Vortex Glass Dynamics in Josephson Junction Arrays

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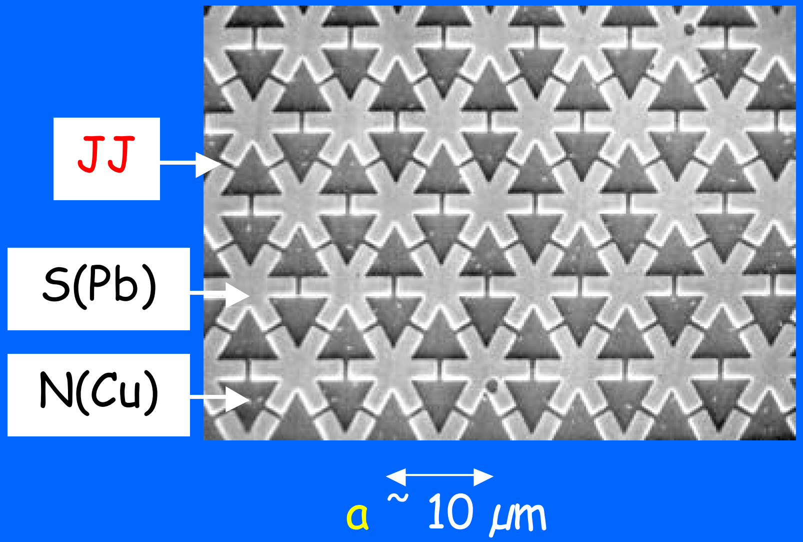
Triangular array of SNS JJ



Basic Issue

Flux noise [$S_\phi(\omega)$] and impedance [$Z(\omega)$] measurements performed on nominally unfrustrated ($f=0$) regular arrays reveal features inconsistent with predictions of the BKT theory

Hidden disorder + Residual frustration

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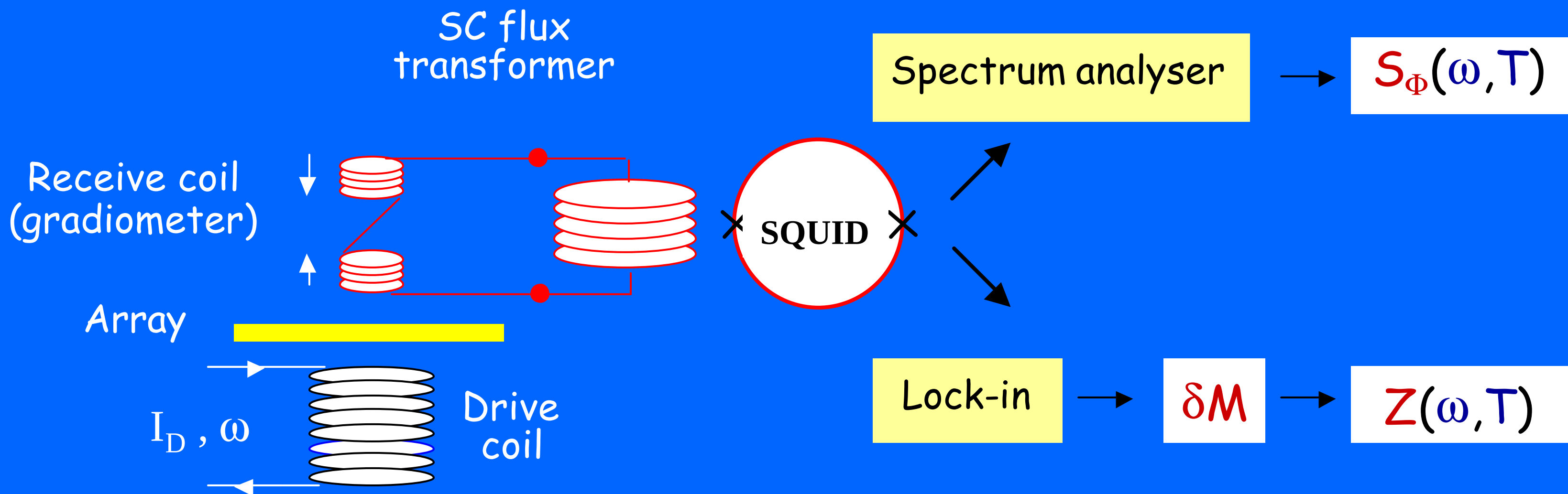
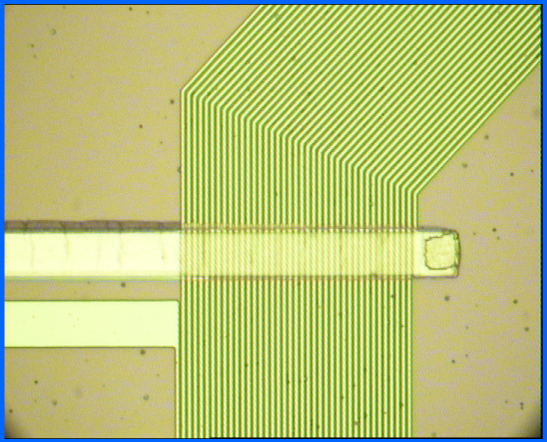
Glass-like dynamics

AC Sheet Impedance $Z(\omega,T)$ and Flux Noise Spectrum $S_{\Phi}(\omega,T)$





Spectral range: 0.1 Hz – 20kHz
Inductance sensitivity: ~ 1 pH

40 turns receive microcoil of 0.7 μm wide and 0.7 μm spaced Nb wires

Relation between $S_\Phi(\omega,T)$ and $Z(\omega,T)$

A. First attempt by Kim and Minnhagen, PRB (1999):

- ignore screening effects (mutual influence of field and current fluctuations)
- incorrect calculation of the magnetic field created by the currents

B. More «universal» approach by S.E. Korshunov, PRB (2002):

- no decomposition in vortex and « spin-wave » fluctuations
- $S_\Phi(\omega,T)$ directly related to current correlations $\Rightarrow$ screening effects

$\Downarrow$

Fluctuation-dissipation theorem: $S_\Phi(\omega,T) = 2(k_B T/\omega)\mathrm{Im}(\delta M)$

Regimes of Interest

$\delta M \sim M_s\{1 + [Z(\omega,T)/i\omega m_c]\}^{-1}$ $\qquad$ $S_\Phi(\omega,T) = 2(k_B T/\omega)\mathrm{Im}(\delta M)$

$Z = R_z + i\omega L_z$ $\qquad$ or $\qquad$ $G \equiv Z^{-1} \equiv R_G^{-1} + (i\omega L_G)^{-1}$

M_s and m_c from coil geometry

$\Downarrow$

A. Weak screening: $L_Z/m_c \gg 1$

$$S_\Phi(\omega,T) \propto k_B T\, \mathrm{Re}[Z^{-1}(\omega,T)]$$

B. Strong screening: $L_Z/m_c \ll 1$

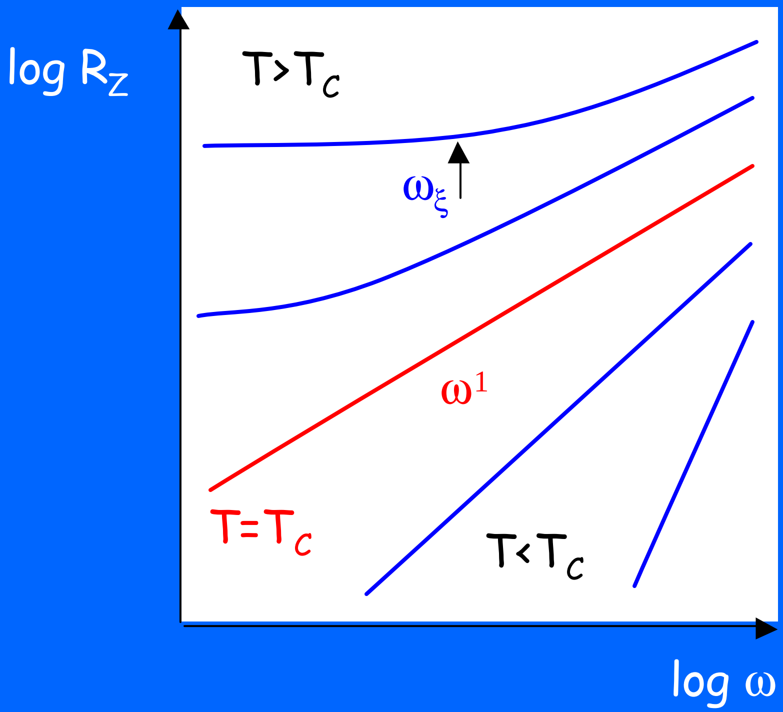
Low temperatures: $R_Z \ll \omega L_Z$

$$S_\Phi(\omega,T) \propto [k_B T/\omega^2] R_Z(\omega,T)$$

Theoretical Predictions for « Ideal » Josephson Junction Arrays at Strictly Zero Frustration (f=0)

AHNS extension of the BKT theory

Ambegaokar et al., PRL (1978)



Vortex-Antivortex (VA) pairs dominate for:

$T=T_c$

$T>T_c$ and $\omega_\xi < \omega < \omega_D$

$$\Rightarrow \quad R_Z(\omega) \propto \omega^{2(T_c/T)-1}$$

$$\omega_\xi \sim \omega_D \exp\{-b/[(T/T_c)-1)]^{1/2}\}$$

$$\omega_D \approx \phi_0^2/R_n k_B T$$

Free vortices (FV) dominate for:

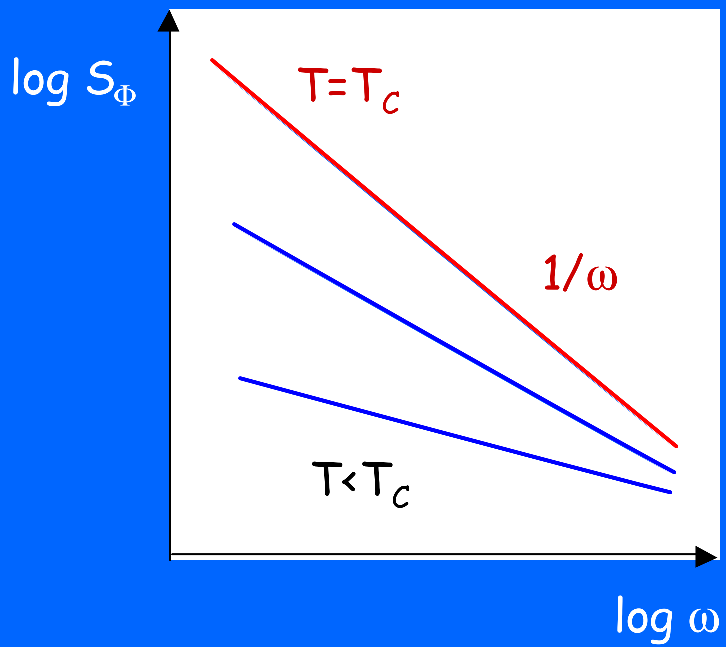
$T>T_c$ and $\omega < \omega_\xi$

$\Rightarrow$ R_Z independent of ω

Consequences for $S_\Phi(\omega)$

$T \leq T_c$

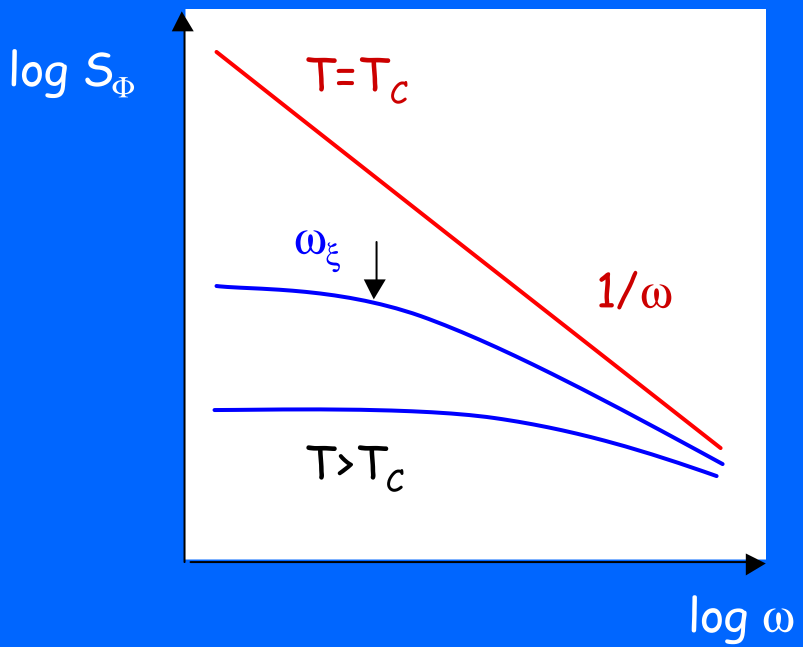
$S_\Phi(\omega) \propto R_Z(\omega)/\omega^2 \propto \omega^{2(Tc/T)-3}$



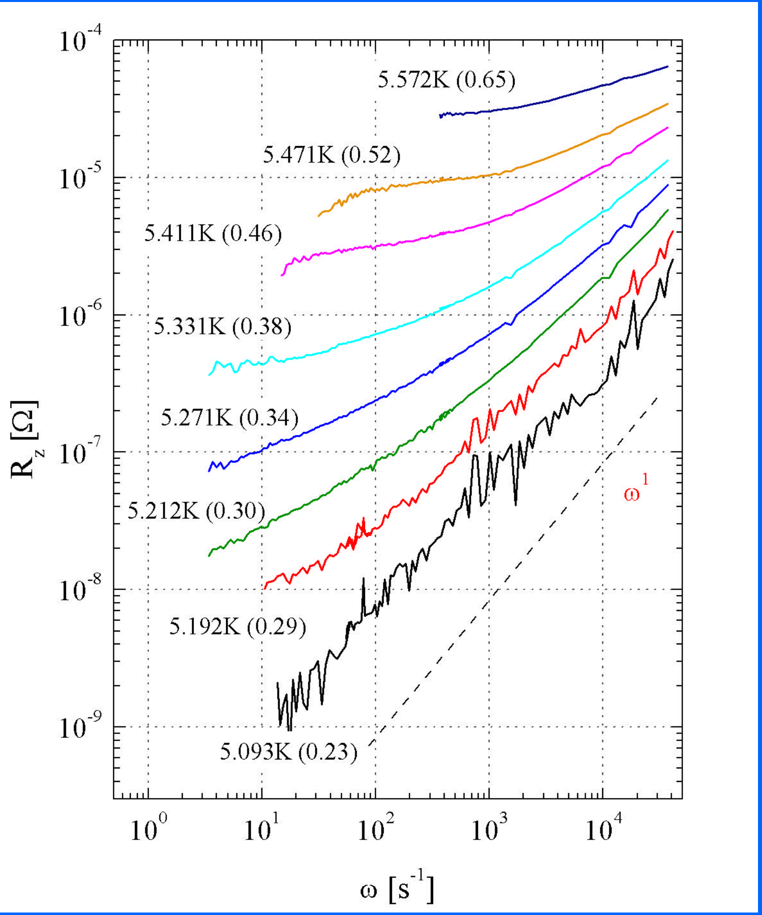
$T \geq T_c$

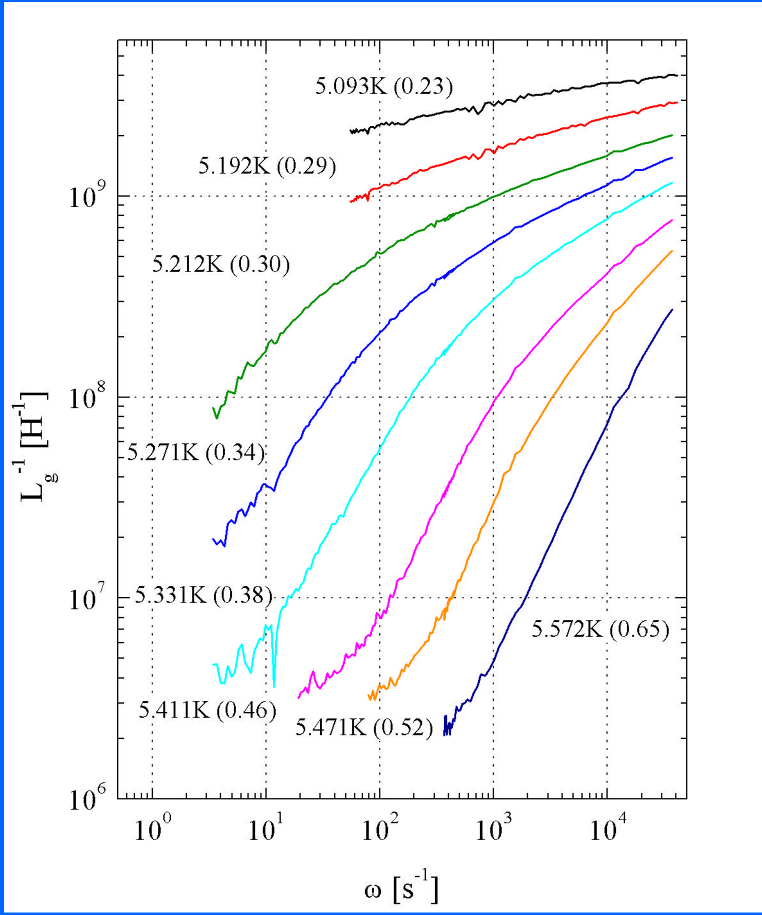
$S_\Phi(\omega) \propto \mathrm{Re}[Z(\omega)^{-1}]$

- $\omega < \omega_\xi$ (FV) $S_\Phi(\omega)$: white noise
- $\omega_\xi < \omega < \omega_D$ (VA) $S_\Phi(\omega) \propto \omega^{-[2(Tc/T)-1]}$



Frequency Dependence - $R_Z(\omega)$ and $L_G^{-1}(\omega)$ Isotherms



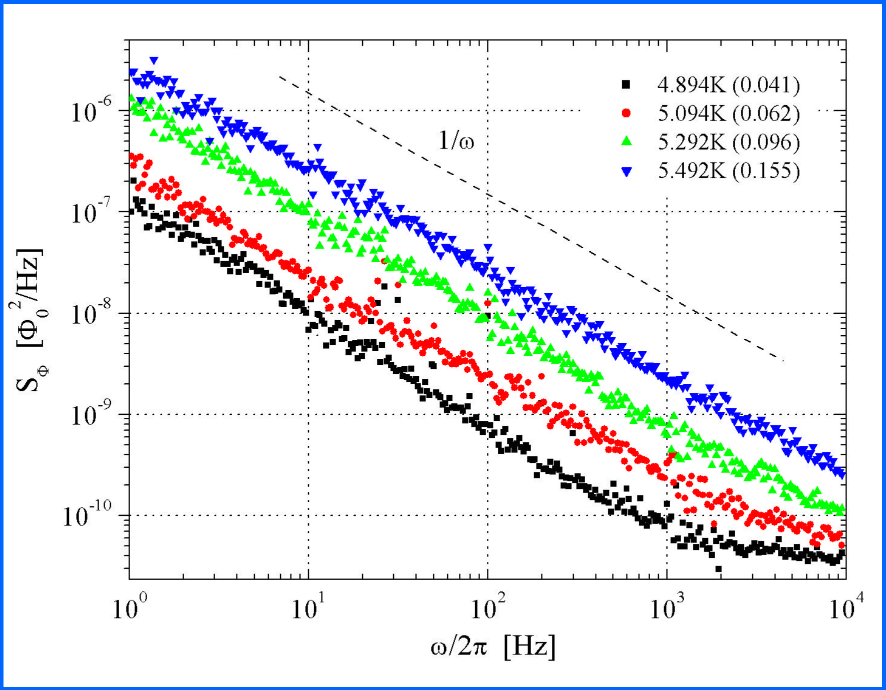


- Low T : $R_Z(\omega) \propto \omega$
- High T: R_Z independent of ω

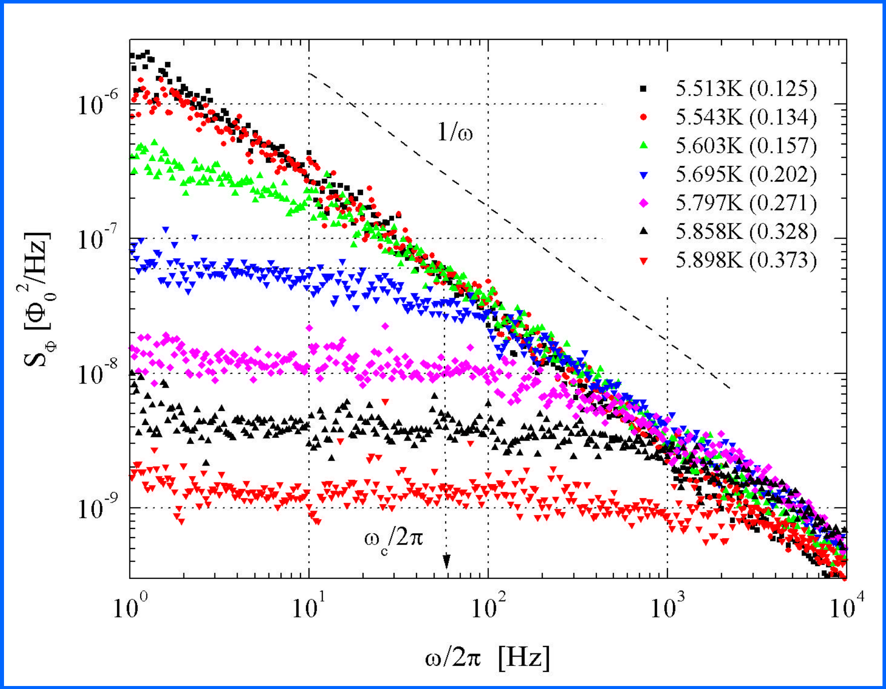
- Low T : $L_G^{-1}(\omega)$ weakly increasing with ω
- High T: strong suppression of $L_G^{-1}(\omega)$

Magnetic Flux Noise Power Spectra - $S_\Phi(\omega)$ Isotherms

Low T



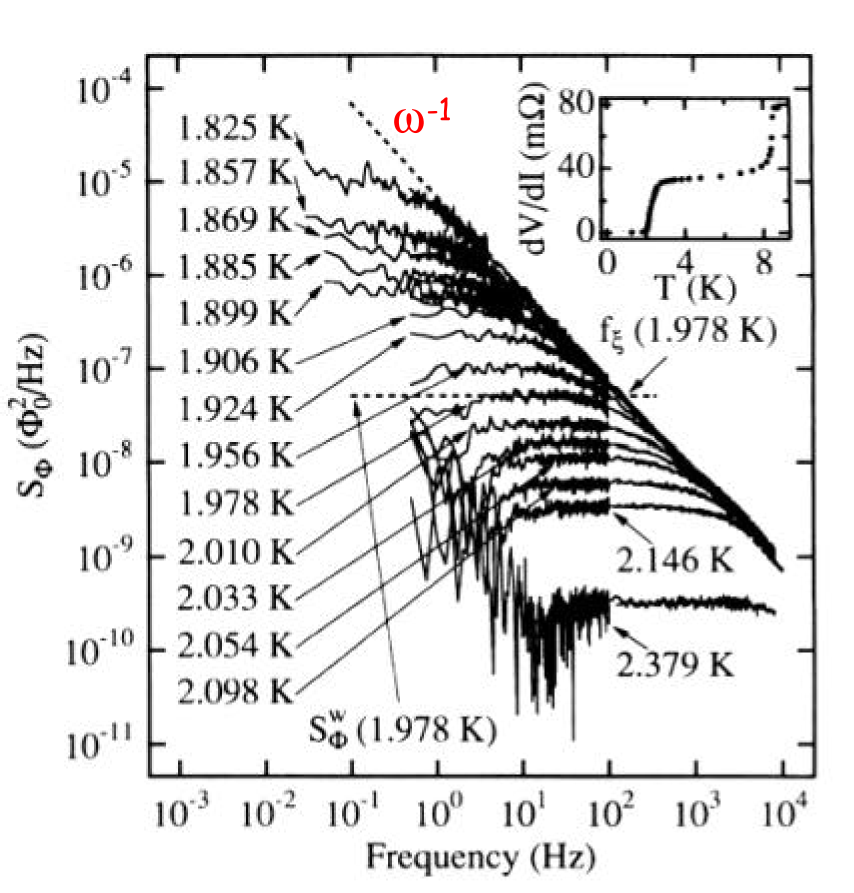
High T



- $1/\omega$ noise over 4 decades in ω
- Consistent with $S_\Phi(\omega) \propto R_Z(\omega)/\omega^2$

- Crossover from $1/\omega$ to white noise
- White noise consistent with R_Z independent of ω at high T

Additional Evidence for 1/ω Magnetic Flux Noise : Shaw et al., PRL (1996)



At high T (« above T_{BKT} ») : Crossover from 1/ω to white noise

Data consistent with dynamic scaling based on the BKT theory

1/ω noise unexplained

Glass-like Vortex Dynamics in « Real » Josephson Junction Arrays: Basic Ingredients

• « Hidden » Disorder

Coupling energy in proximity-effect coupled SNS arrays:

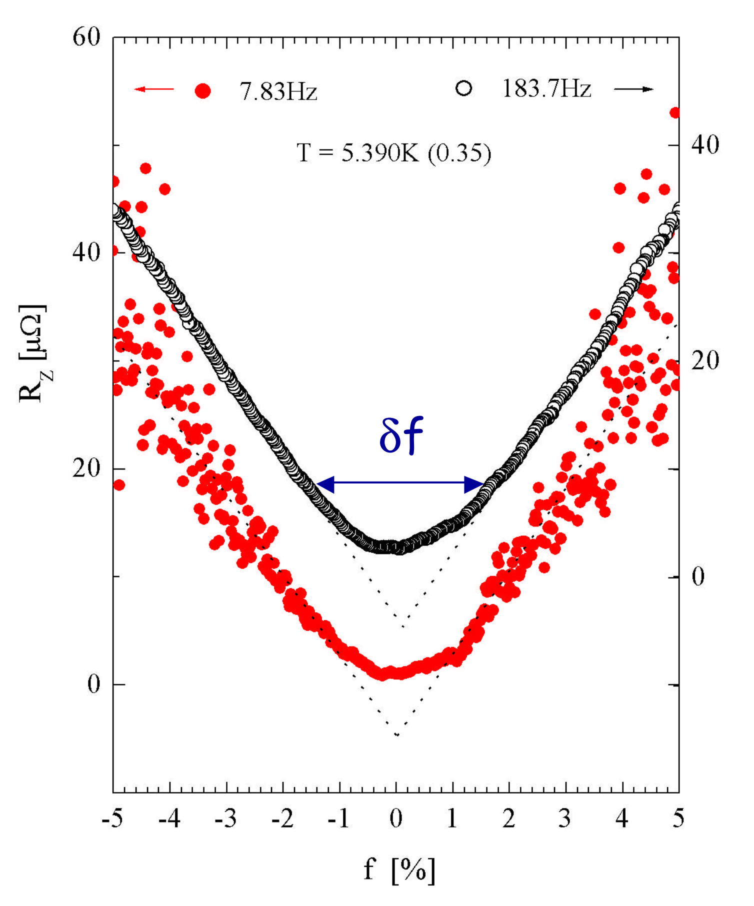
$$E_J \propto \exp[-d/\xi_N]$$

d: length of N bridge , $\xi_N(T)$: coherence length in N

⇒ Weak unavoidable random variations in the junction geometrical parameters introduced by the fabrication process result in strong fluctuations of E_J

Typically: $\Delta d/d$ ~ 3-5 % , $d/\xi_N(T_{CS})$ ~ 16-17 ⇒ $\Delta E_J/E_J$ ~ 50-90 %

• Residual frustration



Deviations from linearity ($R_Z \propto f$) at very small f

Frustration measured by $f = \Phi_P / \phi_0$

Φ_P = magnetic flux per plaquette

Incomplete suppression of ambient magnetic fields (1-10 mG) ⇒ vortices always present in the array

⇒ $\delta f \sim 10^{-3}$-10^{-2}

Thermally created vortices due to finite size effects (L, Λ) : irrelevant for T below T_c

Vortex Glass in two dimensions ?

- Unlike in 3D, in 2D a vortex glass is unstable against plastic flow of thermally created dislocation pairs

- However, «Dynamic freezing» from a liquid to a frozen liquid is possible at sufficiently short time scales

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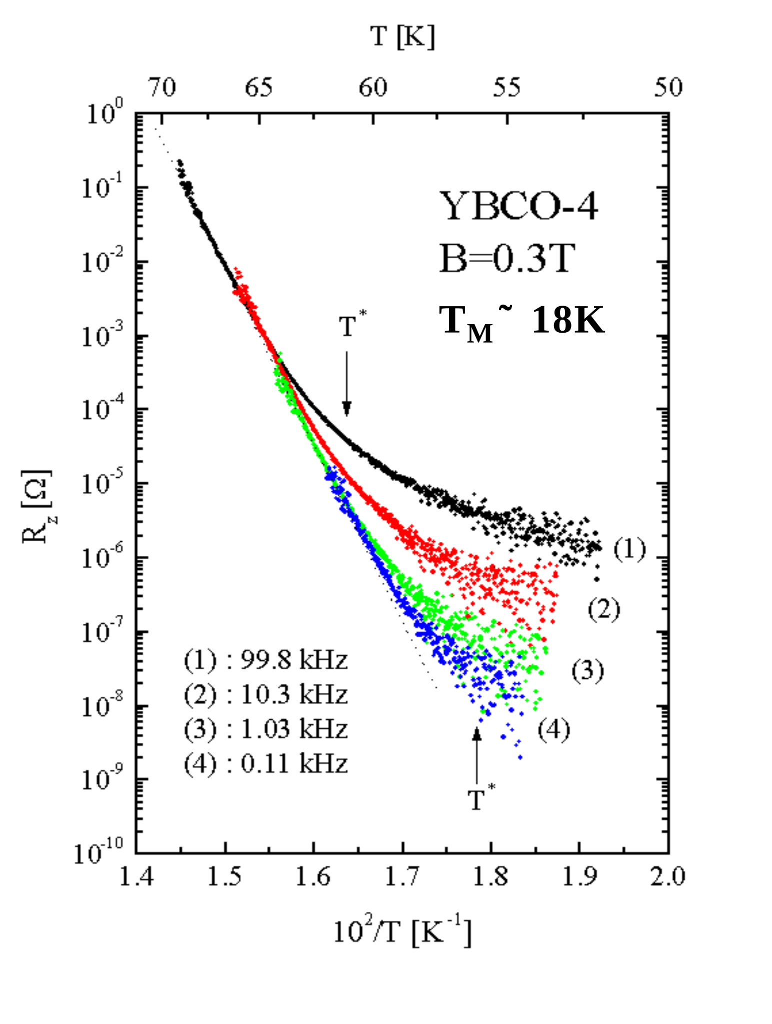
- Regime crossover at a ω-dependent temperature $T^*(\omega)$

- $T^*(\omega)$ well above melting of ideal 2D vortex crystal

M. Calame et al., PRL (2001)



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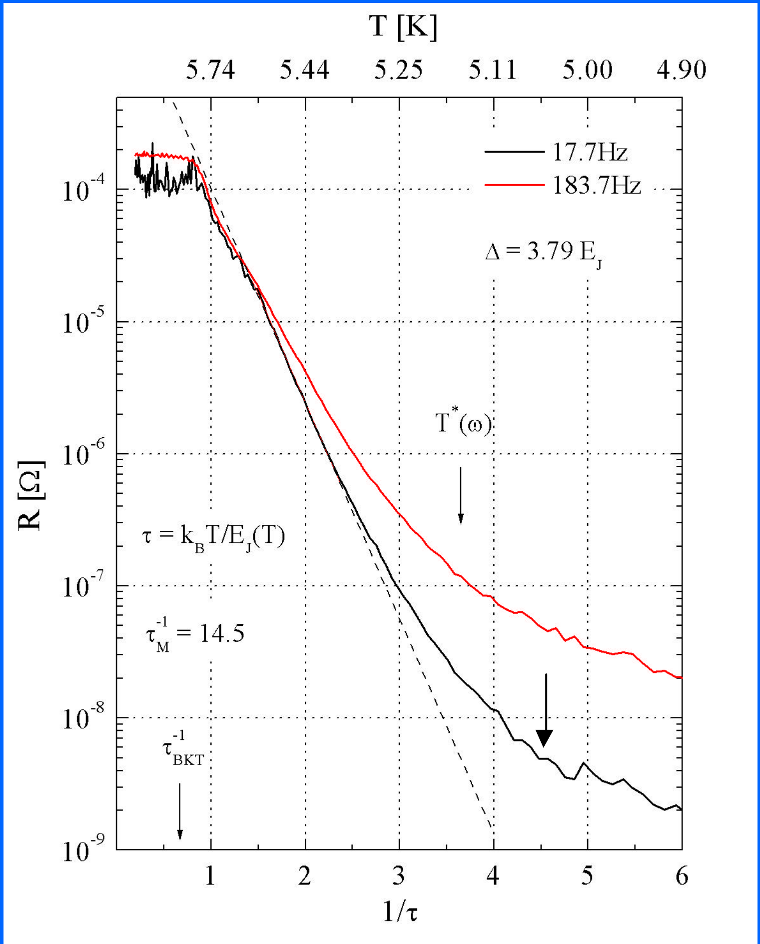
Triangular JJ array

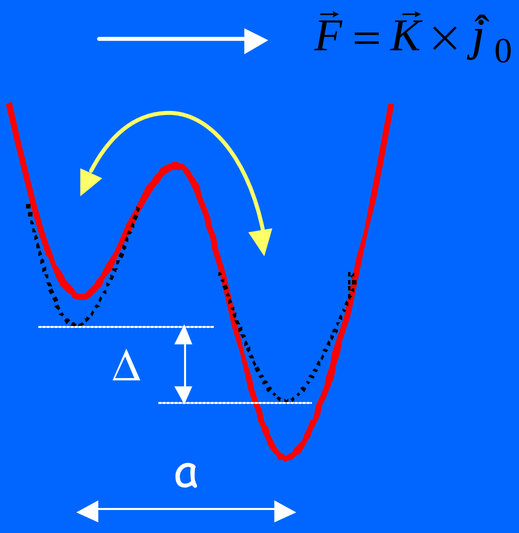


$\tau \equiv k_B T/E_J(T)$

Vortex Hopping in the Frozen Liquid Regime [$T < T^*(\omega)$]

Two-level system approach, Mott and Davies, 1971; Koshelev and Vinokur, 1991

Thermally activated vortex hopping between pairs of metastable states in neighboring plaquettes



Average velocity in a single two-level system

$$\langle v \rangle \sim (a^2\phi_0/k_BT)\cosh^{-2}(\Delta/k_BT)\, i\omega[1 + i\omega\tau(U)]^{-1}K$$

$$\tau(U) = \tau_0 \exp(U/k_BT)$$ U : energy barrier

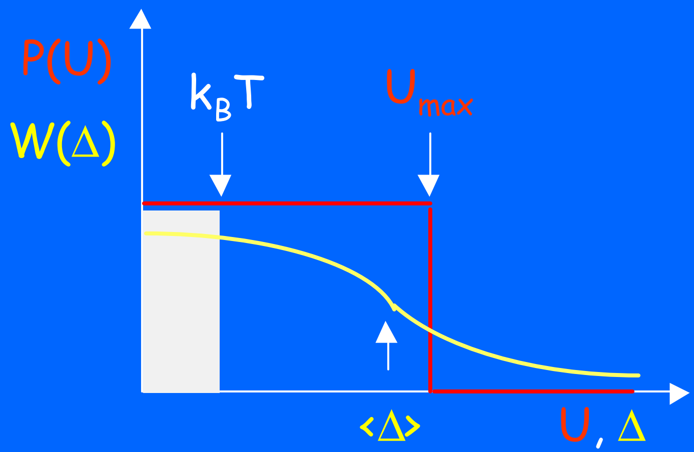
Vortex contribution δZ_v to the impedance Z

$$\delta Z_v \equiv E/K \sim (\phi_0/a^2K)\,\delta f \,\langle\langle v \rangle\rangle$$

$\langle\langle v \rangle\rangle$: vortex velocity averaged over all possible two-level systems, with distributions $W(\Delta)$ and $P(U)$

Comparison of $R_Z(\omega)$ and $S_\Phi(\omega)$ with Theoretical Predictions

Distributions

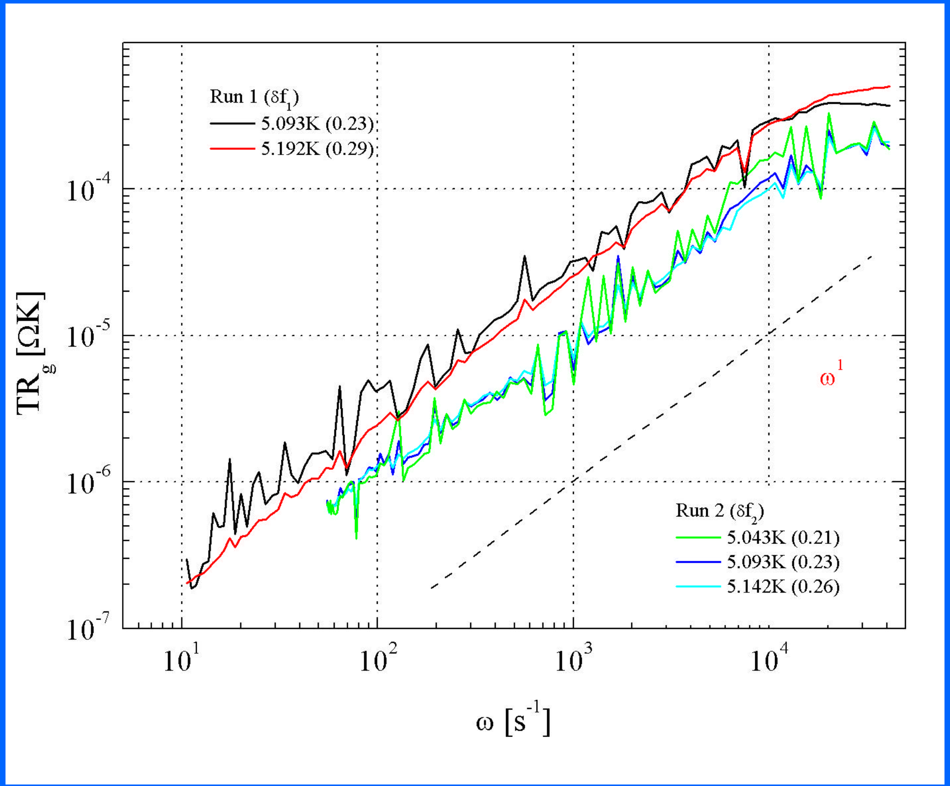


- U_{max}, $\langle\Delta\rangle \sim E_J(T)$
- $\omega\tau_0 \ll 1 \ll \omega\tau_{max}$

⇓

- $R_Z(\omega,T) \sim \delta f \tau \omega L_J(T)$

$\tau \equiv k_B T/E_J(T)$, $L_J(T)=(\phi_0/2p)^2/E_J(T)$



⇒ $R_G(\omega,T) \sim [\omega L_J(T)]^2/R_Z(\omega,T) \sim \delta f^{-1}\omega/T$

Scaling prediction for $S_\Phi(\omega,T)$

Korshunov, PRB (2000)

$$S_\Phi(\omega,T) \sim (k_B T/\omega^2)\, R_Z(\omega,T)$$

⇓

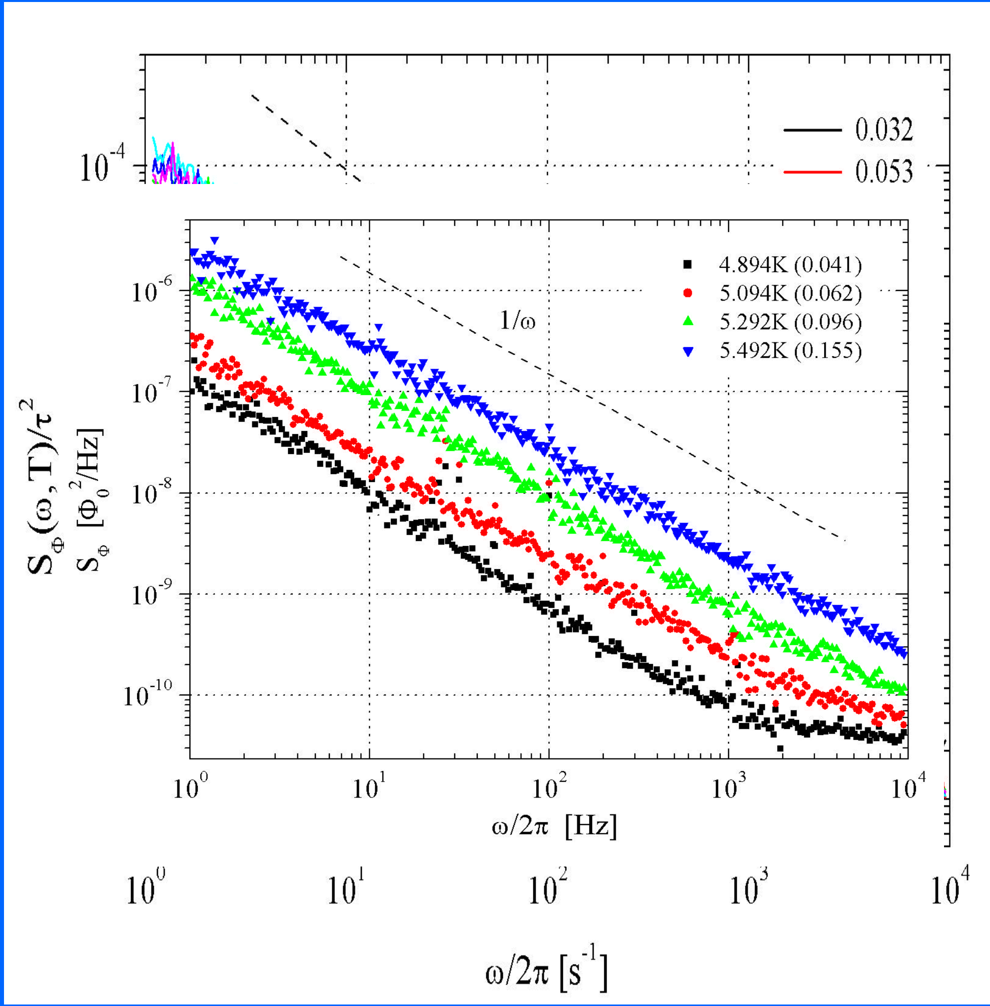
$$S_\Phi(\omega,T) \sim \tau^2/\omega$$

$$\tau \equiv k_B T/E_J(T)$$

⇓

$$S_\Phi(\omega,T)/\,\tau^2 \sim 1/\omega$$

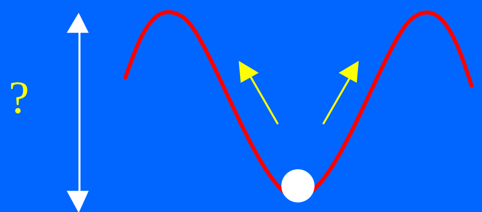
independent of T



Thermally Activated Vortex Motion in the Liquid State [T > T*(ω)]

Barrier limited diffusion of noninteracting particles

Martinoli et al., Physica B (91)
Coffey and Clem, PRL (91)

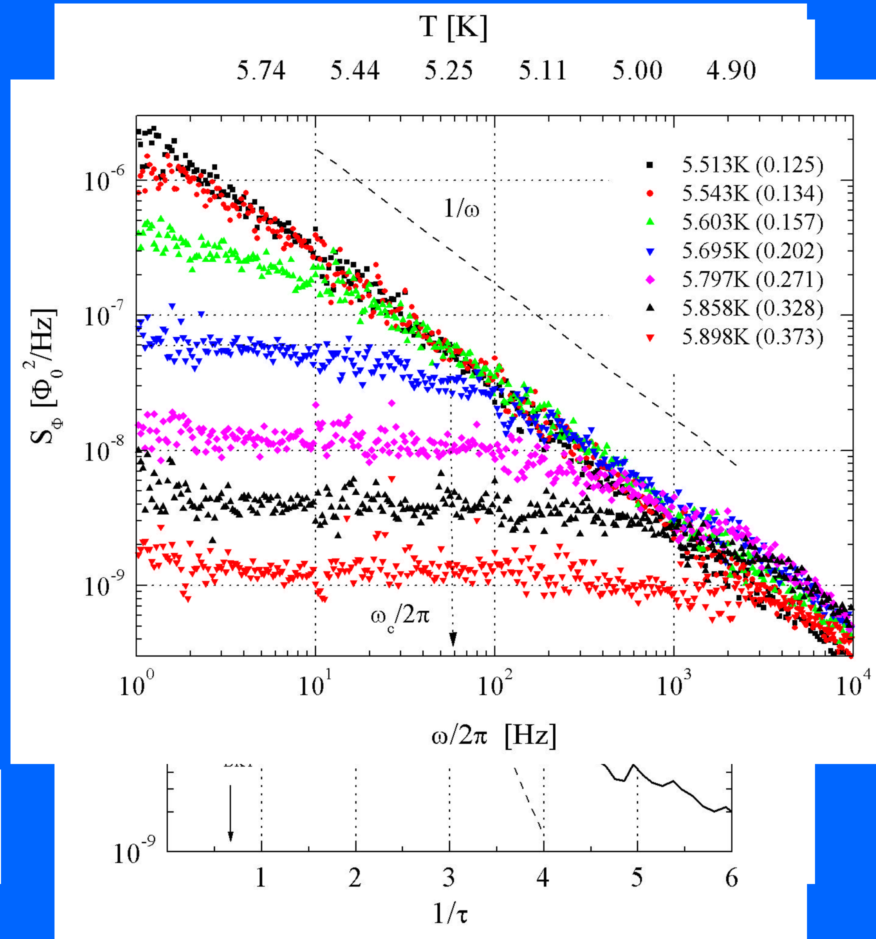


$$R_Z(T) \propto \delta f \exp(-\Delta/k_B T)$$

$$\Downarrow$$

$$S_\Phi(T) \sim (k_B T)/R_Z(T) \sim (k_B T)\exp(\Delta/k_B T)$$

- Crossover $[S_\Phi(T)]_{white} \Leftrightarrow [S_\Phi(\omega,T)]_{1/\omega}$



$\Rightarrow$ $\omega_C \sim \omega_D \exp(-\Delta/k_B T)$

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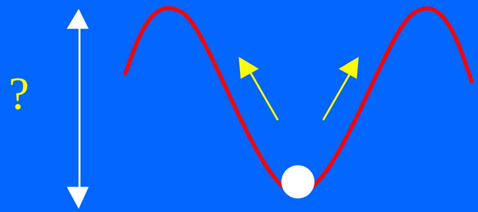

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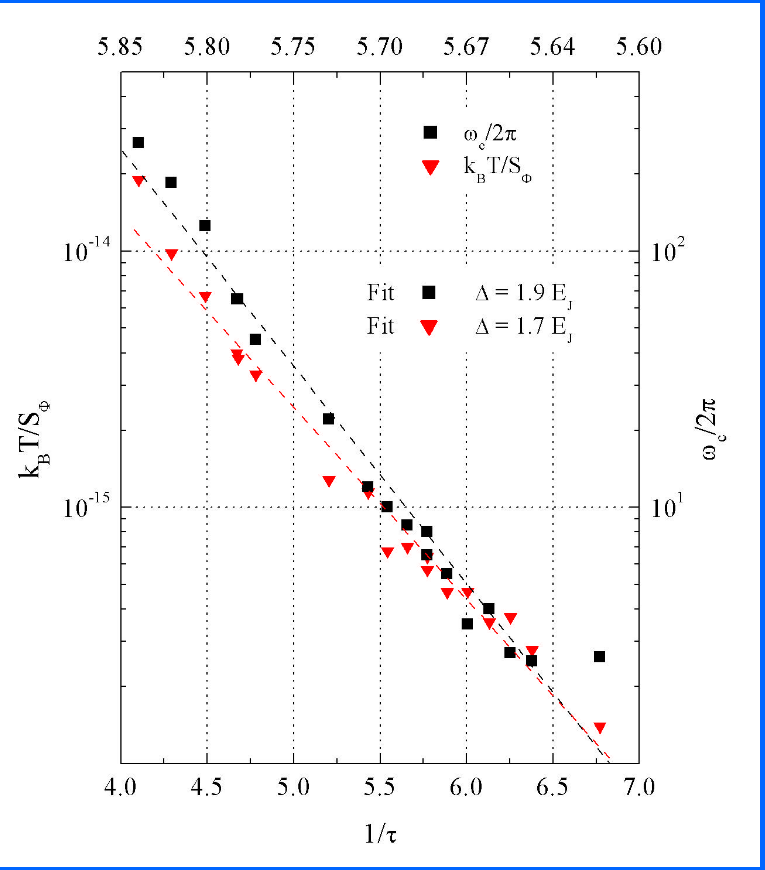


- Crossover $[S_\Phi(T)]_{white} \Leftrightarrow [S_\Phi(\omega,T)]_{1/\omega}$

$$\Rightarrow \quad \omega_C \sim \omega_D \exp(-\Delta/k_B T)$$

Energy Barriers

The energy barriers Δ extracted from both $R_Z(T)$ and $S_\Phi(T)$ in the vortex liquid regime [T > T*(ω)] are much higher [Δ ~ (2-4)E_J] than that predicted for a triangular array of infinite size (Δ ~ 0.04E_J)

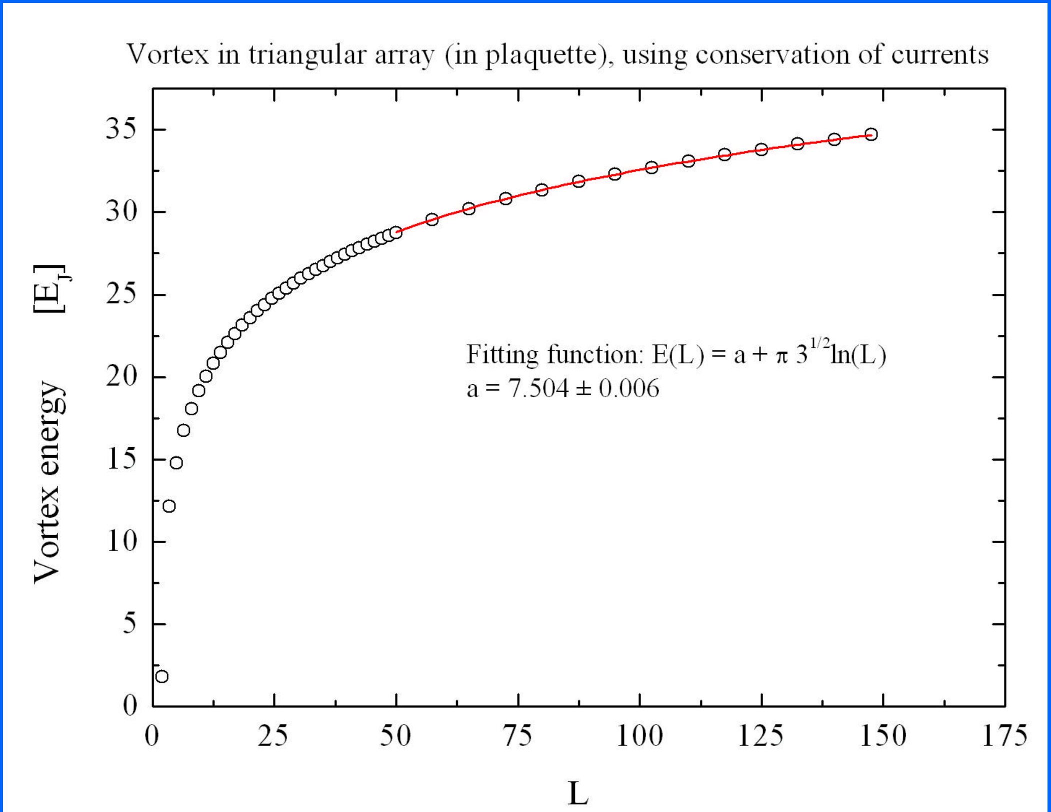
Possible explanations

1. Vortex diffusion controlled by surface barriers
Burlachkov et al., PRB (94)

In 2D $\Rightarrow$ $\Delta \sim (p/2)v3E_J\ln(C/\delta f)$, $C \sim 0.2$

$\Delta \sim (2\text{-}4)E_J$ $\Rightarrow$ $\delta f \sim (10\text{-}5)\%$

2. ? is not an energy barrier, but rather the energy needed to create the core of thermally excited vortices which dominate the dynamic response at high temperatures



Lobb et al., PRB (83)

S. Candia et al., 2003

Conclusions

Low-temperature glass-like features observed in flux noise spectra and impedance measurements performed on regular nominally unfrustrated arrays of SNS Josephson junctions can be explained by a simple vortex hopping model based on « hidden » disorder in the couling energy and residual frustration due to incomplete suppression of ambient magnetic fields.

Energy barriers extracted from flux noise spectra and resistance data in the high-temperature vortex liquid regime are much higher than the « bulk » value predicted by theory. Surface barrier mechanism? Vortex core mechanism?