Interaction, transmission distribution and electromagnetic environment

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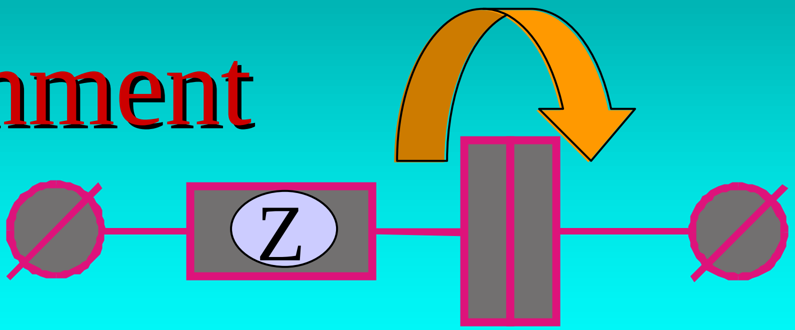
Prerequisites:

- Tunneling in the e.m. environment
- Tunneling in Luttinger liquids
- *Weak* interaction limit in Luttinger liquids

Tunneling in the e.m. environment



- Late 80’s, early 90’s
- Suppression of tunneling rates
- Inelastic tunneling: P(E) – probability to loose energy E
- Anomalous Ohm’s law:
- Correlators of voltage fluct.

$$\frac{d^2 I}{dV^2} = \frac{e}{R_T} P(eV)$$

$$G(V) \propto (eV, kT)^{a}; \quad a = \frac{2Z}{R_Q}$$

$$t \to t \exp(i\Phi); < \exp(i\Phi) >= \exp(i\Phi^{slow}) \exp(\frac{-<\Phi^2_{fast}>}{2})$$

$$<\Phi^2_{fast}> \propto \int \frac{dw}{w} = \frac{a}{2} \log$$

Tunneling in Luttinger liquids

- Early 90’s:Kane and Fisher
- Suppression of tunneling rates due to inelastic processes
- Anomalous exponent
- Analogy with E.M. wrong Z? No, Z_{eff} felt by moving electron

- No new state of matter –just tricky transmission lines
- Correlators of voltage again

$$G(V) \propto (eV, kT)^{a};$$

$$a = Interaction\ in\ L.L.$$

$$\mathrm{Re}\ normalization, \quad \Phi - state\ of\ L.L.$$

$$t \to t\exp(i\Phi); <\exp(i\Phi)> = \exp(i\Phi^{slow})\exp\left(\frac{-<\Phi_{fast}^{2}>}{2}\right)$$

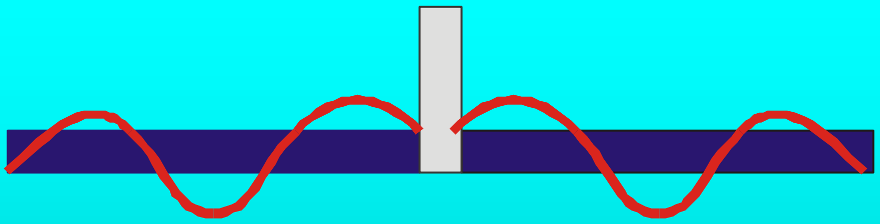
$$<\Phi_{fast}^{2}> \propto \int\frac{dw}{w} = \frac{a}{2}\log$$

Weak interaction limit

- Glazman, Matveev ‘93 – no mainstream activity
- **Heresy:** Elastic processes are responsible for anomalous exponents.
- An alternative? Non-Luttinger behavior?
- Nice about it: everything is according to Landauer
- Suitable for any transparency !!!!

- Tricky limit: weak interaction, big effect

$a \ll 1; \quad a \log \gg 1$



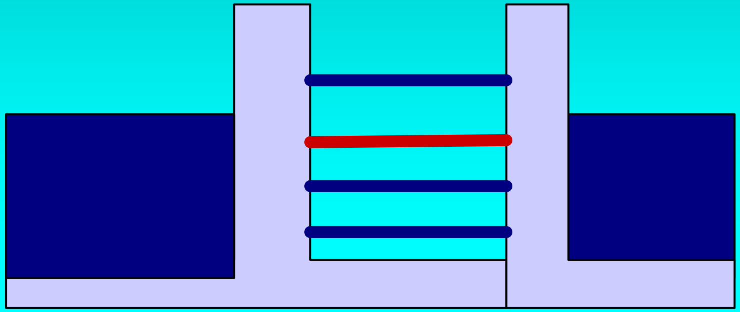
$$\frac{dT_0}{d\log(1/E)} = -a \cdot T_0(1-T_0)$$

Outline

- Resonant tunneling in L.L. in weak interaction limit
  - Results of Kane and Fisher in Landauer setup
  - Quantitative crossover
- Interaction corrections to Full Counting Statistics of an arbitrary mes. conductor
- **or** weak Coulomb blockade of two mes.conductors
  - F.C.S just highlights transmission distribution
  - That evolve with energy …
- Different problems, interrelated answers

Resonant tunneling in L.L.

- Formulation of the problem: two tunneling barriers
- Resonant levels in between: let’s take the one closest to Fermi energy
- Kane and Fisher: another exponent, incoherent tunneling (?)
- Recent doubts
- Let’s take a limit of weak interaction



$$G(T) = \frac{\Gamma(T)}{T} = \frac{T^{a/2}}{T}$$

Res.Tunneling: the method

- Essential: transparency depends on energy
- The method has to be modified
- New renormalization equation for tr. amp.
- Crossover energy
- Above
- Below

$$T_0(E) = \frac{\Gamma_L \Gamma_R}{(E-\Delta)^2 + (\Gamma_L + \Gamma_R)^2 / 4}$$

$$t(E) = t^{(\Lambda)}(E) + \int_{|E|}^{\Lambda} \frac{dE'}{4E'} a \left(r_L(E) r_L^*(E') + r_R^*(E) r_R(E') \right)$$

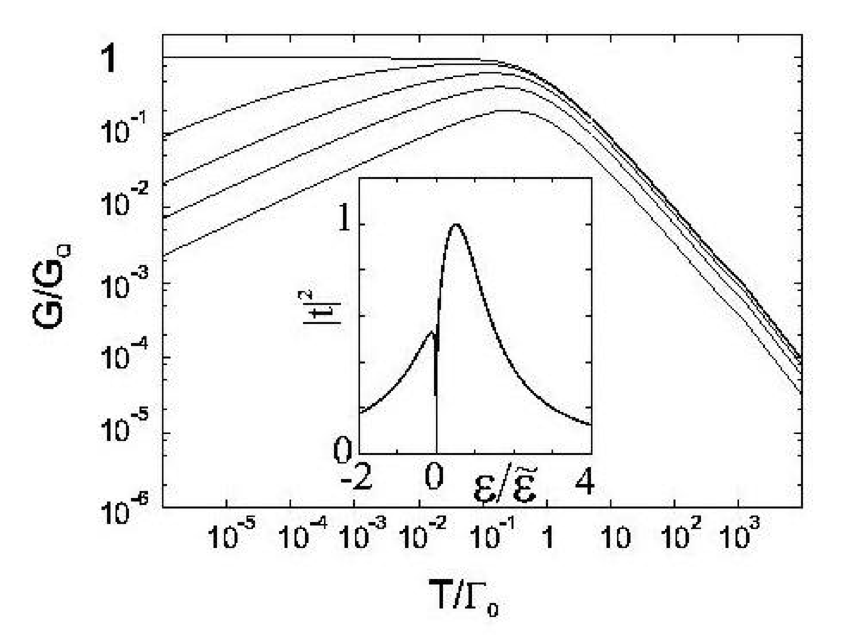
$$\Gamma(\tilde{e}) = \tilde{e} \square (e / \tilde{e})^{a/2}$$

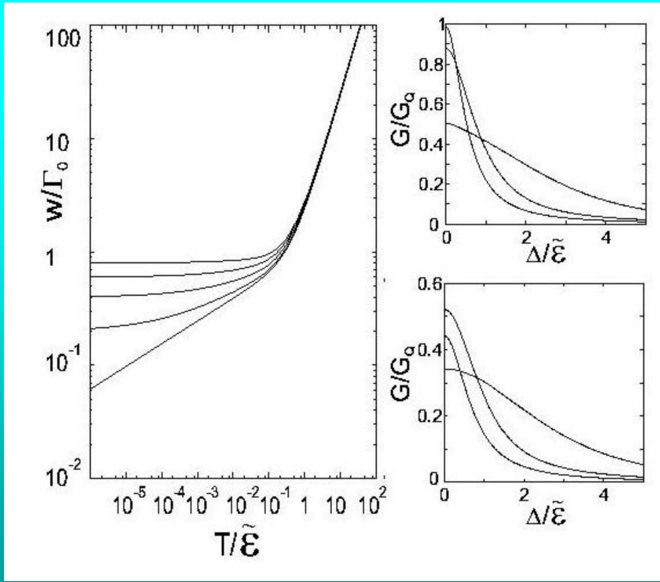
$$\frac{dT_0}{d\log(1/E)} = -a \cdot T_0 (1 - T_0)$$

Res. Tunneling: the results

- Crucial difference between Symmetric an Asymmetric peaks

- Symmetric: height saturates, width decreases
- Asymmetric: width saturates, height decreases





Interaction correction to full counting statistics

- Invention of *Levitov at al.*
- *No interaction = scattering approach.*
- FCS decodes transmission eigenvalues!

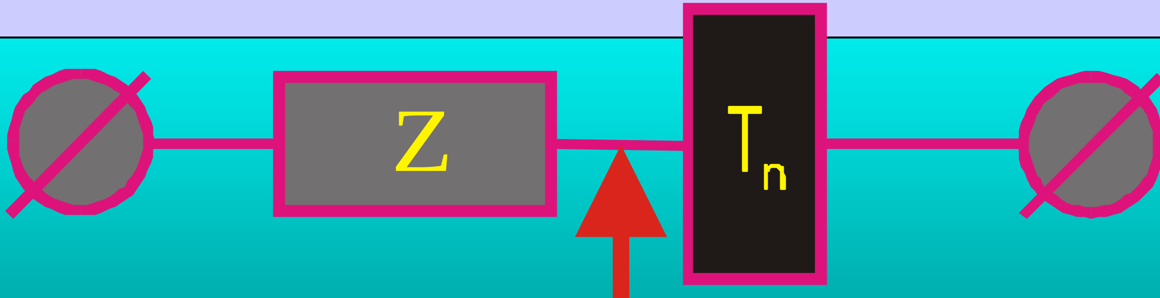
$$P_t(N) = \int_0^{2p} d\,c\ \exp(-S(c, t))\exp(-iN\,c)$$

$$S(c) = t\sum_n \int_0^{eV} dE\,\log(1 + T_n(\exp(i\,c) - 1))$$

- Interaction ?
- Strong interaction = Orthodox Coulomb Blockade
- Weak interaction = e.m. environment

C.S: the method

- Keldysh action: path integral
- Two blocks: the mesoscopic conductor + external resistor
- Low frequencies: *classical* effect
- Freq. $> eV, k_B T$: interaction correction
- Perturbation in Z << R_Q



$$f^+(t), f^-(t)$$

C.S.: the answer

- Two terms – inelastic and elastic
- *Log* divergencies – only in the elastic term!

$$dS = a\,gX \cdot (f_L(E)(1 - f_R(E + w)) +$$

$$a \cdot Y \cdot (f_L(E) - f_R(E)) \cdot \int \frac{dw}{w}$$

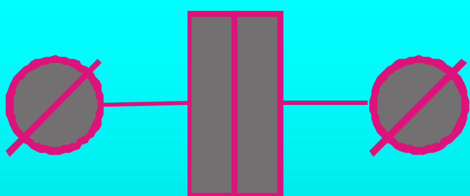
- Idea: renormalization of transmissions!
- Painful but strict proof

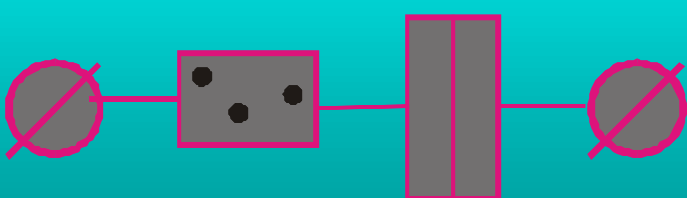
$$\frac{dT_n}{d\log(1/E)} = -a \cdot T_n(1 - T_n)$$

Transmission distributions

- Open channels T approx. 1 (Imry, long time ago)
- Two classes of transmission distributions

- No open channels





- **Sqrt. divergency of the density of the open channels**

$$\rho(T) = \frac{G}{2\pi G_Q} \frac{1}{\sqrt{T(1-T)}}$$

two b.c.

$$\rho(T) = \frac{G}{2G_Q} \frac{1}{\sqrt{T^2(1-T)}}$$

diffusive

$$\rho(T) = \frac{G}{\pi G_Q} \frac{1}{\sqrt{T^3(1-T)}}$$

two t.j.

Evolution of transmission distribution

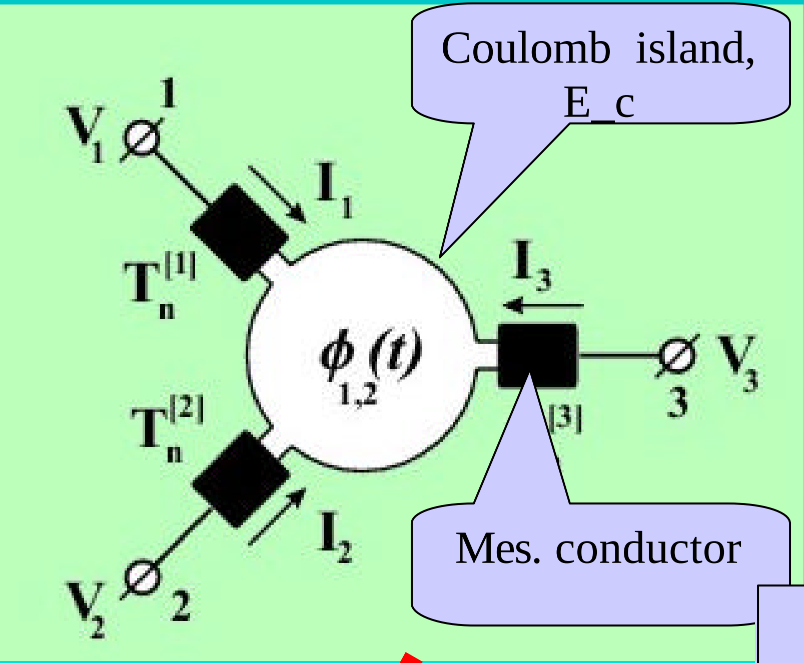
- What happens if Energy goes down ? $a \log(1/e)$? 1
- First insight: T -> 0, all becomes a tunnel junction a
- Second insight: singularity at T=1: any exponent available
- Correct insight: two possible exponents: a and $a/2$

- If there are open channels
- The distribution assumes a certain limiting form

$$r(T) ; e^{a/2} \frac{1}{\sqrt{T^3(1-T)}}$$

two t.j.
in series

Weak Coulomb Blockade and F.C.S

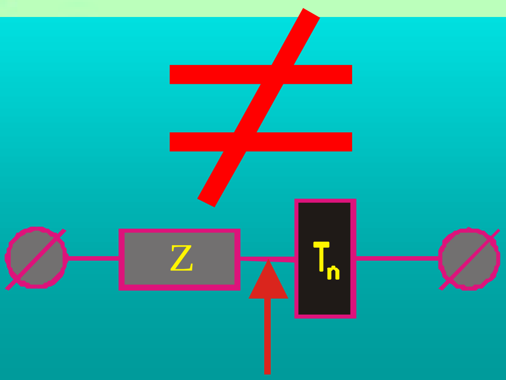


- Weak means: $G >> G_Q$
- Keldysh action: path integral
- Applying the same philosophy: concentrate on log corrections to elastic scattering
- "functional" RG Skvortsov 2001
- Valid in energy window

$$\hbar / t_{escape} \le E \le \hbar / RC$$

$$\frac{dT_n}{d\log(1/E)} = -a \cdot T_n (1 - T_n)$$

$$a = \frac{2}{\sum_{n,j} T_n^{(j)}}$$

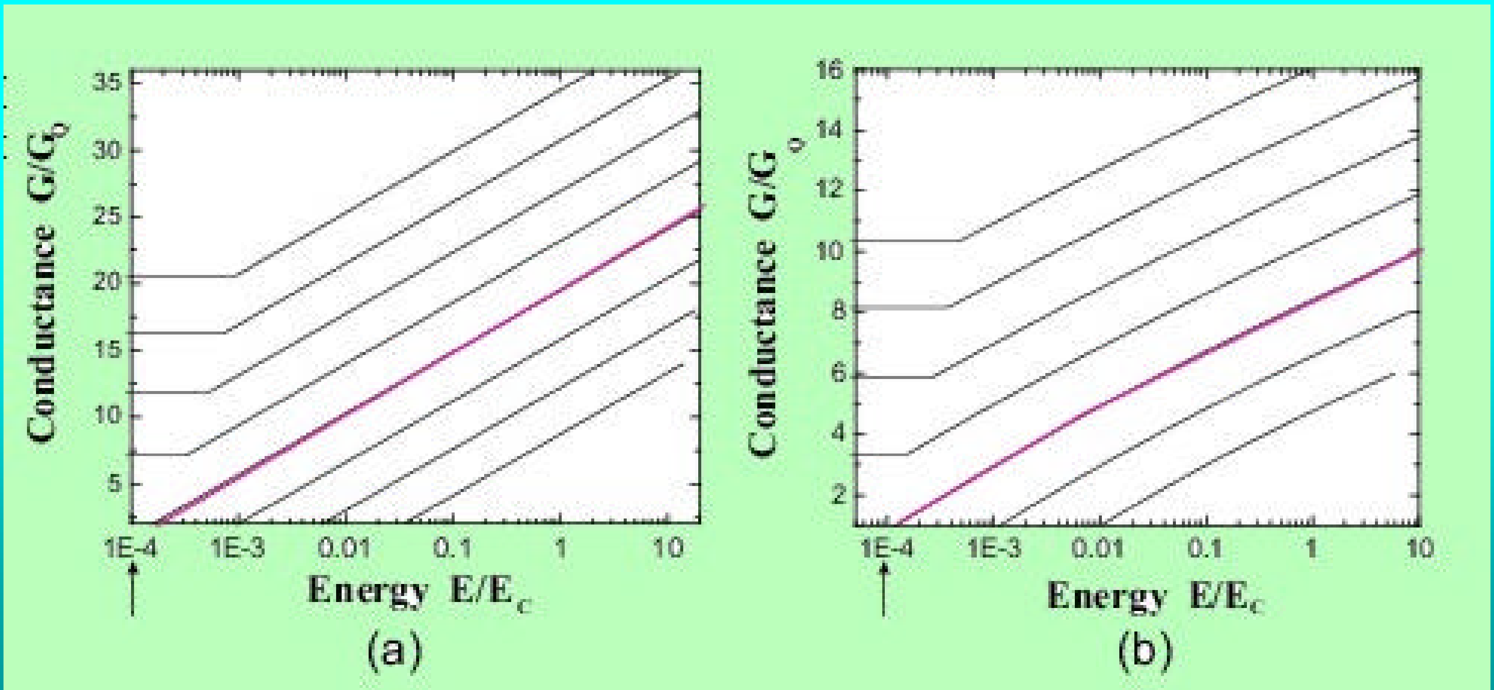


Blockade or no blockade?

Energy goes down ?

- A: conductance hits G_Q
- B: energy hits E_th

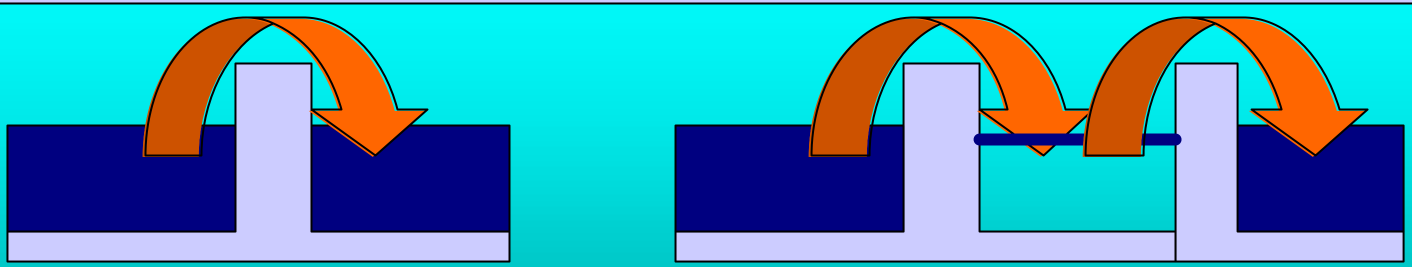
$$\tilde{E}_C \propto \prod_n R_n^{1/2} \cdot E_C$$



Synthesis and conclusions

- No interaction: variety of possible conduction types
- Interaction reveals important "law"
- Two types of conduction:
  - Direct tunneling
  - (symmetric) resonant tunneling

a



$\frac{a}{2}$

- Many conductors do have a "middle"
- Disorder brings symmetric resonances