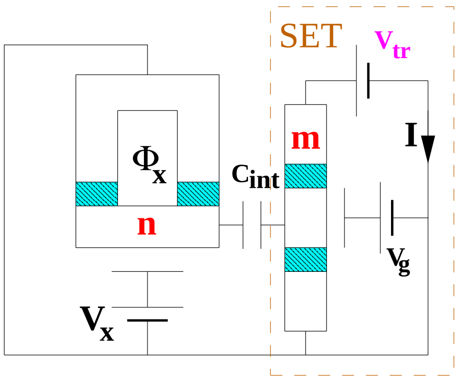
$$U(\Phi) = -E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) - I\Phi$$

Sources of Dissipation

- Intrinsic sourses:
  - $1/f$: background charges, fluxes, crit. current - strong
  - quasiparticles - frozen
  - electromagnetic radiation - shielding
  - nuclear spins - inhomogeneous level broadening
  - . . .



- External (artificial) sources:
  - control circuitry
  - detectors - on-off



- Other errors:
  - leakage out of 2-state Hilbert space
  - errors in manipulations / preparation
  - unknown couplings
  - . . .

Weak coupling: Golden rule

weak noise!

$$H = -\tfrac{1}{2}B_z\,\sigma_z - \tfrac{1}{2}B_x\,\sigma_x + \sigma_z X + H_{\rm bath}\ , \qquad X \equiv \Sigma_a\,\lambda_a x_a$$

Rotation to eigenbasis

$$H = -\tfrac{1}{2}\Delta E\,\sigma_z + (\cos\eta\,\sigma_z - \sin\eta\,\sigma_x)\,X + H_{\rm bath}$$

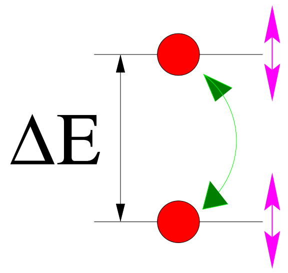
$$\uparrow \qquad\qquad \uparrow$$

longitudinal and transverse coupling to bath

$$\Delta E \equiv \sqrt{B_z^2 + B_x^2} \qquad \tan\eta = B_x/B_z$$

$$\Gamma_\downarrow = \hbar^{-2}\ \sin^2\eta\ \langle X_\omega^2\rangle_{\omega=\Delta E/\hbar}$$

$$\Gamma_\uparrow = \hbar^{-2}\ \sin^2\eta\ \langle X_\omega^2\rangle_{\omega=-\Delta E/\hbar}$$



relaxation: $\dfrac{1}{T_1} = \Gamma_{\rm rel} = \hbar^{-2}\ \sin^2\eta\ S_X(\omega = \Delta E/\hbar)$

$$S_X(\omega) \equiv \langle X_\omega^2\rangle + \langle X_{-\omega}^2\rangle = 2\hbar J(\omega)\coth\frac{\hbar\omega}{2k_{\rm B}T}\ ; \quad \langle X_\omega^2\rangle \equiv \int dt\, e^{i\omega t}\,\langle X(t)X(0)\rangle$$

dephasing: $\dfrac{1}{T_2} = \Gamma_\varphi = \dfrac{1}{2T_1} + \hbar^{-2}\ \underline{\cos^2\eta\ S_X(\omega = 0)}$

Expt.: $1/f$ (low-frequency) noise $\Rightarrow$ <u>strong dephasing</u>

<u>suppress dephasing</u>:

- $\cos\eta = 0$ [purely transverse coupling]
- echo techniques Nakamura et al. '02, Vion et al. '02

$1/f$ noise: models

$$H = -\frac{1}{2}\Delta E\ \sigma_z + (\cos\eta\ \sigma_z - \sin\eta\ \sigma_x)\ X + H_{\rm bath} \qquad\qquad S_X(\omega) = \frac{E_{1/f}^2}{|\omega|}$$

- Eq. models: sub-Ohmic spin-boson model

$$J(\omega) = \left(\frac{\pi}{2}\alpha\hbar\omega_0^{1-s}\right)\omega^s\ , \quad s < 1$$

$$S_X(\omega) = 2\hbar J(\omega)\coth\frac{\hbar\omega}{2k_{\rm B}T_{bath}} \propto \frac{1}{|\omega|}\ \text{if}\ \begin{cases} s = 0, & \omega \ll T_{bath} \\ s = -1, & \omega \gg T_{bath} \end{cases} \text{[adjustable]}$$

fixes antisymmetrized correlations

- model by 2-state fluctuators: Paladino et al. '02; non-gaussian effects
- gaussian $X(t)$ with spectrum $S_X(\omega) = E_{1/f}^2/|\omega|$

Longitudinal ($\eta = 0$) 1/f noise, "classical" treatment

$$\langle\sigma_+(t)\rangle \propto \langle e^{2i\int_0^t dt'\,X(t')}\rangle = e^{-2\int_0^t dt'\int_0^t dt''\langle X(t')X(t'')\rangle} =$$

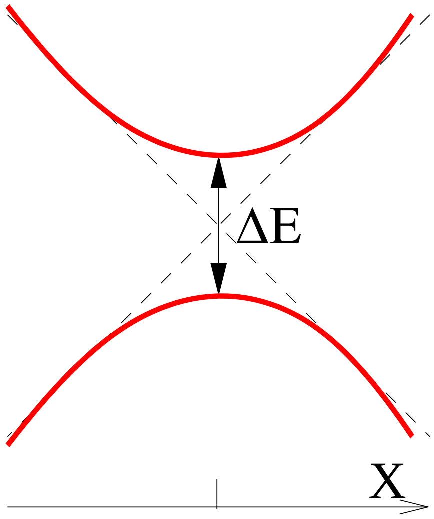
$$= e^{-\int\frac{d\omega}{2\pi}S_X(\omega)\frac{\sin^2(\omega t/2)}{(\omega/2)^2}}$$

cf. Cottet et al. '01

$$\langle\sigma_+(t)\rangle \propto \exp(-E_{1/f}^2 t^2|\ln t\omega_{\rm ir}|)\ , \qquad T_2^{-1} \approx E_{1/f} \approx (\text{few ns})^{-1}$$

Non-gaussian effects for <u>transverse</u> or <u>nonlinear</u> coupling

$1/f$ noise, transverse coupling



$\eta = \pi/2$ protects from $1/f$ noise

$$H = -\frac{1}{2}\Delta E\, \sigma_z - \sigma_x X + H_{\mathrm{bath}}$$

Golden rule for $\quad E_{1/f} \ll \Delta E$

$$T_1^{-1} = S_X(\Delta E) = \frac{E_{1/f}^2}{\Delta E} \qquad T_2^{-1} = \frac{T_1^{-1}}{2}$$

Next order (beyond Golden rule) is relevant

At low frequencies (adiabatic approx.) $H = -\frac{1}{2}\Delta E(X)\, \sigma_z + H_{\mathrm{bath}}$

$$\Delta E(X) = \sqrt{\Delta E^2 + 4X^2} \approx \Delta E + \frac{2X^2}{\Delta E}$$

$$S_{X^2}(\omega) = 2\int \frac{d\omega'}{2\pi} \left\{ \langle X^2_{\omega-\omega'} \rangle \langle X^2_{\omega'} \rangle + (\omega \to -\omega) \right\} \sim \frac{E_{1/f}^4}{|\omega|} \ln \frac{|\omega|}{\omega_{\mathrm{ir}}}$$

again $1/f$ noise!!

New scale

$$\tilde{E}_{1/f} \equiv \frac{E_{1/f}^2}{\Delta E}$$

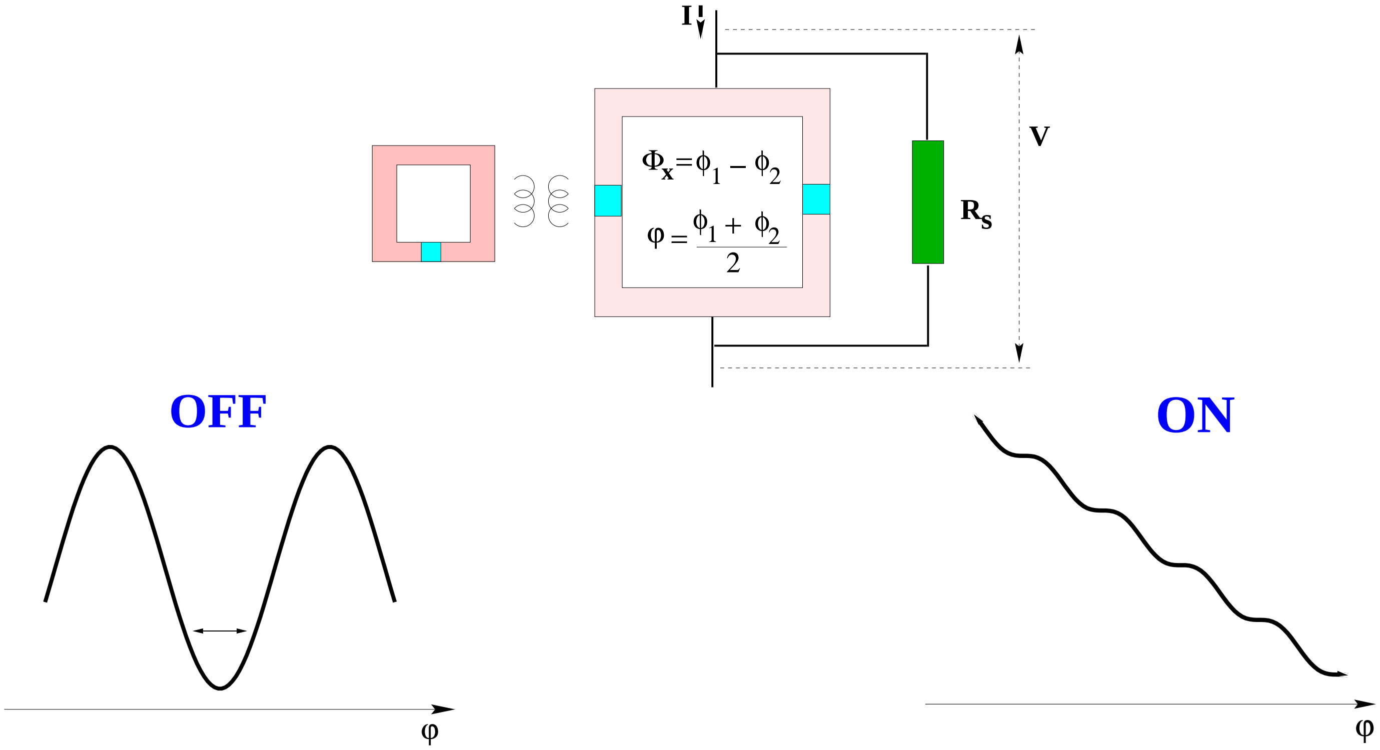
$$T_2^{*\;-1} = a\; \tilde{E}_{1/f} \qquad a \sim \ln \frac{\tilde{E}_{1/f}}{\omega_{\mathrm{ir}}}$$

$$T_1/T_2 = 1/2 + a$$

(≈ 3 in exp. of Saclay group)

Nonlinear coupling

Dephasing by <u>quantum detector</u> (symmetric dc-SQUID detector) in off-state



$$H = \frac{\epsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x - \frac{\Phi_0}{2\pi} I_c(\Phi_x)\cos\varphi - \frac{\Phi_0}{2\pi}\delta I_c \hat{\sigma}_z \cos\varphi$$

Overdamped oscillator: $\cos\varphi \approx 1 - \varphi^2/2$

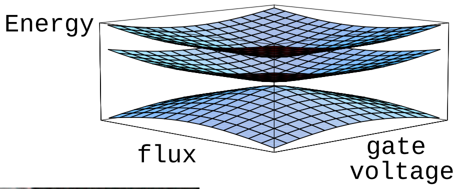
$$\langle \varphi_\omega^2 \rangle = \frac{1}{\omega}\frac{\mathrm{Re}Z(\omega)}{R_\mathrm{K}}\left(\coth\frac{\omega}{2T} + 1\right) \qquad Z(\omega) = \left(\frac{1}{R_s} + i\omega C + \frac{2\pi I_c}{i\omega\Phi_0}\right)^{-1}$$

$$\Gamma_\varphi^{\mathrm{off}} \propto \int d\omega \langle \varphi_\omega^2 \rangle \langle \varphi_{-\omega}^2 \rangle = \begin{cases} \left(\frac{\delta I_c}{I_c}\right)^2 \frac{T^3}{\alpha^2 E_\mathrm{J}^2} & T < \alpha E_\mathrm{J} \\ \left(\frac{\delta I_c}{I_c}\right)^2 \frac{T^2}{\alpha E_\mathrm{J}} & T > \alpha E_\mathrm{J} \end{cases} \qquad \alpha \equiv \frac{R_s}{R_\mathrm{K}}$$

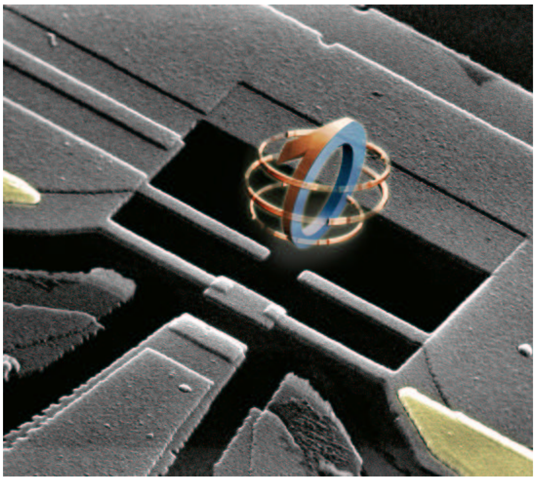
Recent experiments

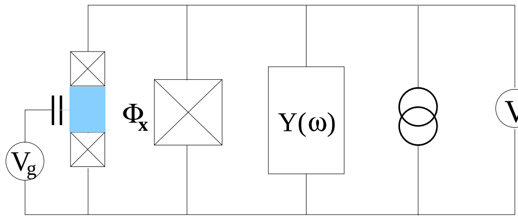
Charge-flux qubit with phase read-out (Saclay design):

$E_J \sim E_C$



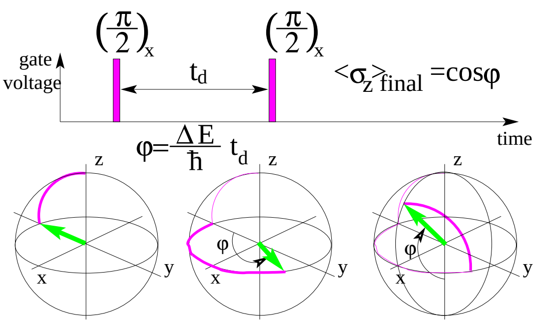
Vion et al. '02

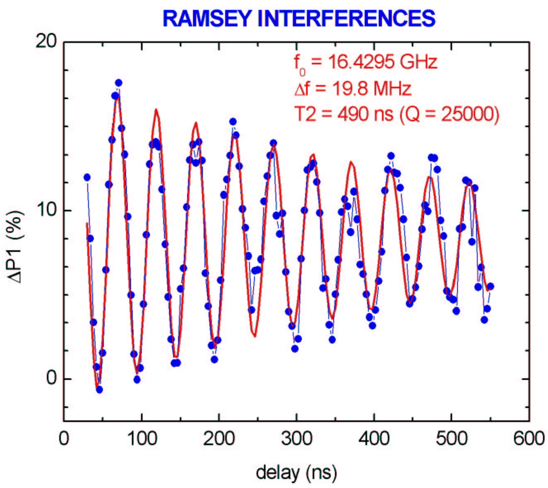




Symmetry point: $\Delta E_{\rm ch}(V_{\rm g0}) = 0$ and $\partial E_{\rm J}(\Phi_{\rm x0})/\partial \Phi_{\rm x} = 0$:

$$\mathcal{H} = -\frac{1}{2} E_{\rm J}(\Phi_{\rm x0})\,\sigma_x - \frac{1}{2}\frac{\partial \Delta E_{\rm ch}}{\partial V_{\rm g}}\,\delta V_{\rm g}\,\sigma_z - \frac{1}{4}\frac{\partial^2 E_{\rm J}}{\partial \Phi_{\rm x}^2}\,\delta\Phi_{\rm x}^2\,\sigma_x$$





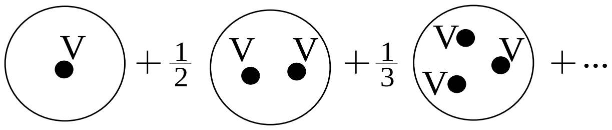
Nonlinear coupling

$\mathcal{H} = -\frac{1}{2}\Delta E\,\hat{\sigma}_z + \hat{\sigma}_z \tilde{V}(t) + H_{\rm bath}$ later $\tilde{V} = \frac{1}{2E_0}X^2$, $X(t)$ — gaussian

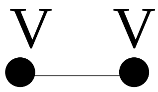
$$\langle \sigma_+(t) \rangle = \left\langle \tilde{T} \exp\left(\frac{i}{\hbar}\int_0^t \tilde{V}\,dt\right)\; T\exp\left(\frac{i}{\hbar}\int_0^t \tilde{V}\,dt\right)\right\rangle$$

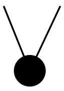
Linked-cluster expansion:

$\ln\langle\sigma_+(t)\rangle = \sum_{n=1}^{\infty}\frac{1}{n}F_n =$

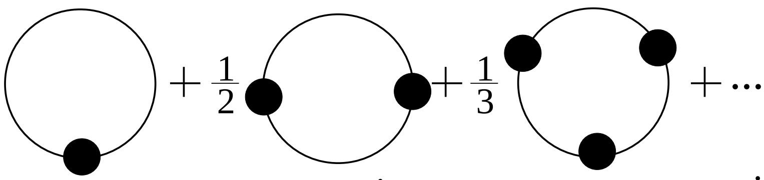


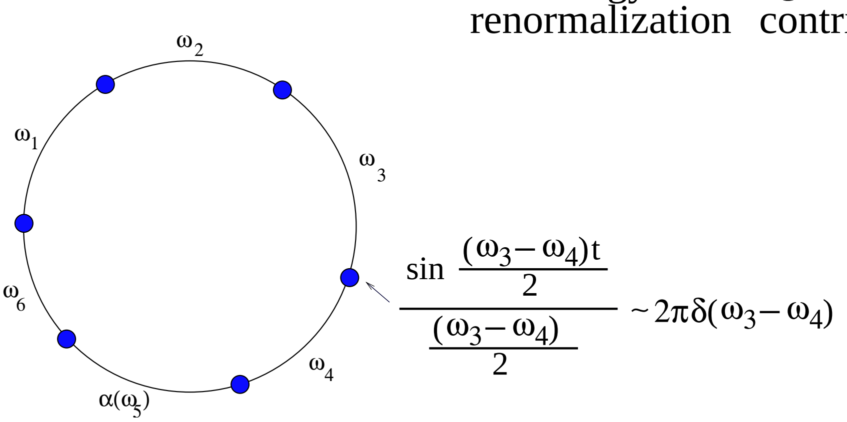
If $\tilde{V}$ — gaussian, only linked cluster:



If $\tilde{V} = \frac{1}{2E_0}X^2 =$  :

$\ln\langle\sigma_+(t)\rangle = \sum_{n=1}^{\infty}\frac{1}{n}F_n =$



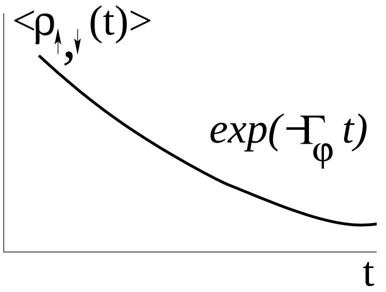


——— $= -i\langle T_{\rm K} X(t) X(0)\rangle$

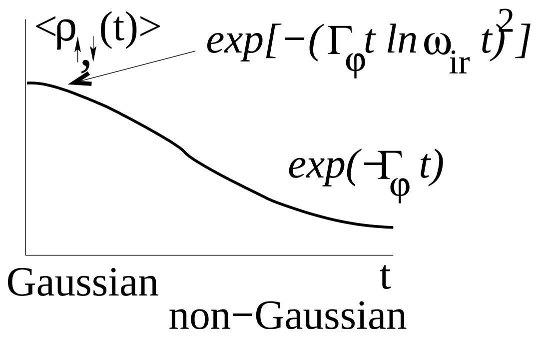
Quadratic coupling: various spectra ($1/f$, Ohmic)

- Results:
- decay laws of coherence:

Ohmic noise:



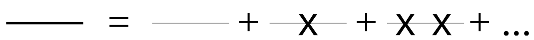
$1/f$ noise:



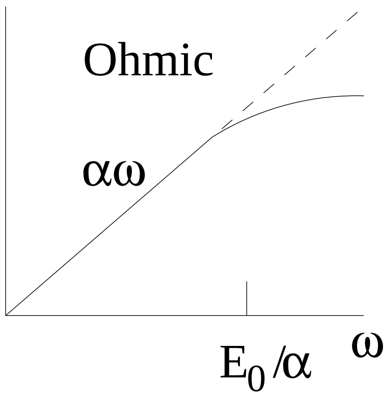
- dephasing times:

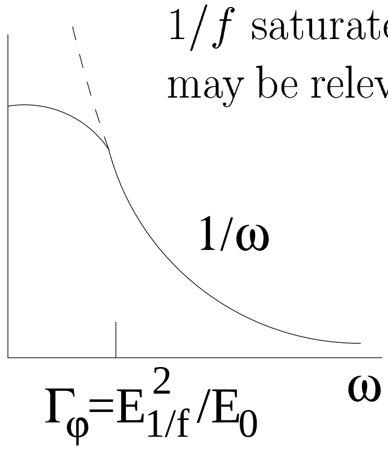
$$\Gamma_\varphi = \frac{\alpha^2 T^3}{E_0^2} \qquad\qquad \Gamma_\varphi = \frac{E_{1/f}^2}{E_0} \ln \frac{E_{1/f}^2}{E_0 \omega_{\rm ir}}$$

- Renormalization of noise:



higher orders (non-gaussian) relevant ⇔ strong renormalization





$1/f$ saturates at low freq.
may be relevant for spin-echo

Gaussian at: not too short times short times

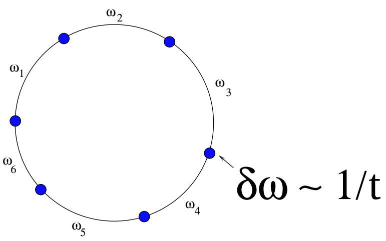
- Noise with short-time correlations ⇒ gaussian (=only lowest order needed) and $\propto \exp(-\Gamma_\varphi t)$

(Ohmic: $\tau_c \sim \hbar/k_{\rm B}T$)

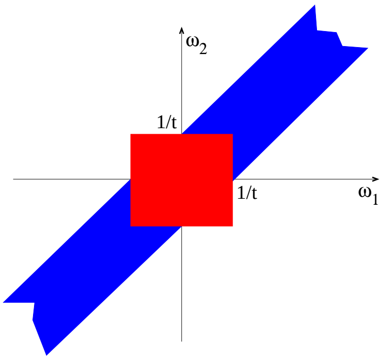
At $t \gg \tau_c$: $\langle\sigma_+(t)\rangle \propto \exp(-t\cdot S(\omega=0))$

$\Gamma_\varphi = \alpha^2 T^3/E_0^2$ eff. suppressed by cooling down

Calculation: 1/f noise, quadratic longitudinal coupling



$$\Gamma_\varphi \equiv \frac{E_{1/f}^2}{E_0}$$



$$F_n = \left[4it\Gamma_\varphi \ln(\omega_{\rm ir} t)\right]^n$$

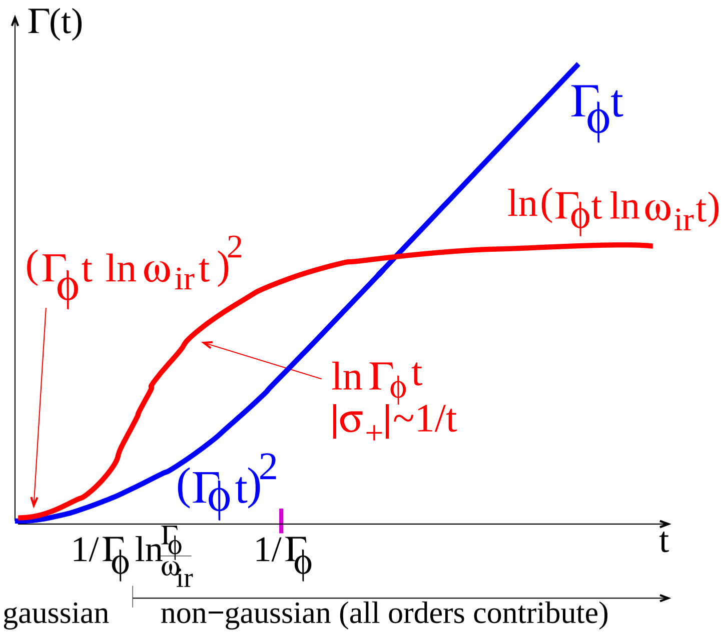
$$F_n = 2t\,(-2i\Gamma_\varphi)^n \int_{1/t}^{\infty} \frac{d\omega}{2\pi}\,\omega^{-n}$$

diverging series can be resummed formally

Justification: periodic b.c.
for det-regularization

$$\ln \sigma_+(t) = -\ln\left(1 - 4it\Gamma_\varphi \ln(\omega_{\rm ir} t)\right)$$

$$\ln \sigma_+(t) = -2t \int_{1/t}^{\infty} \frac{d\omega}{2\pi} \ln\left(1 + \frac{2it\Gamma_\varphi}{\omega}\right)$$



$$|\sigma_+(t)| \equiv e^{-\Gamma(t)}$$

dephasing time: $1/\Gamma_\varphi \ln \frac{\Gamma_\varphi}{\omega {\rm ir}}$

Discussion

- higher orders: typically give effects at longer times:
  $X^3/E_0^2 \;\rightarrow\; S_{X^3} \sim (E_{1/f}^3/E_0^2)\ln^2\frac{|\omega|}{\omega_{ir}}/|\omega|$
- reduction $X_\perp \rightarrow X_\parallel^2$
- why X^2 (cf. Leggett 80's)
  - comparison to X
  - non-gaussian effects
- treatment of slow modes $\omega_{ir} < \omega < 1/t$:
  - neglect (spin-echo)
  - long-time correlations governed by $\langle X_\omega^2\rangle_0$
  - long-time correlations governed by $\langle X_\omega^2\rangle_{\rm ren}$

Summary

- transverse $1/f$ noise: subleading order relevant
  reduction to quadratic longitudinal coupling
- quadratic coupling to Ohmic noise:
  – gaussian at relevant times ($\sim \tau_\varphi$)
  – $\tau_\varphi^{-1} \propto T^3$ - eff. suppression by cooling
- quadratic coupling to $1/f$ noise:
  – gaussian at short times; $\langle\sigma_+(t)\rangle \propto \exp(-t^2\ln^2 t)$
  – nongaussian at longer times $\langle\sigma_+(t)\rangle \propto \exp(-t)$
  – intermediate asymptotics
- gaussian properties and noise renormalization
- model of 2-state fluctuators: gaussian treatment of X^2 justified
  and explains correction to the Golden rule [Paladino et al. '02]