From diffusive to ballistic mesoscopics: interaction effects in magnetotransport of 2D electrons

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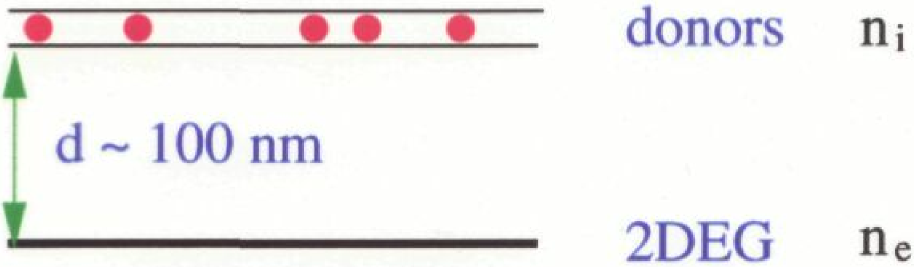
2D electron gas in strong magnetic fields

Standard realization: GaAs/AlGaAs heterostructures

Disorder: charged donors



Typical experimental parameters:

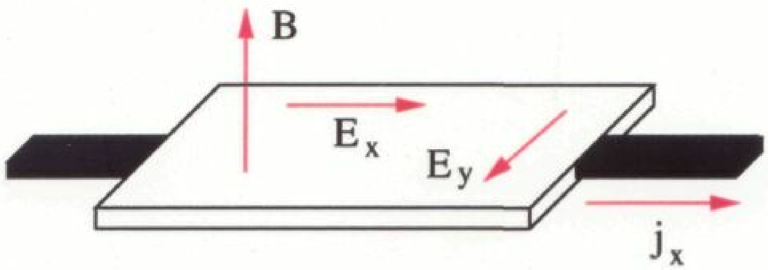
$n_e \sim \; n_i \sim (1 \div 3) \cdot 10^{11}\,\mathrm{cm}^{-2}, \qquad d \sim 100\,\mathrm{nm}$

$\longrightarrow \; k_F d \sim 10 \gg 1$ weak smooth disorder

$\tau/\tau_s \sim (k_F d)^2 \gg 1$

τ, τ_s – transport and single-particle relaxation times

Magnetotransport:



Resistivity tensor:

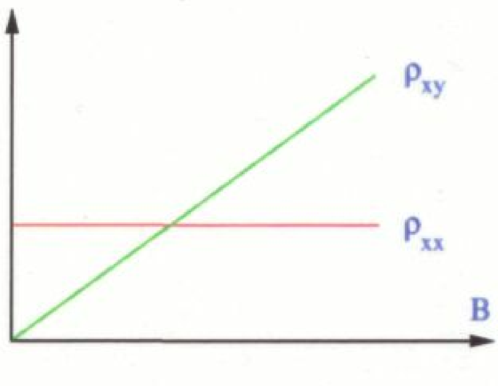
$\rho_{xx} = E_x/j_x$

$\rho_{yx} = E_y/j_x$ Hall resistivity

Classically (Drude–Boltzmann theory):

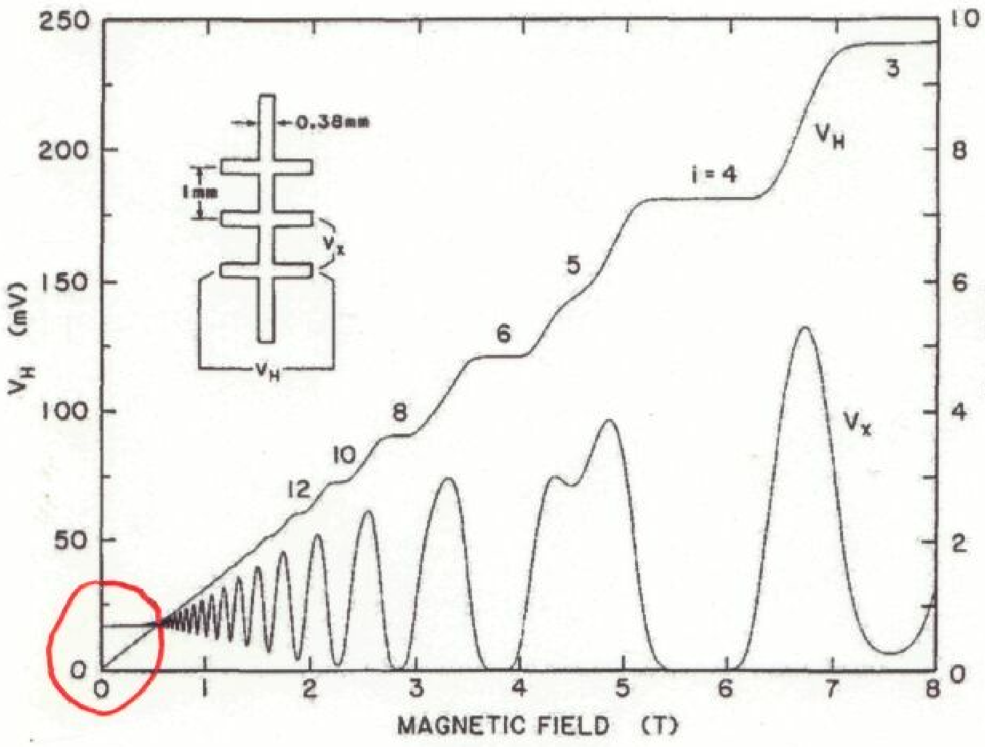
$$\rho_{xx} = \frac{m}{e^2 n_e \tau} \quad \text{independent of } B$$

$$\rho_{yx} = -\frac{B}{n_e e c}$$



Landau quantization

$\longrightarrow$ Shubnikov – de Haas oscillations and QHE



Disorder damping of SdHO $\propto \exp\{-\pi/\omega_c\tau_s\}$

$$1/\tau \ll \omega_c \ll 1/\tau_s$$

$\omega_c = \frac{eB}{mc}$

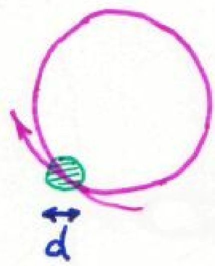
classically strong fields but SdHO suppressed

$\longrightarrow$ quasiclassics applies

Classical magnetoresistance

memory effects neglected in the Boltzmann equation

strongly enhanced by magnetic field
(returns induced by the cyclotron motion)



$\longrightarrow$ classical magnetoresistance

important for:

- random magnetic field

  $\longrightarrow$ magnetoresistance of composite fermions near $\nu = 1/2$

- rare strong scatterers (disordered antidot arrays)

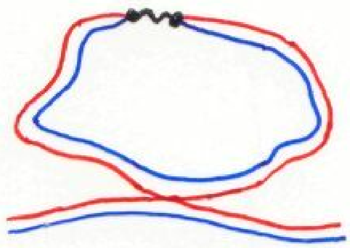
But: small in a smooth random potential with typical parameters for $B \lesssim 0.5\,\mathrm{T}$

Also: essentially T-independent at low T

Quantum correction to magnetoresistivity

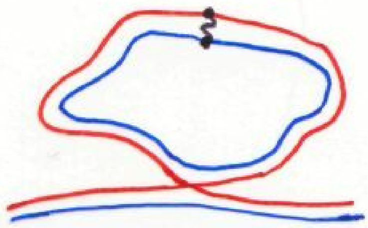
e-e interaction $\longrightarrow$
quantum correction to resistivity

Altshuler, Aronov '79



$\Delta\sigma_{xx} = \frac{e^2}{2\pi\hbar} \ln \frac{k_B}{\hbar} T\tau$

$T\tau \ll 1$ – diffusive regime



Hartree term $\longrightarrow$ factor $(1 - \frac{3}{2}F)$

$r_s \ll 1 \longrightarrow F \sim r_s \ln r_s^{-1} \ll 1$

$\frac{\Delta\sigma_{xx}}{\sigma_{xx}} \propto \int_\tau^{T^{-1}} dt\, \mathcal{D}(t)$ return probability

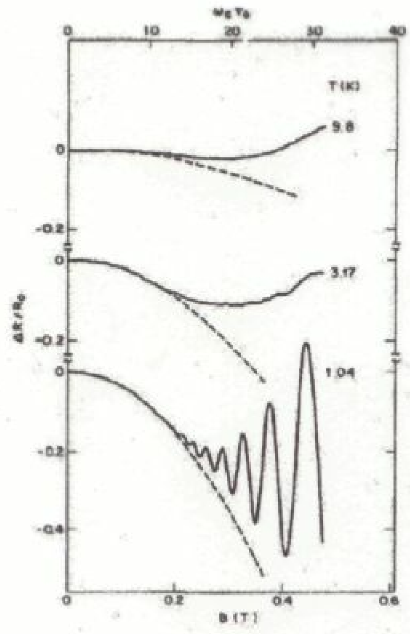
Houghton, Senna, Ying '82, Girvin, Jonson, Lee '82:

- this is valid also at $\omega_c\tau \gg 1$
- $\Delta\sigma_{xy} = 0$

$$\longrightarrow \quad \frac{\Delta\rho_{xx}(B)}{\rho_0} = \frac{(\omega_c\tau)^2 - 1}{\pi k_F l} \ln T\tau$$

$\omega_c = \frac{eB}{mc}$

interaction-induced T-dependent quantum $\Delta\rho_{xx}(B)$



Experiment:
Paalanen, Tsui, Hwang '83

Agreement with the theory?

But: $T \sim 1 \div 10\,\mathrm{K}$
$1/\tau \sim 0.3\,\mathrm{K}$

High-mobility samples: $1/\tau \sim 50\,\mathrm{mK}$ (while $1/\tau_s \sim 3\,\mathrm{K}$)

$\Longrightarrow T > 1/\tau$ for experimentally relevant temperatures

$\longrightarrow$ ballistic regime.

$T < 1/\tau_s \quad \longrightarrow$ multiple small-angle scattering processes

Diffusive theory is not applicable

ad hoc proposals:

- Paalanen, Tsui, Hwang '83

$$\ln T\tau \longrightarrow \ln T\tau_s\,, \qquad \text{validity} \longrightarrow T\tau_s \ll 1$$

- Choi, Tsui, Palmateer '86

$$\ln T\tau \longrightarrow \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{1}{T\tau}\right) \simeq -\frac{\pi^2}{2T\tau}\,, \qquad T\tau \gg 1$$

Theory is needed!

Interaction correction in the ballistic regime

Gold, Dolgopolov '86 Temperature-dependent screening

Zala, Narozhny, Aleiner '01 Friedel oscillations

$\longrightarrow$ renormalization of the collision integral

White-noise disorder

- $\Delta\sigma_{xx}$ at $B = 0$

$$\Delta\sigma_{xx} \sim \frac{e^2}{\pi\hbar} T\tau \,, \qquad\qquad T\tau \gg 1$$

- $\Delta\rho_{xy}/B$ at $B \to 0$
- in-plane magnetic field: magnetoresistance due to Zeeman effect

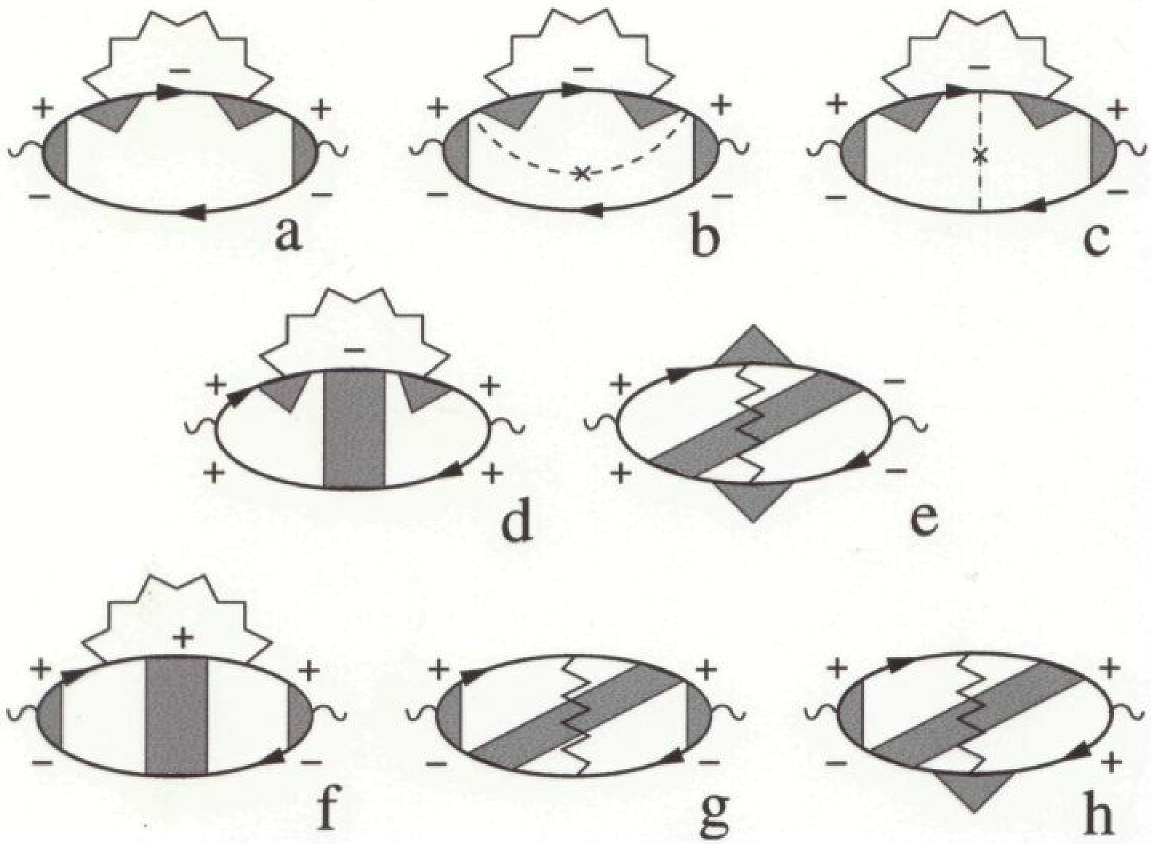
Magnetoresistance in a transverse field - ?

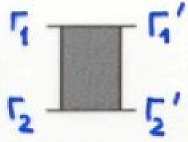
- smooth disorder – ?
- strong B ($\omega_c\tau \gg 1$) – ?

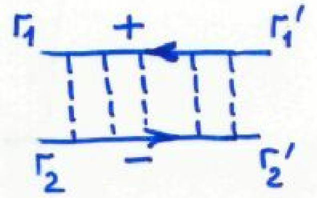
$\longrightarrow$ this work

Correction to the tunneling DOS:
Rudin, Aleiner, Glazman '97

Ballistic-diffuson diagrammatics




ballistic diffuson $\mathcal{D}(\omega; \mathrm{r}_1, \mathrm{r}_2; \mathrm{r}_1', \mathrm{r}_2')$ =



Wigner transformation $\longrightarrow$ $\mathcal{D}(\omega; \mathrm{r}, \mathrm{n}; \mathrm{r}'\mathrm{n}')$

n – velocity direction, $\mathbf{n}^2 = 1$

describes quasiclassical propagation of an electron in the phase space

diffusive regime: $\mathcal{D}(\omega, q) = \frac{2\pi\nu}{D_B q^2 - i\omega}$

ballistic regime: much more complicated

Strategy: derive general expression for $\Delta\sigma_{\alpha\beta}$ in terms of $\mathcal{D}$.

Interaction correction: general formula

$$\Delta\sigma_{\alpha\beta} = -2e^2 v_F^2 \nu \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\partial}{\partial\omega} \left\{ \omega \coth\frac{\omega}{2T} \right\} \times \int \frac{d^2\mathrm{q}}{(2\pi)^2} \,\mathrm{Im}\,[\, U(\omega,\mathrm{q})\; B_{\alpha\beta}(\omega,\mathrm{q})\,]$$

Short-range interaction: $U(\omega,\mathrm{q}) = V_0$

Coulomb interaction:

$$U(\omega,\mathrm{q}) = \frac{1}{2\nu} \frac{\kappa}{q + \kappa[1 + i\omega\langle\mathcal{D}(\omega,q)\rangle]}$$

inverse screening length $\kappa = 4\pi e^2 \nu$

If only small–angle impurity scattering present $\longrightarrow$

$$B_{\alpha\beta}(\omega,\mathrm{q}) = \frac{T_{\alpha\beta}}{2}\langle\mathcal{D}\mathcal{D}\rangle + T_{\alpha\gamma}\left(\frac{\delta_{\gamma\delta}}{2}\langle\mathcal{D}\rangle - \langle n_\gamma \mathcal{D} n_\delta\rangle\right) T_{\delta\beta} - 2T_{\alpha\gamma}\langle n_\gamma \mathcal{D} n_\beta \mathcal{D}\rangle - \langle \mathcal{D} n_\alpha \mathcal{D} n_\beta \mathcal{D}\rangle,$$

$$\hat{T} = \frac{\tau}{1+\omega_c^2\tau^2}\begin{bmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{bmatrix} = \frac{\hat{\sigma}}{e^2 v_F^2 \nu}$$

$\langle\ldots\rangle$ – averaging over velocity direction $\mathrm{n} = (\cos\phi, \sin\phi)$,

e.g. $\langle n_x \mathcal{D} n_x\rangle = (2\pi)^{-2}\int d\phi_1 d\phi_2 \cos\phi_1 \mathcal{D}(\omega,\mathrm{q};\phi_1,\phi_2)\cos\phi_2$

diagrams a, b, c $\longrightarrow$ term I

a, f, g $\longrightarrow$ II

h $\longrightarrow$ III

d, e $\longrightarrow$ IV

Arbitrary (short- or/and long-range) disorder:

term I $\longrightarrow$

$$\pi\nu T_{\alpha\gamma}\left[\langle D S_{\gamma\delta} D\rangle - 2\langle D n_\gamma W n_\delta D\rangle\right] T_{\delta\beta}$$

$W(\vec{n},\vec{n}')$ — scattering cross-section

$$S_{\gamma\delta} = \begin{bmatrix} W(\vec{n},\vec{n}') & \omega_c/2\pi\nu \\ -\omega_c/2\pi\nu & W(\vec{n},\vec{n}') \end{bmatrix}$$

In particular:

white-noise disorder: $W(\vec{n},\vec{n}') = \dfrac{1}{2\pi\nu\tau}$

Limiting cases:

- Diffusive regime $T\tau \ll 1$

  $B = 0 \longrightarrow$ Altshuler, Aronov

  $B \neq 0 \longrightarrow$ Houghton, Senna, Ying;
  Girvin, Jonson, Lee

- $B = 0$, white-noise disorder, arbitrary $T\tau$

  $\longrightarrow$ Zala, Narozhny, Aleiner

$B = 0$, ballistic regime $T\tau \gg 1$:

Return after one scattering event $\longrightarrow$ $\delta\sigma_{xx}(T) \propto W(2k_F)T\tau$

- White-noise disorder: $\delta\sigma_{xx}(T) \sim T\tau$

– agrees with Gold, Dolgopolov; Zala, Narozhny, Aleiner

- Smooth disorder, correlation length $d \gg k_F^{-1}$:

$\delta\sigma_{xx}(T) \propto e^{-k_F d} T\tau + e^{-(T\tau)^{1/2}}$

– exponentially suppressed!

But: strong B $\longrightarrow$ multiple cyclotron returns after

$n = 1, 2, \ldots$ revolutions.

Magnetoresistance in a smooth disorder

"Ballistic diffuson":

$$\left[-i\omega + iv_{\mathrm{F}}q\cos\phi + \omega_c\frac{\partial}{\partial\phi} - \frac{1}{\tau}\frac{\partial^2}{\partial\phi^2}\right]\mathcal{D}(\omega, q; \phi, \phi') = 2\pi\delta(\phi-\phi')$$

Strong magnetic field $\omega_c\tau \gg 1 \quad \longrightarrow$

$$\mathcal{D}(\omega, q; \phi, \phi') = \exp[-iqR_c(\sin\phi - \sin\phi')]$$
$$\times \left[\frac{(1 - i(qR_c/\omega_c\tau)\cos\phi)(1 - i(qR_c/\omega_c\tau)\cos\phi')}{D_B q^2 - i\omega} + \sum_{n\neq 0}\frac{e^{in(\phi-\phi')}}{D_B q^2 - i(\omega - n\omega_c) + n^2/\tau}\right]$$

$D_B = R_c^2/2\tau$ – diffusion constant in strong magnetic field

- $T \gg \omega_c \quad \longrightarrow \quad \Delta\sigma_{\alpha\beta}$ exponentially suppressed

- $T \ll \omega_c \quad \longrightarrow \quad$ characteristic $D_B q^2,\ \omega \ll \omega_c$

$\longrightarrow$ keep only the first term in $\mathcal{D} \quad \longrightarrow$

$$B_{xx}(\omega, q) = \frac{J_0^2(qR_c)}{(\omega_c\tau)^2}\frac{D_B\tau q^2}{(D_B q^2 - i\omega)^3}$$

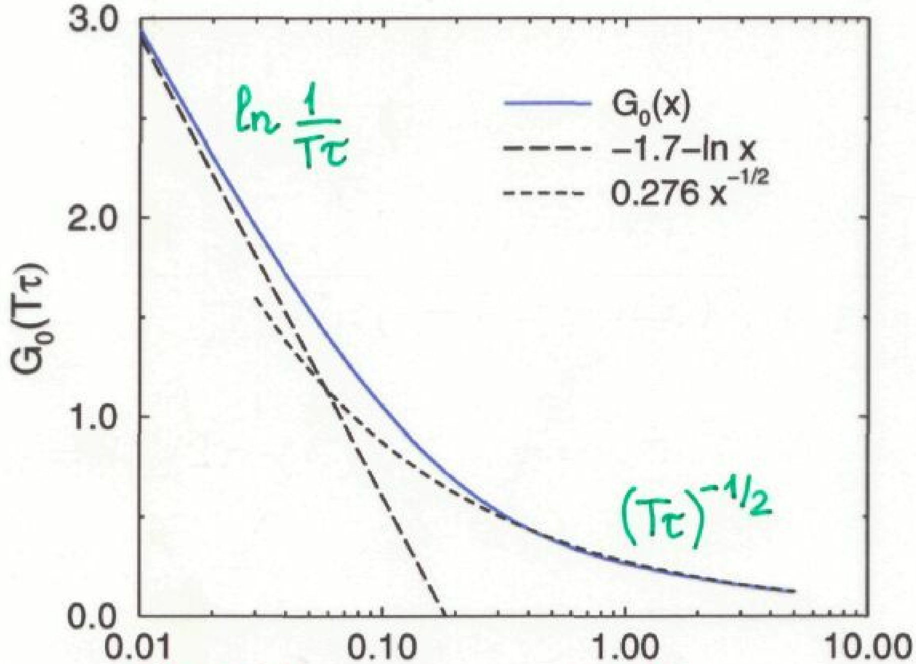
Diffusion of the guiding center

Short-range interaction

$$\Delta\sigma_{xx} = -\frac{e^2}{2\pi^2}\nu V_0 G_0(T\tau)$$

$$G_0(x) = \pi^2 x^2 \int_0^\infty \frac{dz}{z^3}\frac{\exp[z]}{\sinh^2(\pi x/z)}\left[I_0(z)(1-z)+zI_1(z)\right]$$

$$G_0(x) = \begin{cases} -\ln x + \text{const}, & x \ll 1 \\ c_0 x^{-1/2}, & x \gg 1 \end{cases} \qquad c_0 = \frac{3\zeta(3/2)}{16\sqrt{\pi}} \simeq 0.276$$



Crossover at numerically small $T\tau \sim 0.1$!

Hartree term: opposite sign and 2 times larger due to spin

$$\frac{\Delta\sigma_{xy}}{\sigma_{xy}} \ll \frac{\Delta\sigma_{xx}}{\sigma_{xx}} \quad \longrightarrow \quad \frac{\Delta\rho_{xx}}{\rho_0} = (\omega_c\tau)^2\frac{\Delta\sigma_{xx}}{\sigma_0}$$

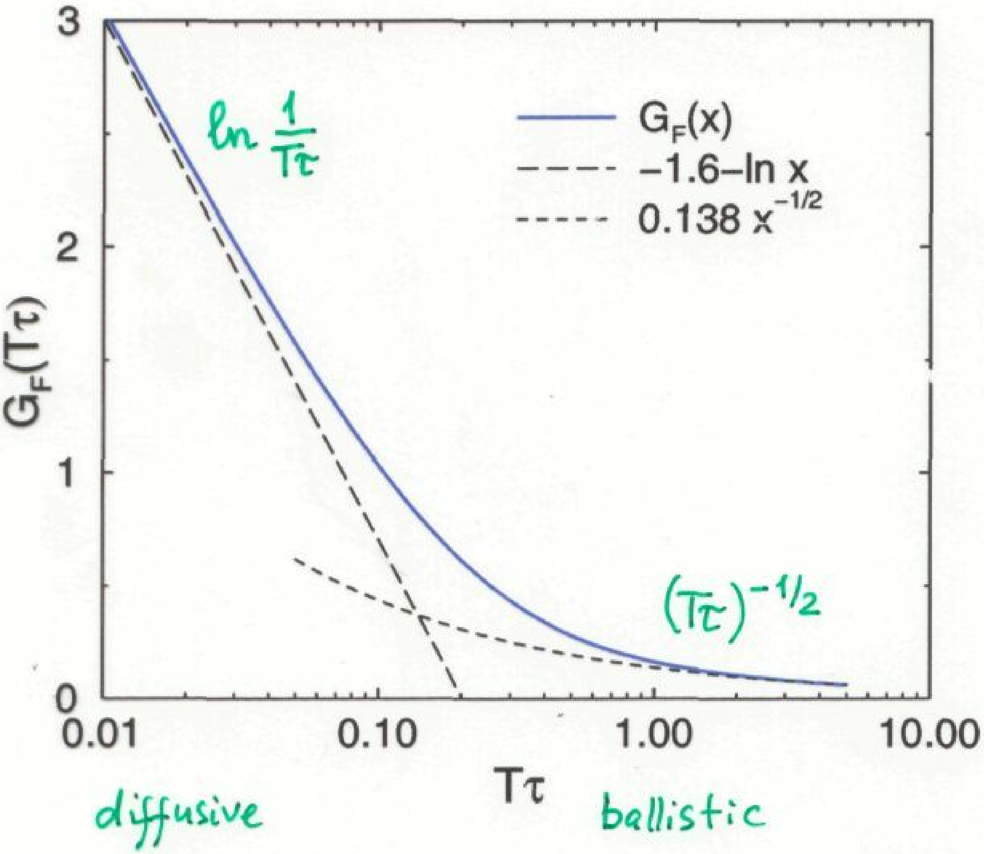
Coulomb interaction

$$\frac{\Delta\rho_{xx}(B)}{\rho_0} = -\frac{(\omega_c\tau)^2}{\pi k_F l} G_F(T\tau)$$

$$G_F(x) = \frac{1}{4x^2}\int_0^\infty dz z^3 J_0^2(z)$$

$$\times \sum_{n=1}^{\infty} \frac{n(3n[1-J_0^2(z)] + [3-J_0^2(z)]z^2/2x)}{(n+z^2/2x)^3(n[1-J_0^2(z)]+z^2/2x)^2}$$

$$G_F(x) = \begin{cases} -\ln x + \text{const}, & x \ll 1 \\ (c_0/2)x^{-1/2}, & x \gg 1 \end{cases} \qquad c_0 = \frac{3\zeta(3/2)}{16\sqrt{\pi}} \simeq 0.276$$



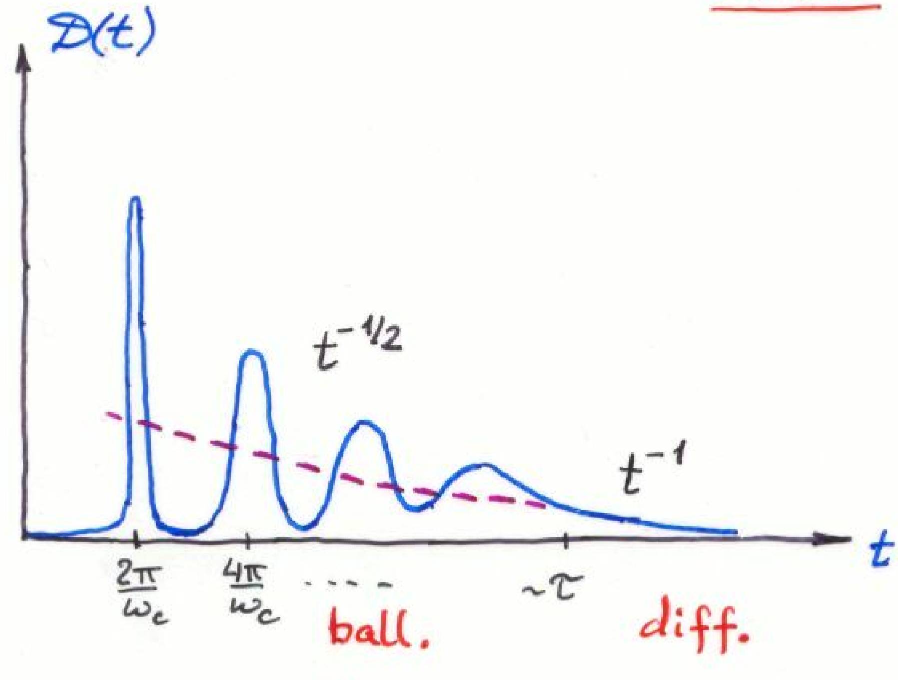
Qualitative picture

$$\Delta\sigma_{xx} \propto \int \frac{d\omega}{2\pi} \frac{\partial}{\partial\omega}\left(\omega \coth\frac{\omega}{2T}\right) \int \frac{d^2q}{(2\pi)^2} \frac{J_0^2(qR_c)\, D_B q^2}{(D_B q^2 - i\omega)^3}$$

$$\propto \int_0^\infty dt\, \mathcal{D}(t) \left[\frac{\pi T t}{\sinh \pi T t}\right]^2 \sim \int_0^{T^{-1}} dt\, \mathcal{D}(t)$$

$$\mathcal{D}(t) = \left\langle \mathcal{D}(r=0, t, \varphi, \varphi') \right\rangle = \int \frac{d^2q}{(2\pi)^2} J_0^2(qR_c)\, e^{-D_B q^2 t}$$

— return probability



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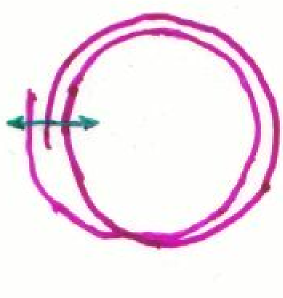
1D diffusion



$$\Delta\sigma_{xx} \propto \begin{cases} \ln \frac{1}{T\tau}, & T \ll \frac{1}{\tau} \\ (T\tau)^{-1/2}, & T \gg \frac{1}{\tau} \end{cases}$$

Hartree contribution, $\kappa/k_F \ll 1$

$U(\omega, q) \quad \longrightarrow \quad -(\frac{3}{2}+\frac{1}{2})\, U(0, 2k_F \sin\frac{\phi'-\phi}{2})$

singlet (1/2) $\longrightarrow$ mixing with the exchange term

Diffusive regime

- singlet fully absorbed
- $\Delta\rho^H_{xx} = -\frac{3}{2}F\,\Delta\rho^F_{xx}, \qquad F = \frac{1}{2\pi}\frac{\kappa}{k_F}\ln\frac{k_F}{\kappa} \ll 1$

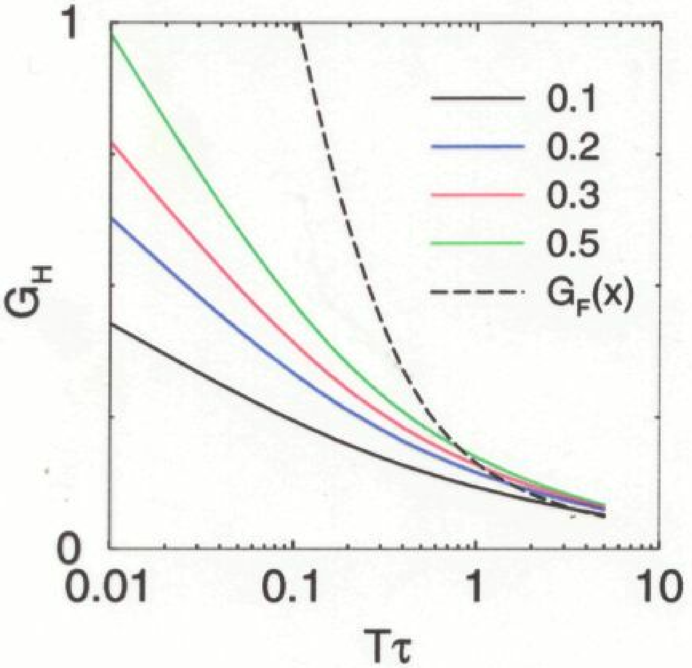
Ballistic regime:

$\longrightarrow$ new energy scale $\quad T_{\rm H} \sim \tau^{-1}(k_F/\kappa)^2$

where $\Delta\rho_{xx} = \Delta\rho^F_{xx} + \Delta\rho^H_{xx}$ changes sign:

- $T \ll T_{\rm H}$ exchange dominates, negative MR
- $T \gg T_{\rm H} \qquad \Delta\rho^H_{xx} = -2\Delta\rho^F_{xx}$

$\longrightarrow$ positive MR, the same $(T\tau)^{-1/2}$ dependence



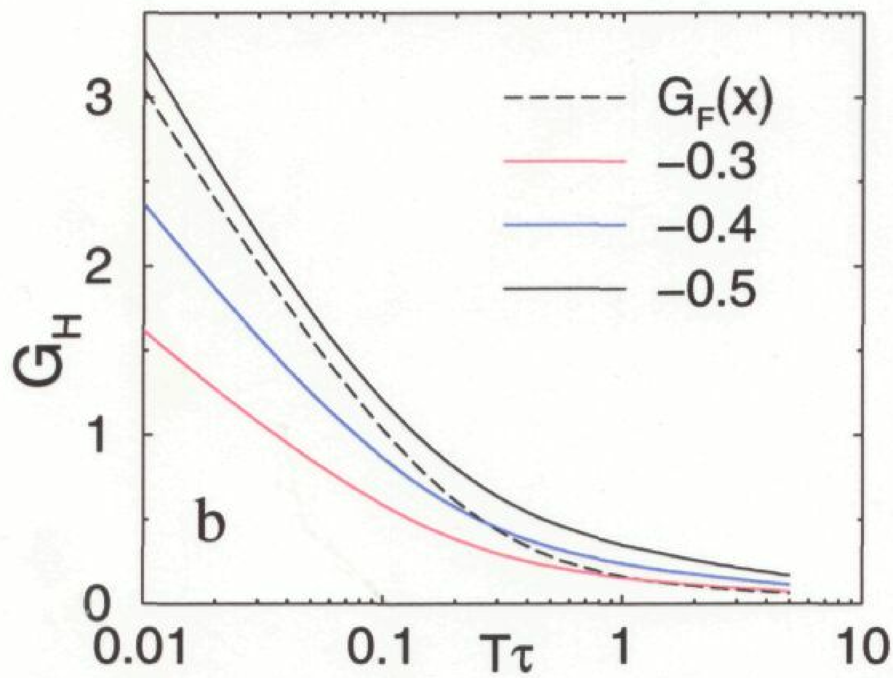
Strong interaction

- κ/k_F is not small:
  $\longrightarrow$ strong Fermi-liquid renormalization
- determined by angular harmonics $F_m^{\rho,\sigma}$ of the Fermi-liquid functions $F^{\rho,\sigma}(\theta)$
- Diffusive regime, $T\tau \ll 1$:

$$G_{\rm H}(T\tau) = 3(1 - \ln(1 + F_0^\sigma)/F_0^\sigma) \ln T\tau$$

- Ballistic limit, $T\tau \gg 1$:

$$G_{\rm H}(T\tau) = -\left(3\sum_m \frac{F_m^\sigma}{1+F_m^\sigma} + \sum_{m\neq 0} \frac{F_m^\rho}{1+F_m^\rho}\right)\frac{c_0}{2}(T\tau)^{-1/2}$$

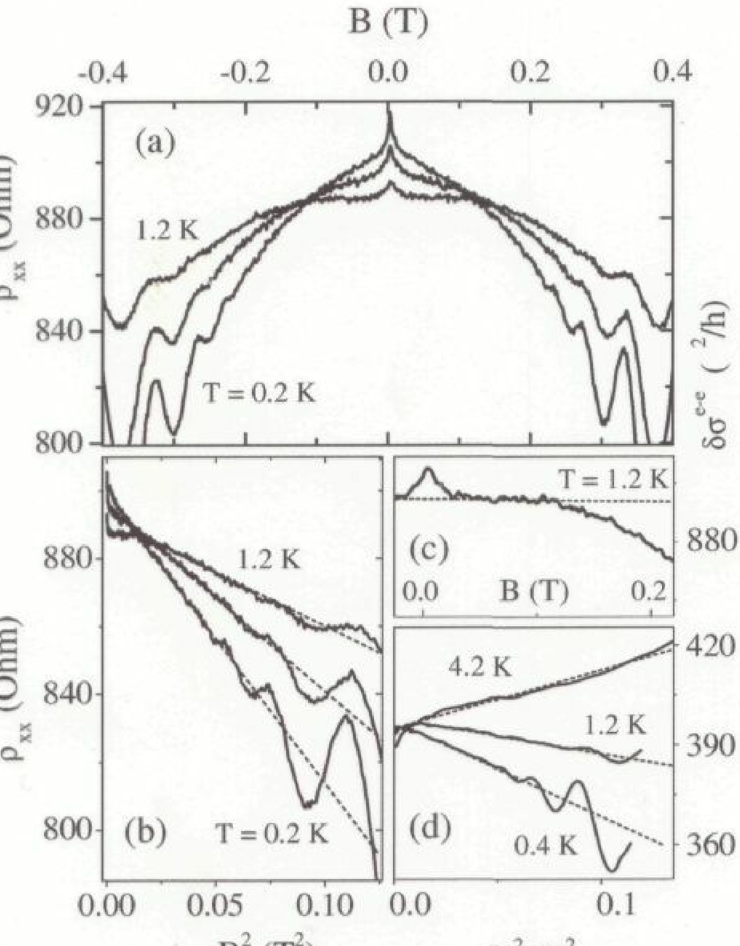


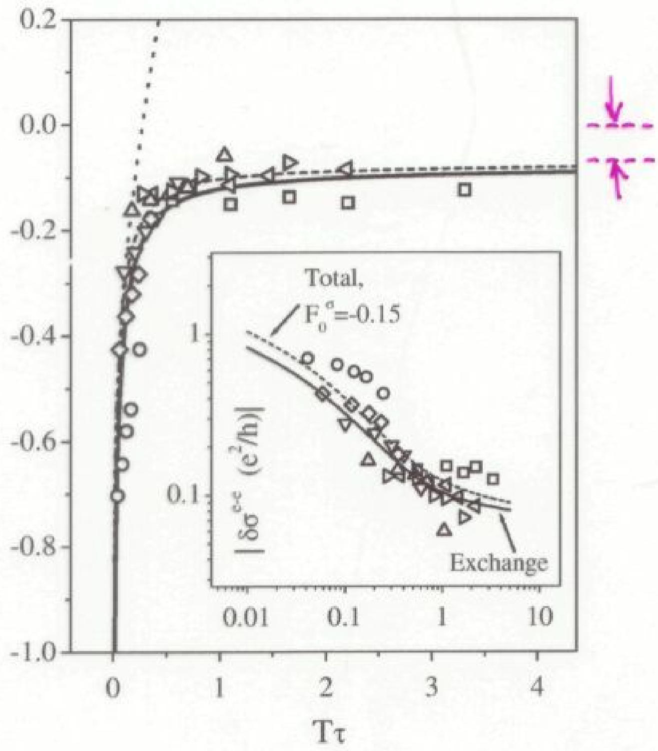
Experiment

A.K. Savchenko group (Exeter, UK) cond-mat/0207662

PRL 2003

n - GaAs $T\tau \sim 0.1 \div 5$





Small constant shift, attributed to classical memory effects

Mixed disorder

- smooth random potential: $\tau_{sm}/\tau_{sm,s} \sim (k_F d)^2 \gg 1$

\+

- short-range impurities (white noise): τ_{wn}

assume $\tau_{sm,s} \ll \tau_{wn} \ll \tau_{sm}$

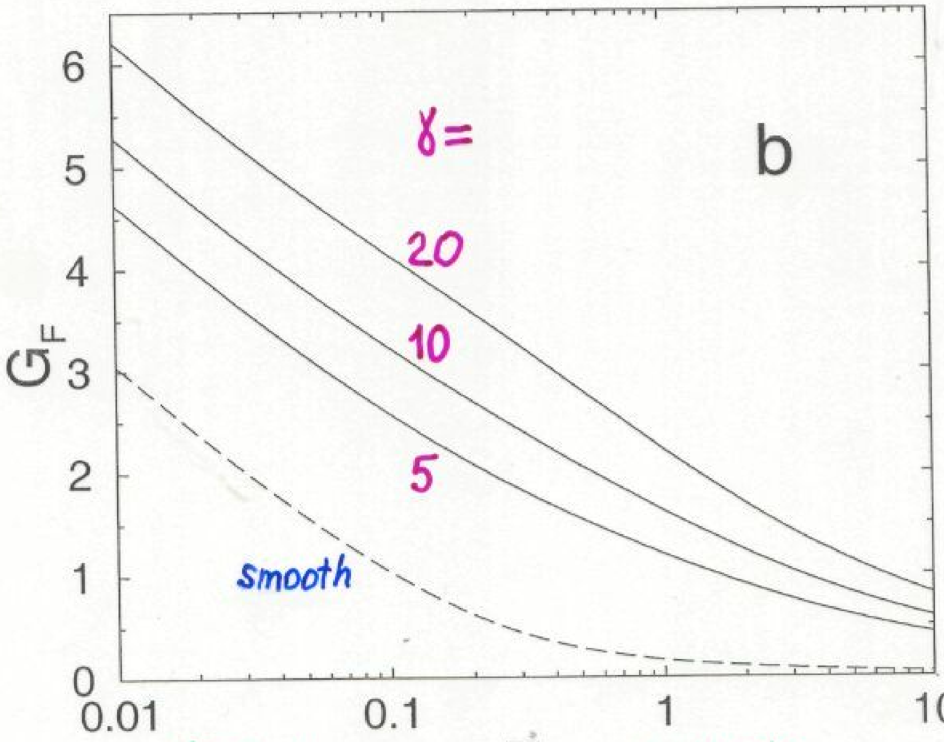
$\rightarrow \tau^{-1} = \tau_{sm}^{-1} + \tau_{wn}^{-1} \simeq \tau_{wn}^{-1}$

$$\frac{\delta\rho_{xx}^F(B)}{\rho_0} = -\frac{(\omega_c\tau)^2}{\pi k_F l} G_F^{mix}(T\tau, \gamma)$$

$$\gamma = \frac{\tau_{sm}}{\tau} \gg 1$$

$$G_F^{mix}(x,\gamma) \simeq \begin{cases} -\ln x + (\gamma/2)^{1/2}, & x \ll 1 \\ 4\gamma^{1/2} \cdot \frac{c_0}{2} x^{-1/2}, & x \gg 1 \end{cases}$$

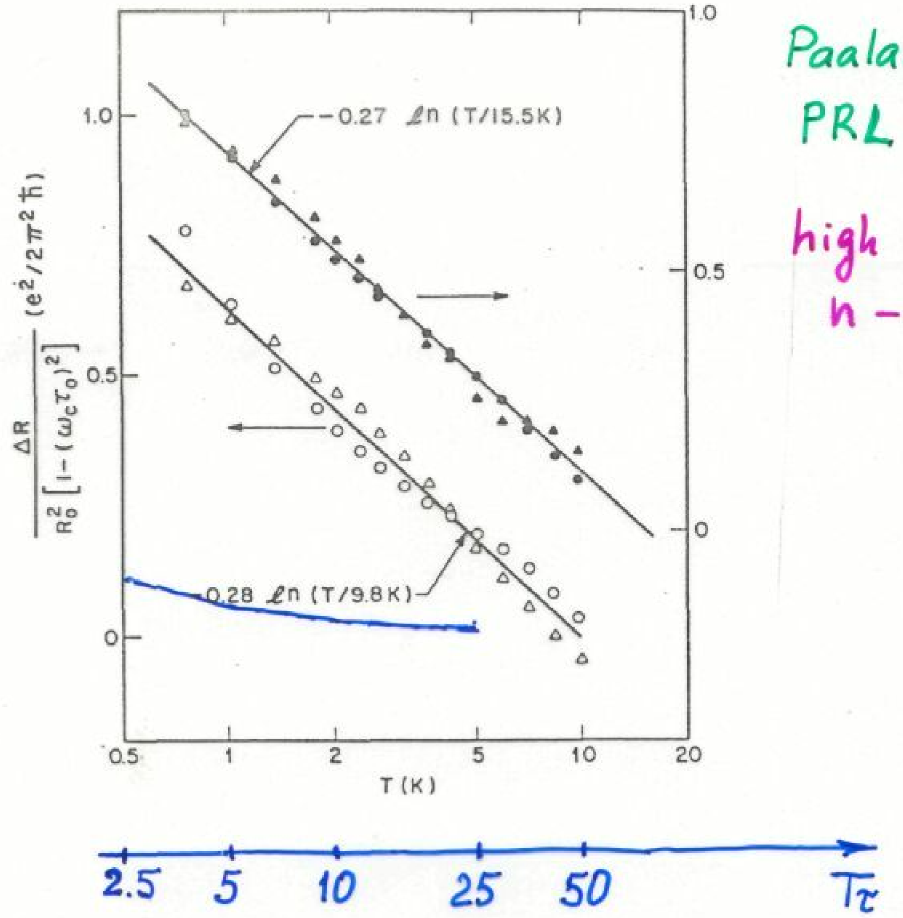
strongly enhanced!

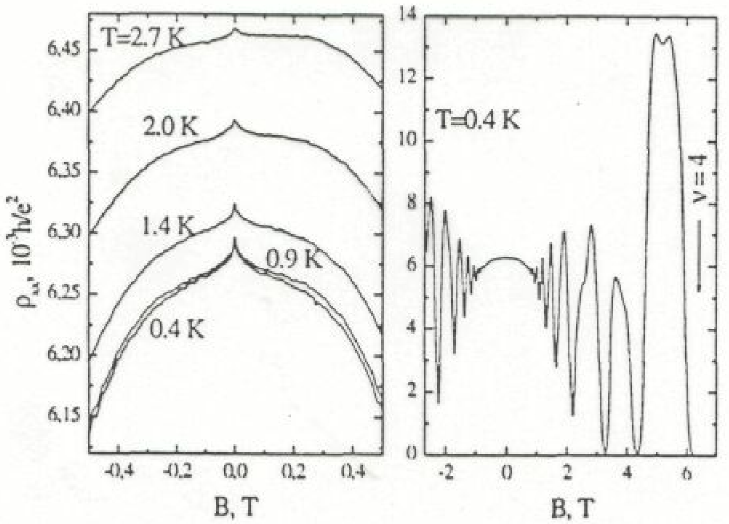


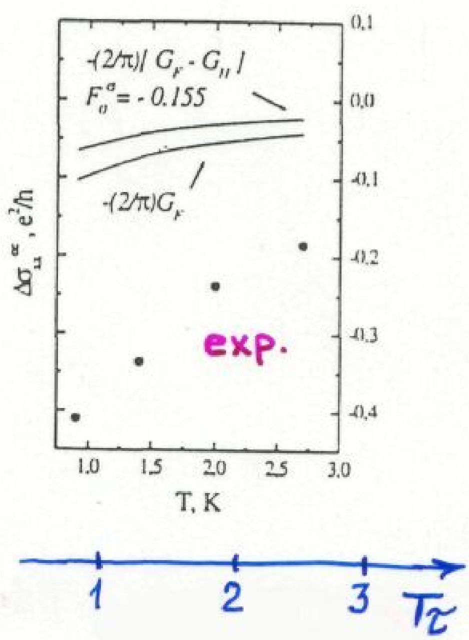
diffusive $T\tau$ ballistic

Paalanen, Tsui, Hwang, PRL 1983

high mobility n – GaAs / AlGaAs







Olshanetsky, Renard, Kvon, Portal, Woods, Zhang, Harris, cond-mat/0303269

n – Si / SiGe

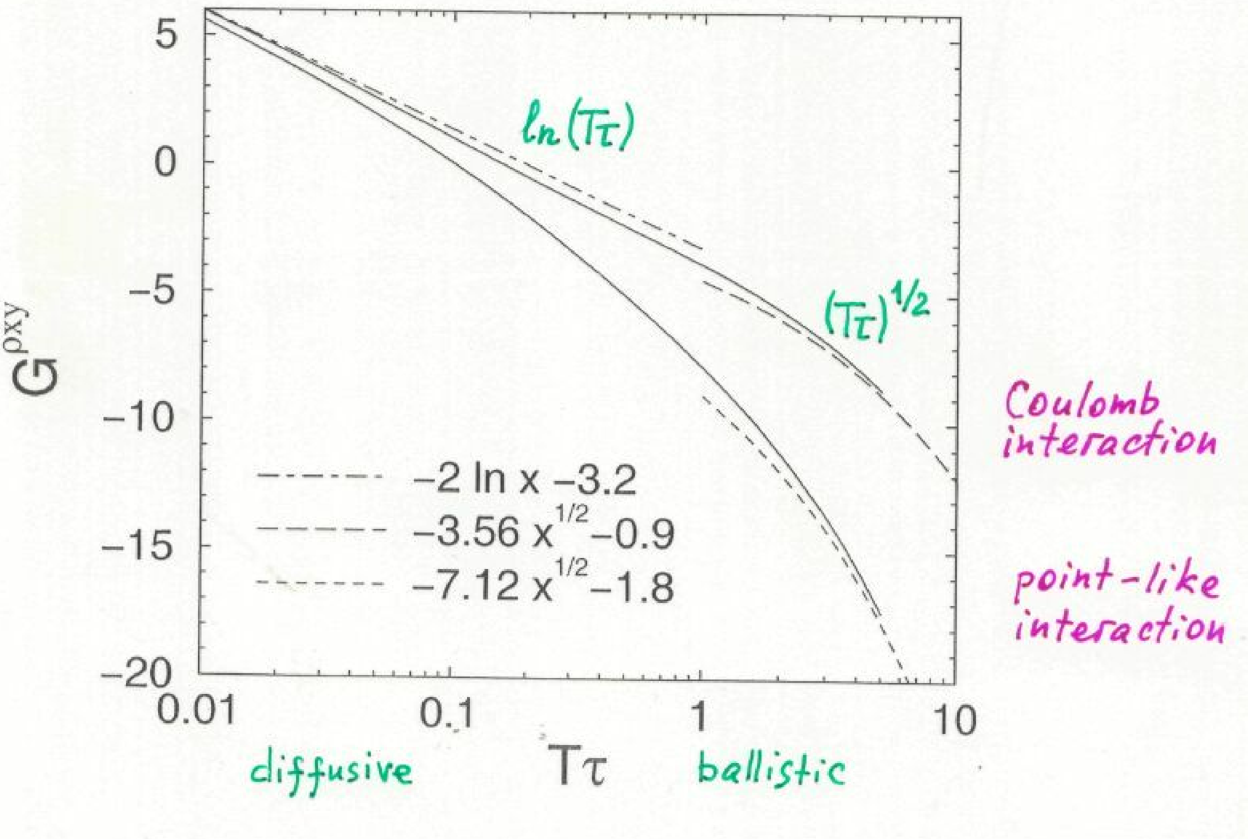
Hall resistivity

Interaction → T-dependent slope of the Hall resistivity

$$\frac{\delta\rho_{xy}^F}{\rho_{xy}} = \frac{1}{\pi k_F \ell} \cdot G_F^{\rho_{xy}}(T\tau)$$

$$G_F^{\rho_{xy}}(x) \simeq \begin{cases} -2\ln x + \text{const}, & x \ll 1 \\ -\frac{11}{2} c_1 x^{1/2}, & x \gg 1 \end{cases}$$

$$c_1 = -\frac{\sqrt{\pi}}{4}\zeta(1/2) \simeq 0.647$$



Summary

- Ballistic-diffuson diagrammatics →
  Interaction correction to $\sigma_{\alpha\beta}$ in terms of the classical phase-space propagator $D(\omega; r, n; r', n')$:
  General formula valid for arbitrary $T\tau$, $\omega_c\tau$, $k_F d$.
- $T\tau \gtrsim 1$ ballistic regime → character of disorder important!
- magnetoresistance in smooth disorder due to cyclotron returns:

$$\frac{\delta\rho_{xx}}{\rho_0} \simeq -\frac{(\omega_c\tau)^2}{\pi k_F l} G(T\tau) \qquad G(x) \sim \begin{cases} \ln\frac{1}{x}, & x \ll 1 \\ x^{-1/2}, & x \gg 1 \end{cases}$$

$$\frac{\delta\rho_{xy}}{\rho_{xy}} \simeq \frac{1}{\pi k_F l} G^{\rho_{xy}}(T\tau) \qquad G^{\rho_{xy}}(x) \sim \begin{cases} \ln\frac{1}{x}, & x \ll 1 \\ -x^{1/2}, & x \gg 1 \end{cases}$$

- mixed disorder (smooth + short-range), $\tau_{wn} \ll \tau_{sm}$
  → effect enhanced in the ballistic regime by a factor $4(\tau_{sm}/\tau_{wn})^{1/2} \gg 1$
- Further applications and generalizations:
  - modulated systems (lateral superlattices)
  - other dimensionalities
  - quantizing magnetic fields
  - thermoelectric phenomena
  - phonon-induced contribution to resistivity