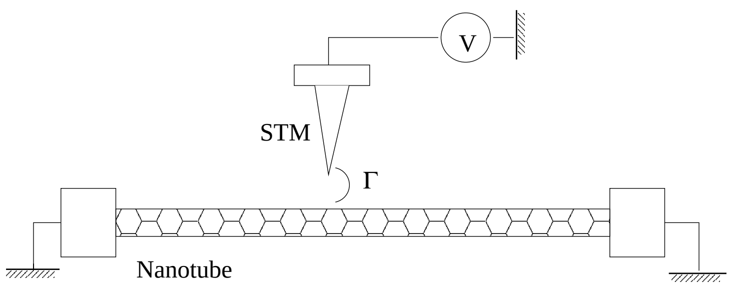
ENTANGLEMENT IN CORRELATED ELECTRONIC SYSTEMS

A. Crépieux, R. Guyon, P. Devillard and T. Martin
Phys. Rev. B 67, 205408 (2003)

N. Chtchelktchev, G. Lesovik, A. Lebedev

- entanglement in systems of electrons
- electron injection in a nanotube
- charge and spin current and time fluctuations (noise)
- effective charges and entanglement.



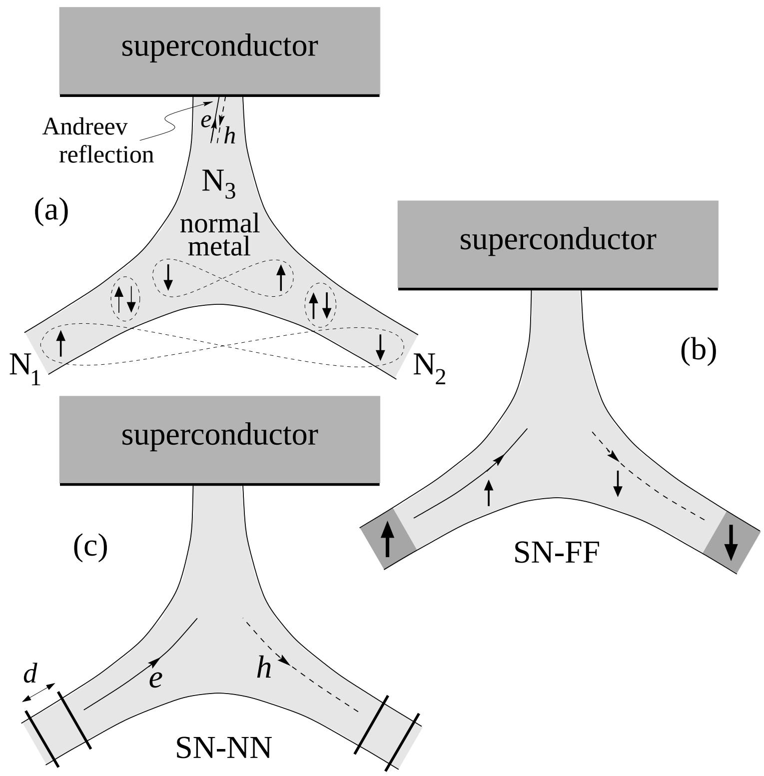
Entanglement from a superconductor source

G. Lesovik T. Martin and G. Blatter,

Eur. Phys. J. B 24, 287 (2001)

M.S. Choi, C. Bruder, D. Loss; P. Recher, E. Sukhorukov, D. Loss; P. Recher, D. Loss;...(quantum dots + Coulomb blockade)

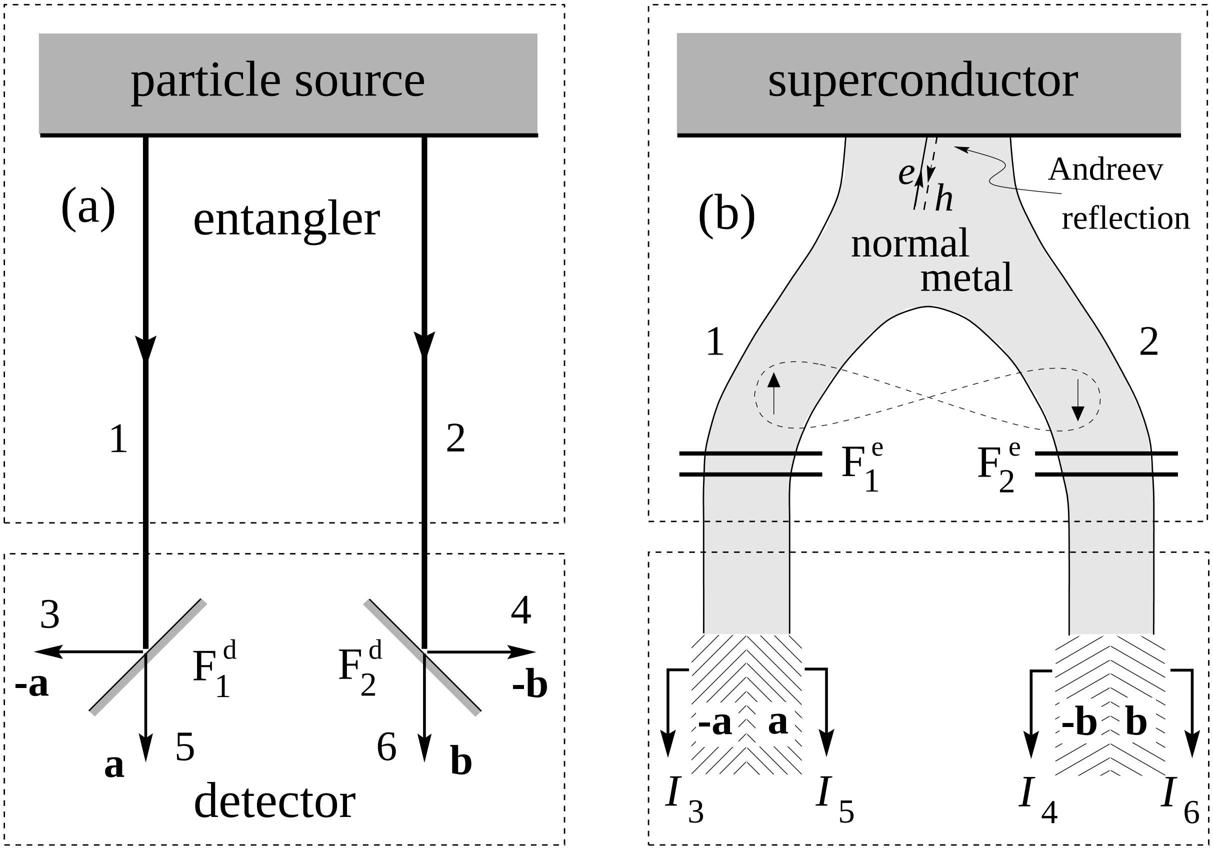
Phys. Rev. B's (2000-2002)



Crossed-Andreev Transport with spin or energy filters

- Positive noise correlations
- Analogous to that of a two terminal device $S \sim R(1-R)$ (R Andreev probability).
- Perfect correlation between 1 and 2.
- Energy (spin) filters select $\pm E_0$ ($\uparrow$ and $\downarrow$) in 1 and 2.

Bell inequality test in solid state devices
N. Chtchelkatchev, G. Blatter, G. Lesovik, T. Martin,
Phys. Rev. B 66, 161320 (2002)



NON-LOCALITY IN QUANTUM TRANSPORT

- $N_{1/2}(\vec{a}/\vec{b},t,\tau)$ Particle number accumulated in $[t,t+\tau]$.
- Compute number correlators in terms of noise.
- Plug in inequality derived from a local theory.
- Maximal violation of Bell inequality (low flux).

Noise in the fractional quantum Hall effect

With Schottky's relation, identify a fractional charge e/3 :

$$S = e^* \langle I \rangle = \frac{e}{3} \langle I \rangle$$

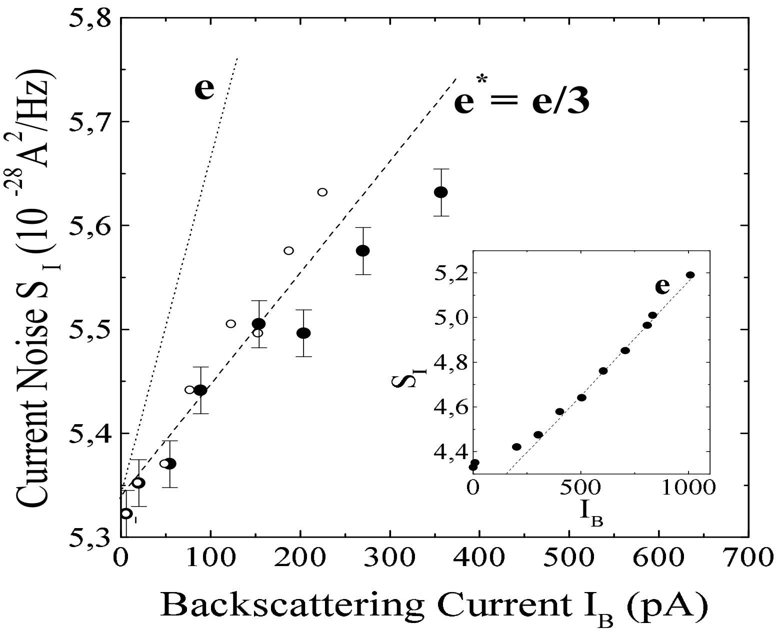


Figure 2

Ref. : Saminadayar et al., PRL 79, 2526 (1997)

Nanotube: no back scattering current, idendification of charges ?

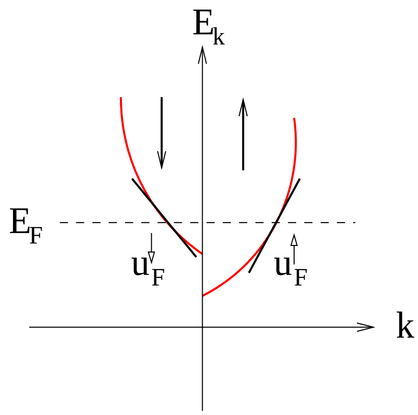
MODEL

Nanotube

1D conductor with Coulomb interactions
$\longrightarrow$ Luttinger liquid theory
Fermionic operator: $\Psi_{r\alpha\sigma}(x,t)$

Tip

metallic, possibly magnetic



Tip fermion operator: $c_\sigma(0,t)$

Tunnel hamiltonian

$$H_T(t) = \Gamma(t) \sum_{r\alpha\sigma} \Psi^+_{r\alpha\sigma}(0,t) c_\sigma(t) + h.c.$$

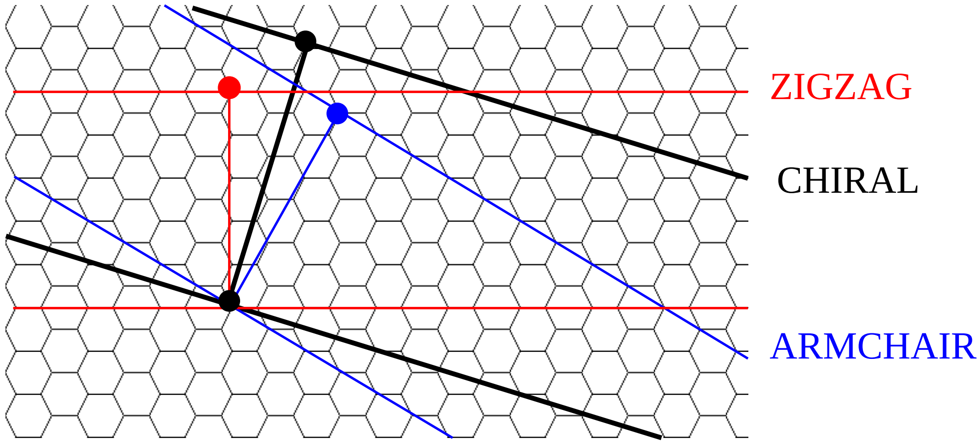
with :

$$\Gamma(t) = \Gamma \exp\left(i\frac{eV}{\hbar}t\right) = \Gamma \exp\left(i\omega_0 t\right)$$

(Peierls substitution)

NANOTUBE

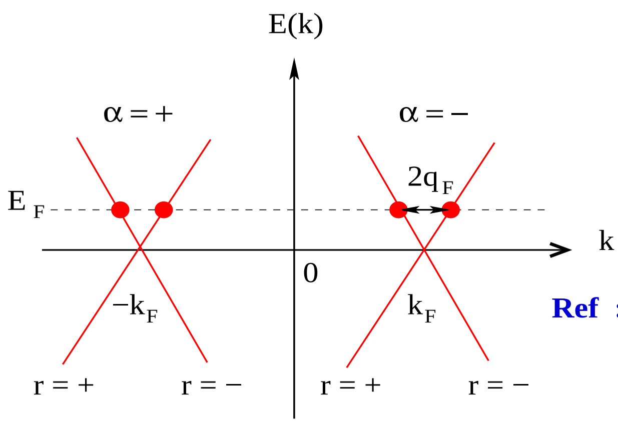
Plane filled with hexagons (graphite), rolled along given axis



typical diameter $\sim$ nm ; length $\sim \mu m$

$\longrightarrow$ 1D conductor but with 2 modes (α)

Ref. : Hamada et al., PRL 68, 1579 (1992)



Ref : Egger et Gogolin
Eur. Phys. J. B 3, 281 (1998)

$$\begin{cases} r &= \pm \quad \text{(branche)} \\ \alpha &= \pm \quad \text{(mode)} \\ \sigma &= \pm \quad \text{(spin)} \end{cases}$$

$$\begin{cases} j &= \text{c (charge) ou s (spin)} \\ \delta &= \pm \quad \text{(flavor)} \end{cases}$$

BOSONISATION

Luttinger liquid Hamiltonian :

$$H_N(t) = \sum_{j\delta} \frac{v_{j\delta}}{2} \int dx \left(K_{j\delta}^N \left(\partial_x \phi_{j\delta}(x,t)\right)^2 + \frac{1}{K_{j\delta}^N} \left(\partial_x \theta_{j\delta}(x,t)\right)^2 \right)$$

4 possibilities for $j\delta$:

- $j\delta =$ c+ with $K_{c+}^N = 1/\sqrt{1 + 4V_0(\mathbf{k}=0)/\pi v_F} < 1$
- $j\delta =$ c- with $K_{c-}^N = 1$
- $j\delta =$ s+ with $K_{s+}^N = 1$
- $j\delta =$ s- with $K_{s-}^N = 1$

where V_0 correspond to forward Coulomb scattering:

$$\begin{aligned} H_{int}(t) = \frac{1}{2} \sum_{r\alpha\sigma r'\alpha'\sigma'} &\int dx \int dx' V_0(x - x') \\ &\times \Psi^\dagger_{r\alpha\sigma}(x,t) \Psi_{r\alpha\sigma}(x,t) \Psi^\dagger_{r'\alpha'\sigma'}(x',t) \Psi_{r'\alpha'\sigma'}(x',t) \end{aligned}$$

Bosonized fermion operator :

$$\Psi_{r\alpha\sigma}(x,t) = \frac{M_{r\alpha\sigma}}{\sqrt{2\pi a}} \, e^{irq_F x + i\alpha k_F x + i\varphi_{r\alpha\sigma}(x,t)}$$

Bosonic field operator :

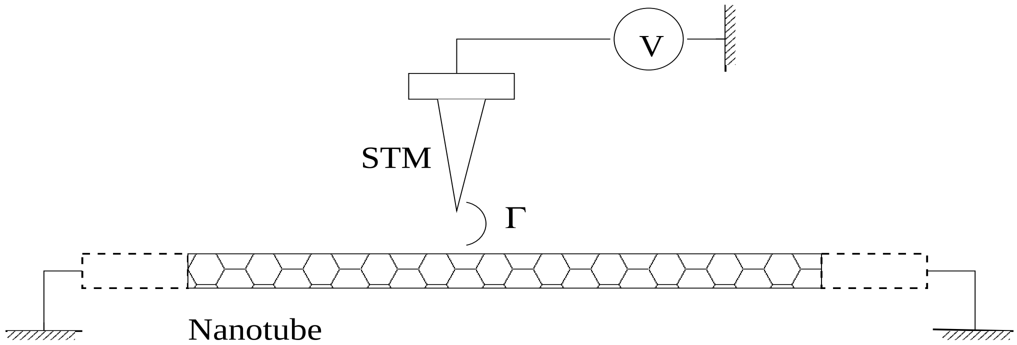
$$\begin{aligned} \varphi_{r\alpha\sigma} = \sqrt{\frac{\pi}{2}} \sum_{j\delta} (&\phi_{c+} + r\theta_{c+} + \alpha\phi_{c-} + r\alpha\theta_{c-} \\ &+ \sigma\phi_{s+} + r\sigma\theta_{s+} + \sigma\alpha\phi_{s-} + r\sigma\alpha\theta_{s-}) \end{aligned}$$

Commutation relations :

$$[\theta_{j\delta}(x), \phi_{j\delta}(x')] = i\pi \, \mathrm{sgn}(x - x')$$

$$[\theta_{j\delta}(x), \partial_{x'}\phi_{j\delta}(x')] = i\pi \, \delta(x - x')$$

NANOTUBE SETUP



Green's function:

$$G^{\theta\theta}_{j\delta}(x,t;x',t') = \langle \theta_{j\delta}(x,t)\theta_{j\delta}(x',t')\rangle$$

Differential equation:

$$\left(\frac{\omega^2}{v^N_{j\delta}K^N_{j\delta}} - \partial_x \frac{v^N_{j\delta}}{K^N_{j\delta}}\partial_x\right) G^{\theta\theta}_{j\delta}(x,x';\omega) = 4\pi\delta(x-x')$$

Ref. : Maslov and Stone, PRB 52, 5539 (1995)

Solution :

$$G^{\theta\theta}_{j\delta}(x,t;x',t') = -\frac{K^N_{j\delta}}{8\pi}\sum_r \ln\left(1 + i\frac{v_F(t-t')}{a} + ir\frac{K^N_{j\delta}(x-x')}{a}\right)$$

$\longrightarrow$ same for $G^{\phi\phi}_{j\delta}$; $G^{\phi\theta}_{j\delta}$; $G^{\theta\phi}_{j\delta}$

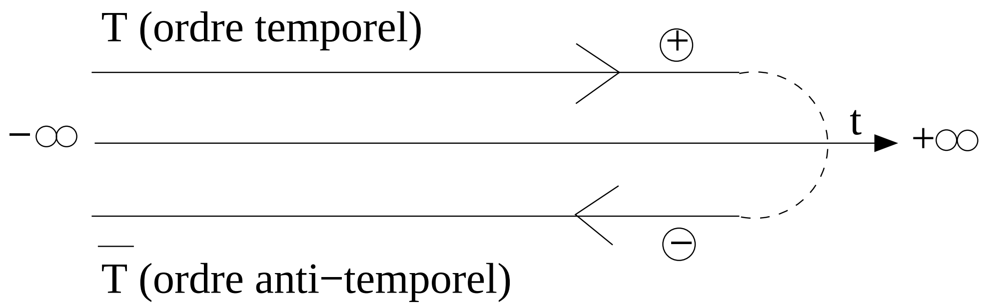
KELDYSH GREEN FUNCTION

Real time Green's function : $G(x,x';t) = \langle A(x,t)B(x',0)\rangle$
Keldysh Green's function matrix:

$$\begin{pmatrix} G^{++}(x,x';t) & G^{+-}(x,x';t) \\ G^{-+}(x,x';t) & G^{--}(x,x';t) \end{pmatrix} = \begin{pmatrix} \langle TA(x,t)B(x',0)\rangle & \langle B(x',0)A(x,t)\rangle \\ \langle A(x,t)B(x',0)\rangle & \langle \overline{T}A(x,t)B(x',0)\rangle \end{pmatrix}$$

with:

$$\langle TA(x,t)B(x',0)\rangle = \Theta(t)\langle A(x,t)B(x',0)\rangle + \Theta(-t)\langle B(x',0)A(x,t)\rangle$$
$$\langle \overline{T}A(x,t)B(x',0)\rangle = \Theta(-t)\langle A(x,t)B(x',0)\rangle + \Theta(t)\langle B(x',0)A(x,t)\rangle$$



$G^{\theta\theta}$ Keldysh function (same for $G^{\phi\phi}$)

$$G^{\theta\theta}_{j\delta(K)}(x,x';t) = \begin{pmatrix} \begin{matrix} t>0 : G^{\theta\theta}_{j\delta}(x,x';t) \\ t<0 : G^{\theta\theta}_{j\delta}(x',x;-t) \end{matrix} & G^{\theta\theta}_{j\delta}(x',x;-t) \\ G^{\theta\theta}_{j\delta}(x,x';t) & \begin{matrix} t>0 : G^{\theta\theta}_{j\delta}(x',x;-t) \\ t<0 : G^{\theta\theta}_{j\delta}(x,x';t) \end{matrix} \end{pmatrix}$$

Keldysh Green functions for $G^{\phi\theta}$ (same for $G^{\theta\phi}$)

$$G^{\phi\theta}_{j\delta(K)}(x,x';t) = \begin{pmatrix} \begin{matrix} t>0 : G^{\phi\theta}_{j\delta}(x,x';t) \\ t<0 : G^{\theta\phi}_{j\delta}(x',x;-t) \end{matrix} & G^{\theta\phi}_{j\delta}(x',x;-t) \\ G^{\phi\theta}_{j\delta}(x,x';t) & \begin{matrix} t>0 : G^{\theta\phi}_{j\delta}(x',x;-t) \\ t<0 : G^{\phi\theta}_{j\delta}(x,x';t) \end{matrix} \end{pmatrix}$$

MODEL FOR THE TIP

To avoid the issue of Klein factors, one chooses the tip to be a chiral LL without interaction ($K = 1$)

Fermionic operator :

$$c_\sigma(t) = \frac{N_\sigma}{\sqrt{2\pi a}}\, e^{i\tilde{\varphi}_\sigma(t)}$$

Chiral Green's function :

$$g_\sigma(t;t') = \langle \tilde{\varphi}_\sigma(t)\tilde{\varphi}_\sigma(t')\rangle = -\frac{1}{2\pi}\ln\left(1 + i\frac{u_F^\sigma(t-t')}{a}\right)$$

CHARGE TUNNEL CURRENT

Average tunnel current :

$$\langle I_T(t)\rangle = \frac{1}{2}\sum_\eta \langle T_C\{I_T(t^\eta)e^{-i\int_C dt_1 H_T(t_1)}\}\rangle$$

where C is the Keldysh contour

Tunnel Hamiltonian and tunneling current:

$$H_T(t) = \Gamma(t)\sum_{r\alpha\sigma}\Psi^+_{r\alpha\sigma}(0,t)c_\sigma(t) + h.c.$$

$$I_T(t) = -e\frac{dH_T(t)}{d(\omega_0 t)} = ei\left(\Gamma(t)\sum_{r\alpha\sigma}\Psi^+_{r\alpha\sigma}(0,t)c_\sigma(t) - h.c.\right)$$

O(Γ^2) Calculation of the current:

$\longrightarrow$ expand the exp to order 1 in Γ

$$\langle I_T(t)\rangle = \frac{e}{2}\sum_{r\alpha\sigma\varepsilon\eta r_1\alpha_1\sigma_1\varepsilon_1\eta_1}\eta\varepsilon\int_{-\infty}^{+\infty}dt_1\Gamma^{(\varepsilon)}(t)\Gamma^{(\varepsilon_1)}(t_1)$$

$$\times\langle T_C\{\Psi^{(\varepsilon)}_{r\alpha\sigma}(0,t^\eta)\Psi^{(\varepsilon_1)}_{r_1\alpha_1\sigma_1}(0,t_1^{\eta_1})\}\rangle\langle T_C\{c_\sigma(t^\eta)^{(-\varepsilon)}c_{\sigma_1}(t_1^{\eta_1})^{(-\varepsilon_1)}\}\rangle$$

Need to compute:

$$C_N = \langle T_C\{\Psi^{(\varepsilon)}_{r\alpha\sigma}(0,t^\eta)\Psi^{(\varepsilon_1)}_{r_1\alpha_1\sigma_1}(0,t_1^{\eta_1})\}\rangle$$

$$= \frac{1}{2\pi a}\langle T_K\{e^{-i\varepsilon\varphi_{r\alpha\sigma}(0,t^\eta)}e^{-i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_1^{\eta_1})}\}\rangle$$

Conditions : $C_N \neq 0$ if $\varepsilon = -\varepsilon_1$, $r = r_1$, $\alpha = \alpha_1$ et $\sigma = \sigma_1$

Introduce **non chiral fields** $\theta_{j\delta}$ et $\phi_{j\delta}$:

$$\varphi_{r\alpha\sigma}(x,t) = \sqrt{\frac{\pi}{2}}\sum_{j\delta} h_{\alpha\sigma j\delta}(\phi_{j\delta}(x,t) + r\theta_{j\delta}(x,t))$$

with coefficients $h_{\alpha\sigma c+} = 1$, $h_{\alpha\sigma c-} = \alpha$, $h_{\alpha\sigma s+} = \sigma$ et $h_{\alpha\sigma s-} = \alpha\sigma$

The correlator becomes:

$$C_N = \frac{1}{2\pi a}\langle T_K\{e^{-i\varepsilon\sqrt{\frac{\pi}{2}}\sum_{j\delta} h_{\alpha\sigma j\delta}(\phi_{j\delta}(0,t^{\eta})+r\theta_{j\delta}(0,t^{\eta}))} \times e^{i\varepsilon\sqrt{\frac{\pi}{2}}\sum_{j\delta} h_{\alpha\sigma j\delta}(\phi_{j\delta}(0,t_1^{\eta_1})+r\theta_{j\delta}(0,t_1^{\eta_1}))}\}\rangle$$

Use the relation :

$$\langle T_K\{e^{iA}e^{-iB}\}\rangle = e^{\langle T_K\{AB\}\rangle - \frac{1}{2}\langle T_K\{A^2\}\rangle - \frac{1}{2}\langle T_K\{B^2\}\rangle}$$

if A et B are linear combinations of bosonic operators

Then :

$$C_N = \frac{1}{2\pi a}e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta\eta_1)}(0,t;0,t_1) + rG^{\phi\theta}_{j\delta(\eta\eta_1)}(0,t;0,t_1) + rG^{\theta\phi}_{j\delta(\eta\eta_1)}(0,t;0,t_1) + G^{\theta\theta}_{j\delta(\eta\eta_1)}(0,t;0,t_1))}$$

The tunneling current becomes :

$$\langle I_T\rangle = -\frac{2ie\Gamma^2}{(2\pi a)^2}\sum_{r\sigma\eta}\eta\int_{-\infty}^{+\infty}\frac{\sin(\omega_0\tau)d\tau}{(1 - i\eta\frac{u_F^{\sigma}\tau}{a})(1 - i\eta\frac{v_F\tau}{a})^{\nu}}$$

or,

$$\int_{-\infty}^{+\infty} d\tau \frac{sin(|\beta|\tau)}{(b - i\eta\tau)^{\lambda}(c - i\eta\tau)^{\mu}} = i\pi\eta\frac{|\beta|^{\lambda+\mu-1}e^{-|\beta|b}}{\mathbf{\Gamma}(\lambda+\mu)}F(\mu, \lambda+\mu, |\beta|(b-c))$$

with $e^{-|\beta|b} \approx 1$ and

$$F(\mu, \lambda+\mu, |\beta|(b-c)) = 1 + \frac{\mu}{\lambda+\mu}\frac{|\beta|(b-c)}{1!} + ... \approx 1$$

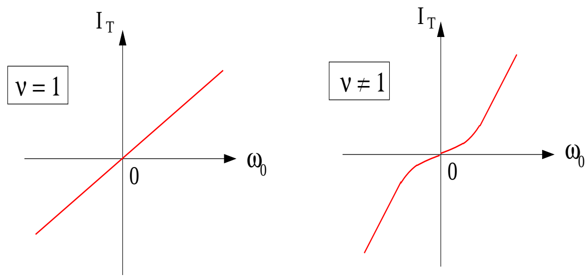
$\mathbf{O}(\Gamma^2)$ tunneling current :

$$\langle I_T \rangle = \frac{2e\Gamma^2}{\pi a} \left(\sum_\sigma \frac{1}{u_F^\sigma} \right) \left(\frac{a}{v_F} \right)^\nu \frac{\mathrm{sgn}(\omega_0)|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}$$

with :

$$\nu = \frac{1}{8} \sum_{j\delta} \left(\frac{1}{K_{j\delta}^N} + K_{j\delta}^N \right) \approx 1,2$$

because $K_{c+}^N \approx 0,28$ et $K_{c-}^N = K_{s+}^N = K_{s-}^N = 1$



$\Longrightarrow$ non-linear I-V characteristics

Exponent in agreement with :

- theory ("bulk tunneling")

  Ref. : Kane et al., PRL 79, 5086 (1997)

- experiments

  Ref. : Wildöer et al., Nature 391, 59 (1998)

CHARGE CURRENT WITHIN THE NANOTUBE

Average current:

$$\langle I(x,t)\rangle = \frac{1}{2}\sum_{\eta}\langle T_C\{I(x,t^{\eta})e^{-i\int_C dt_1 H_T(t_1)}\}\rangle$$

Current operator in the nanotube :

$$\begin{aligned} I(x,t) &= ev_F\sum_{r\alpha\sigma} r\Psi^{+}_{r\alpha\sigma}(x,t)\Psi_{r\alpha\sigma}(x,t) \\ &= 2ev_F\sqrt{\frac{2}{\pi}}\partial_x\phi_{c+}(x,t) \end{aligned}$$

trick:

$$\partial_x\phi_{c+} = \lim_{\gamma\to 0}\frac{1}{i\gamma}\partial_x e^{i\gamma\phi_{c+}}$$

Correlator :

$$C_N = \lim_{\gamma\to 0}\frac{1}{i\gamma}\partial_x\langle T_C\{e^{i\gamma\phi_{c+}(x,t^{\eta})}e^{-i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_1^{\eta_1})}e^{i\varepsilon_1\varphi_{r_1\alpha_1\sigma_1}(0,t_2^{\eta_2})}\}\rangle$$

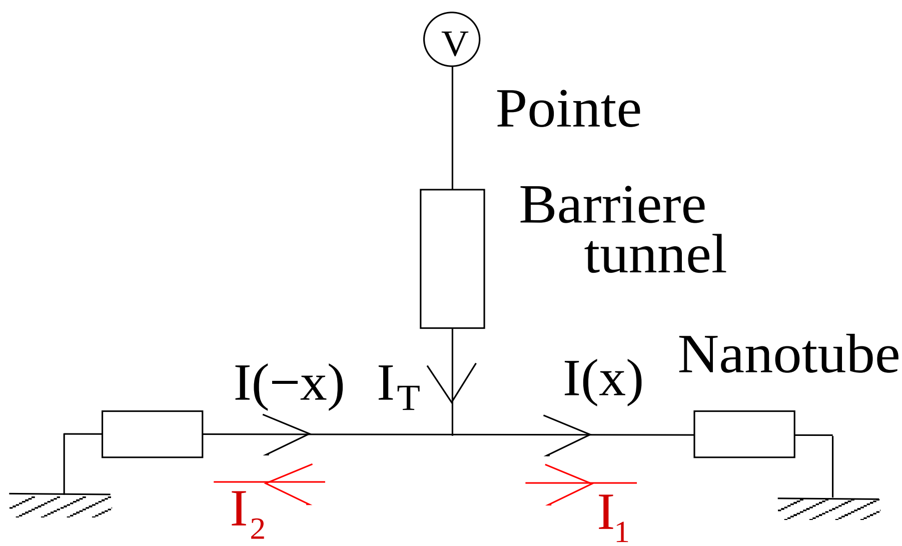
$$\begin{aligned} C_N = -i\varepsilon_1\sqrt{\frac{\pi}{2}}\partial_x\Big(&G^{\phi\phi}_{c+(\eta\eta_1)}(x,t;0,t_1) - G^{\phi\phi}_{c+(\eta\eta_2)}(x,t;0,t_2) \\ &+r_1G^{\phi\theta}_{c+(\eta\eta_1)}(x,t;0,t_1) - r_1G^{\phi\theta}_{c+(\eta\eta_2)}(x,t;0,t_2)\Big) \\ &\times e^{\frac{\pi}{2}\sum_{j\delta}(G^{\phi\phi}_{j\delta(\eta_1\eta_2)}(0,t_1;0,t_2)+G^{\theta\theta}_{j\delta(\eta_1\eta_2)}(0,t_1;0,t_2))} \\ &\times e^{\frac{\pi}{2}\sum_{j\delta}(r_1G^{\phi\theta}_{j\delta(\eta_1\eta_2)}(0,t_1;0,t_2)+r_1G^{\theta\phi}_{j\delta(\eta_1\eta_2)}(0,t_1;0,t_2)))} \end{aligned}$$

O(Γ^2) result:

$$\langle I(x)\rangle = \frac{e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{\mathrm{sgn}(\omega_0)|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}\mathrm{sgn}(x)$$

Current conservation:

$$|\langle I(x)\rangle| = \frac{\langle I_T\rangle}{2}$$



CURRENT FLUCTUATIONS

Noise :

$$S(x,t;x',t') = \frac{1}{2}\sum_{\eta}\langle T_C\{I(x,t^{\eta})I(x',t'^{-\eta})e^{-i\int_C dt_1 H_T(0,t_1)}\}\rangle$$

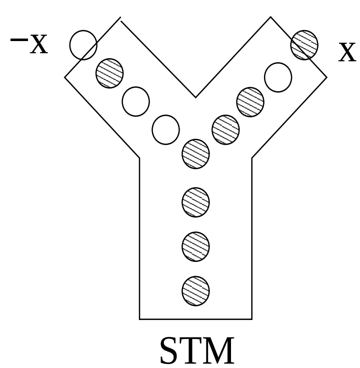
Auto-correlation x' = x :

$$S(x,x,\omega=0) = \frac{1+(K_{c+}^N)^2}{2}e|\langle I(x)\rangle| \qquad (\Gamma^2)$$

Cross-correlations x' = - x :

$$S(x,-x,\omega=0) = -\frac{1-(K_{c+}^N)^2}{2}e|\langle I(x)\rangle| \qquad (\ \Gamma^2)$$

Non interacting three terminal device ⇒ Necessary to go to O(Γ^4)



Ref. : Buttiker, PRL (1990), PRB (1992) Ref. : Martin and Landauer, PRB (1992)

EFFECTIVE CHARGES

Injection of a charge e in a Luttinger liquid:

$$Q_1 = \frac{1 - K_{c+}^N}{2}\, e \quad \text{in one direction}$$
$$Q_2 = \frac{1 + K_{c+}^N}{2}\, e \quad \text{other direction}$$

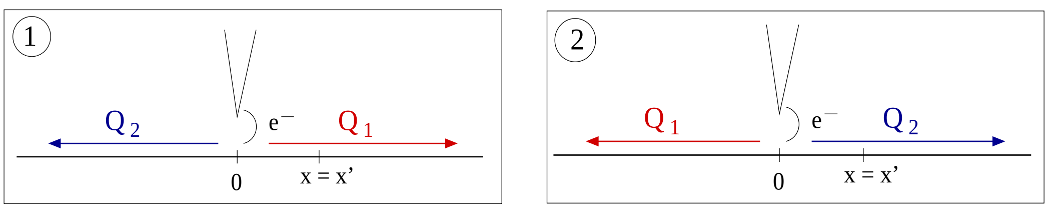
Ref. : Safi, Ann. Phys. Fr. 22, 463 (1997)

Ref. : Imura et al., PRB 66, 035313 (2002)

What does it imply for the noise ?

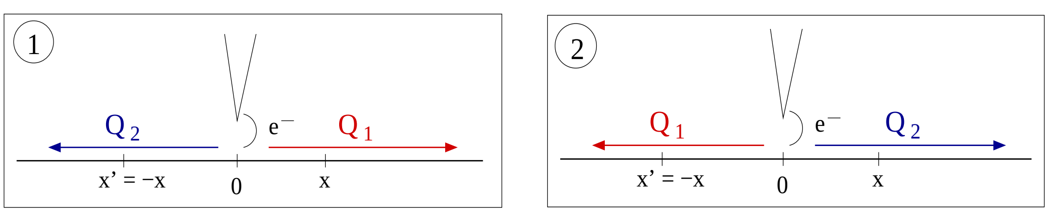
$S(x,x') \propto Q(x)Q(x')$ **?**

Auto-correlation x' = x :



$$S(x,x) \propto \frac{1}{2}\left(Q_1^2 + Q_2^2\right) = \frac{1 + (K_{c+}^N)^2}{4}\, e^2$$

Cross-correlation x' = - x :



$$S(x,-x) \propto \frac{1}{2}\left(Q_1 Q_2 + Q_2 Q_1\right) = \frac{1 - (K_{c+}^N)^2}{4}\, e^2$$

Positive cross-correlations⇒ possibility of entanglement

Add an electron to the nanotube :

$$\sum_{r\alpha} \Psi^\dagger_{r\alpha\sigma}|O_{LL}\rangle = \frac{M_{r\alpha\sigma}}{2\pi a}\sum_{r\alpha} \mathrm{e}^{-i\sum_{j\delta}\sqrt{\frac{\pi}{2K^N_{j\delta}}}h_{\alpha\sigma j\delta}\left(\frac{1+rK^N_{j\delta}}{2}\tilde{\varphi}^+_{j\delta}+\frac{1-rK^N_{j\delta}}{2}\tilde{\varphi}^-_{j\delta}\right)}|O_{LL}\rangle$$

with : $h_{\alpha\sigma c+} = 1$, $h_{\alpha\sigma c-} = \alpha$, $h_{\alpha\sigma s+} = \sigma$, $h_{\alpha\sigma s-} = \sigma\alpha$,

use the underlying chiral bosonic fields :

$$\tilde{\varphi}^r_{j\delta}(x) = \frac{1}{4\sqrt{K^N_{j\delta}}}\sum_{r'\alpha\sigma} h_{\alpha\sigma j\delta}(r + r'K^N_{j\delta}) \times \sum_{(r'k)>0}\sqrt{\frac{1}{|k|L}}\left(d^\dagger_{\alpha\sigma}(k)e^{-ikx} + d_{\alpha\sigma}(k)e^{ikx}\right)e^{-a|k|/2}$$

So:

$$\sum_{r\alpha} \Psi^\dagger_{r\alpha\sigma}|O_{LL}\rangle = \frac{1}{\sqrt{2\pi a}}\sum_{\alpha}\prod_{j\delta}\Big[(\tilde{\psi}^\dagger_{j\delta+})^{Q_{j\delta+}}(\tilde{\psi}^\dagger_{j\delta-})^{Q_{j\delta-}} + (\tilde{\psi}^\dagger_{j\delta+})^{Q_{j\delta-}}(\tilde{\psi}^\dagger_{j\delta-})^{Q_{j\delta+}}\Big]|O_{LL}\rangle$$

charges : $Q_{j\delta\pm} = (1 \pm K^N_{j\delta})/2$,

and:

$$\tilde{\psi}^\dagger_{j\delta\pm}(x) = \exp\left[-i\sqrt{\frac{\pi}{2K^N_{j\delta}}}h_{\alpha\sigma j\delta}\tilde{\varphi}^\pm_{j\delta}(x)\right]$$

SPIN CURRENT AND FLUCTUATIONS

Spin current operator:

$$I_{spin}(x,t) = ev_F \sum_{r\alpha\sigma} r\sigma \Psi^{+}_{r\alpha\sigma}(x,t)\Psi_{r\alpha\sigma}(x,t)$$
$$= 2ev_F\sqrt{\frac{2}{\pi}}\partial_x\phi_{s+}(x,t)$$

Ref. : Balents et Egger, PRB 64, 035310 (2001)

Average spin current :

$$\langle I_{spin}(x)\rangle = \frac{e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{\sigma}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{\mathrm{sgn}(\omega_0)|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}\mathrm{sgn}(x)$$

with $\nu \approx 1, 2$

$\Longrightarrow$ non linear I-V

Auto-correlator x' = x :

$$S_{spin}(x,x) = \frac{1+(K^N_{s+})^2}{2} e|\langle I_{spin}(x)\rangle| = e|\langle I_{spin}(x)\rangle|$$

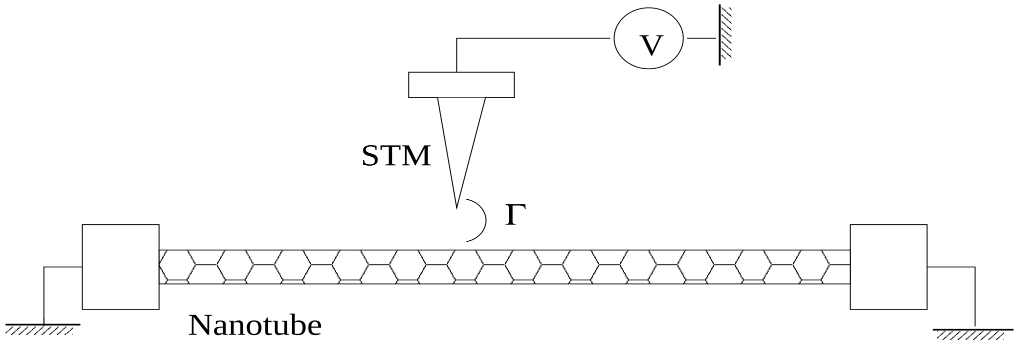
as $K^N_{s+} = 1$ (time reversal symmetry)

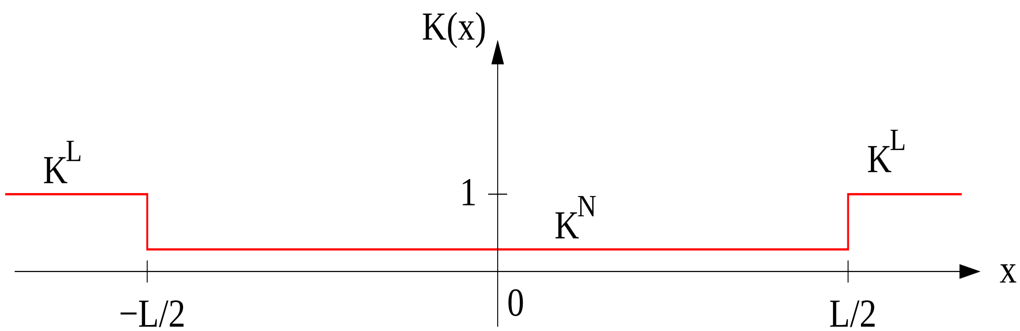
$\Longrightarrow$ Schottky relationship with charge e

Cross-correlations x' = - x :

$$S_{spin}(x,-x) = \frac{1-(K^N_{s+})^2}{2} e|\langle I_{spin}(x)\rangle| = 0 \quad (\text{ordre } \Gamma^2)$$

NANOTUBE WITH CONTACTS





Ref. : Safi and Schulz, PRB 52, 17040 (1995) Ref. : Maslov et Stone, PRB 52, 5539 (1995)

EXPECTED TUNNEL CURRENT AND NOISE

Power counting argument

$$\langle I_T \rangle \propto \int_{-\infty}^{+\infty} \frac{\sin(\omega_0 tau) d\tau}{\left(\frac{a}{v_F} - i\tau\right)^2 \Pi_{j\delta r}\, \Pi_{n=1}^{\infty} \left(1 + \frac{irK_{j\delta}^N \frac{nL}{v_F}}{\frac{a}{v_F} - i\tau}\right)^{\xi_{j\delta rr'n}}}$$

avec :

$$\xi_{j\delta rr'n} = \frac{1}{8}\left(b_{j\delta}^n K_{j\delta}^N + \frac{(-1)^n b_{j\delta}^n}{K_{j\delta}^N} + rr' b_{j\delta}^n \left(1 + (-1)^n\right)\right)$$

expansion of :

$$\prod_{j\delta r_1} \prod_{n=1}^{\infty} \left(1 + \frac{irK_{j\delta}^N \frac{nL}{v_F}}{\frac{a}{v_F} - i\tau}\right)^{-\xi_{j\delta rr'n}}$$

Result :

$$\langle I_T \rangle = \frac{2e\Gamma^2}{\pi a} \left(\sum_\sigma \frac{1}{u_F^\sigma}\right) \left(\frac{a}{v_F}\right)^{\nu'} \frac{\mathrm{sgn}(\omega_0)|\omega_0|^{\nu'}}{\mathbf{\Gamma}(\nu' + 1)} \left(1 + f(K_{j\delta}^N, K_{j\delta}^L) \left(\frac{\omega_0 L}{v_F}\right)^2 + ...\right)$$

with the exponent :

$$\nu' = \frac{1}{8} \sum_{j\delta} \left(K_{j\delta}^L + \frac{1}{K_{j\delta}^L}\right)$$

Schottky :

$$S_T(\omega = 0) = e\langle I_T \rangle$$

CURRENT AND NOISE IN THE NANOTUBE

Current :

$$\langle I(x)\rangle = \frac{e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^{\nu'} \frac{|\omega_0|^{\nu'}\mathrm{sgn}(\omega_0)}{\mathbf{\Gamma}(\nu'+1)}\,\mathrm{sgn}(x)$$
$$\times\left(1+f(K_{j\delta}^N, K_{j\delta}^L)\left(\frac{\omega_0 L}{v_F}\right)^2+\ldots\right)$$

Auto-correlation - Cross-correlator :

$$S(x,x,\omega=0) = \frac{1+(K_{c+}^L)^2}{2}\, e|\langle I_\rho(x)\rangle|$$

$$S(x,-x,\omega=0) = -\frac{1-(K_{c+}^L)^2}{2}\, e|\langle I_\rho(x)\rangle|$$

Contacts = Fermi 1D ($K_{j\delta}^L = 1 \;\; \forall\{j\delta\}$) $\Rightarrow \nu' = 1$:

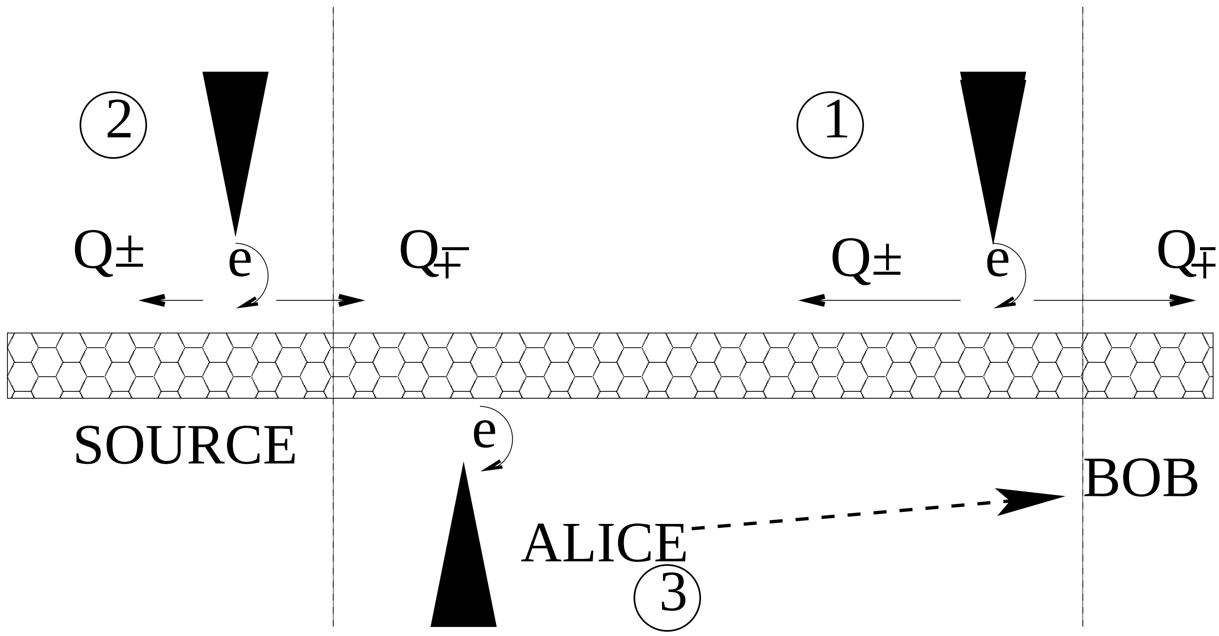
$$\langle I(x)\rangle = \frac{e\Gamma^2\omega_0\,\mathrm{sgn}(x)}{\pi v_F}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(1+f(K_{j\delta}^N, K_{j\delta}^L)\left(\frac{\omega_0 L}{v_F}\right)^2+\ldots\right)$$

$$S(x,x,\omega=0) = e|\langle I_\rho(x)\rangle|$$

$$S(x,-x,\omega=0) = 0 \qquad (\text{ordre} \;\; \Gamma^2)$$

$\Rightarrow$ with contacts : non linear I-V
but no effective charges and cross-correlator = 0

Teleportation of quasiparticle charge states with three STM tips



NON-LOCAL TRANSFER OF CHARGE STATE

- Inject quasiparticle pair shared by Alice and Bob.
- Inject pair with test particle.
- Monitor coincidences in STM currents
- Alice detects an electron → teleportation event.

CONCLUSION

Infinite nanotube :

- $I \propto V^{\nu}$ with expected exponent
- Schottky relation with effective charges:

$$Q_1 = \frac{1 - K_{c+}^N}{2}\, e \quad \text{and} \quad Q_2 = \frac{1 + K_{c+}^N}{2}\, e$$

- cross-correlator $\neq 0$ $\mathrm{O}(\Gamma^2)$
- cross-correlator > 0 : possibility of propagating entangled states.

Table of results

	Without contacts	**with contacts**
Current		
$\langle I(x)\rangle$	$(\omega_0)^\nu$ with $\nu \approx 1,2$	$\omega_0 \left(1 + f(K^L_{j\delta}, K^N_{j\delta}) \left(\frac{\omega_0 L}{v_F}\right)^2 + \cdots\right)$
$\langle I_{spin}(x)\rangle$	$(\omega_0)^\nu$ with $\nu \approx 1,2$	$\omega_0 \left(1 + f(K^L_{j\delta}, K^N_{j\delta}) \left(\frac{\omega_0 L}{v_F}\right)^2 + \cdots\right)$
cross correlations		
$S(x,x)$	$\left(\frac{1+(K^N_{c+})^2}{2}\right) e\vert\langle I(x)\rangle\vert$ $\Rightarrow$ **effective charges** $Q_1 = (1 + K^N_{c+})/2$ $Q_2 = (1 - K^N_{c+})/2$	$e\vert\langle I(x)\rangle\vert$
$S_{spin}(x,x)$	$e\vert\langle I_{spin}(x)\rangle\vert$	$e\vert\langle I_{spin}(x)\rangle\vert$
Cross-correlations		
$S(x,-x)$	$\left(\frac{1-(K^N_{c+})^2}{2}\right) e\vert\langle I(x)\rangle\vert$ > 0 $\Rightarrow$ **entanglement**	0 (O(Γ^2)
$S_{spin}(x,-x)$	0 (O(Γ^2)	0 (O Γ^2)