Density of States in an SNS Junction with Current

P.M. Ostrovsky[1], Yu.V. Nazarov[2]

with a considerable help from

M.A. Skvortsov[1] and M.V. Feigel'man[1]

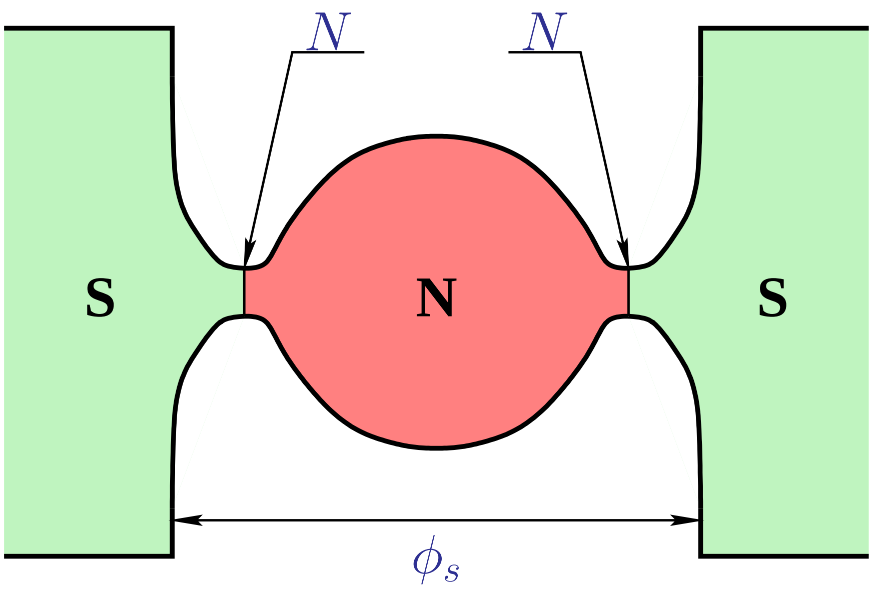
[1] *Landau Institute, Moscow, Russia*

[2] *Department of Applied Physics, Delft University, The Netherlands*

Outline

- Problem definition
- Qualitative picture
- Semiclassical approach
- Sigma-model
- Results

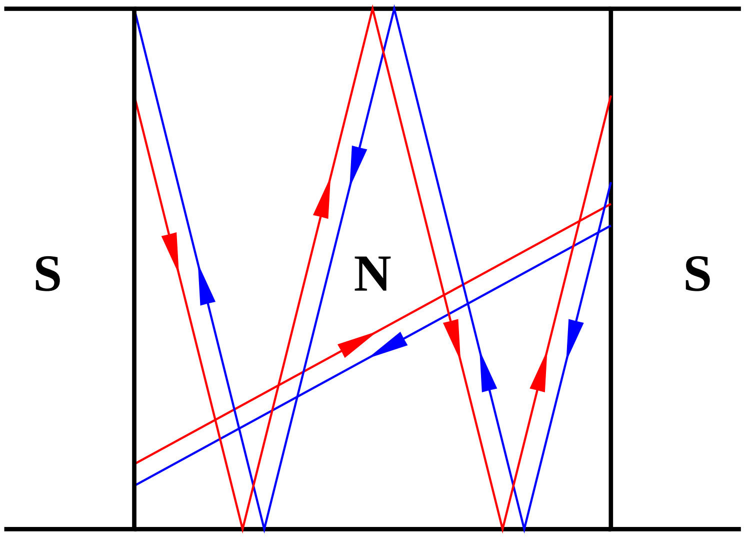
Problem definition



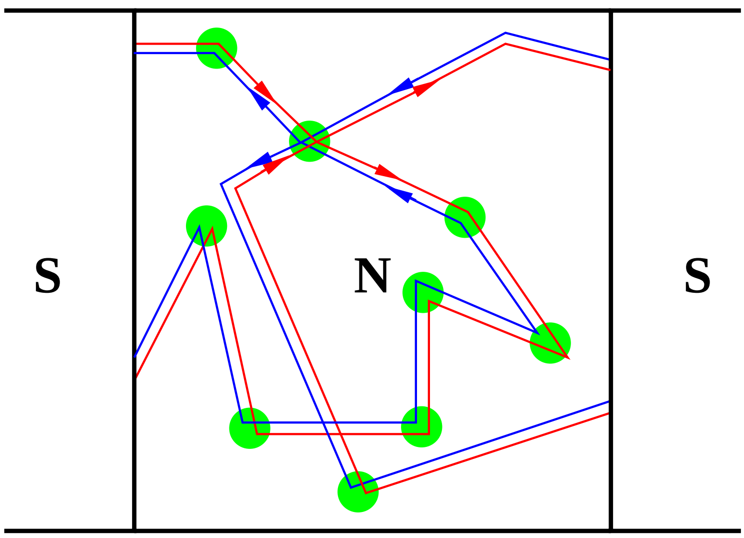
- Symmetrical junction
- Dirty limit: $l \ll L$
- Point contacts — 0D limit
- Robust superconductivity in the leads: $\xi \ll L$
- Large conductance: $N \gg 1$

Qualitative picture

Integrable dynamics: Andreev states with arbitrary low energy exist in a clean rectangular SNS junction. Density of states decreases linearly to zero at Fermi energy.



Chaotic dynamics: low-energy Andreev states in a dirty SNS junction are completely suppressed by interference. Density of states acquire a minigap.



Usadel equation

$$D\nabla\left(\hat{g}_E(\mathbf{r})\nabla\hat{g}_E(\mathbf{r})\right) + iE[\hat{\tau}_z, \hat{g}_E(\mathbf{r})] = 0, \qquad \hat{g}_E^2(\mathbf{r}) = 1.$$

Density of states

$$\langle\rho(E;\mathbf{r})\rangle = \nu\,\mathrm{Re}\,\mathrm{Tr}(\hat{\tau}_z\hat{g}_E(\mathbf{r})).$$

The Usadel action

$$\mathcal{S}_U[\hat{g}] = \pi\nu\int d\mathbf{r}\,\mathrm{Tr}\left(D(\nabla\hat{g})^2 + 4iE\hat{\tau}_z\hat{g}\right) - \frac{N}{2}\sum_{j=1,2}\log(1+\hat{g}_s^{(j)}\hat{g}).$$

Angle parametrization

$$\hat{g} = \hat{\tau}_z\cos\theta + (\hat{\tau}_x\cos\phi + \hat{\tau}_y\sin\phi)\sin\theta.$$

0D equation
$(\theta = \pi/2 + i\psi,\ \gamma = \cos(\phi_s/2),\ \eta = \pi E/(N\delta))$

$$\eta\cosh\psi = \frac{\gamma\sinh\psi}{1+\gamma\cosh\psi}.$$

Integral density of states

$$\langle\rho\rangle = \frac{2}{\delta}\,\mathrm{Im}\sinh\psi.$$

Minigap

The minigap is determined by the maximum of the function

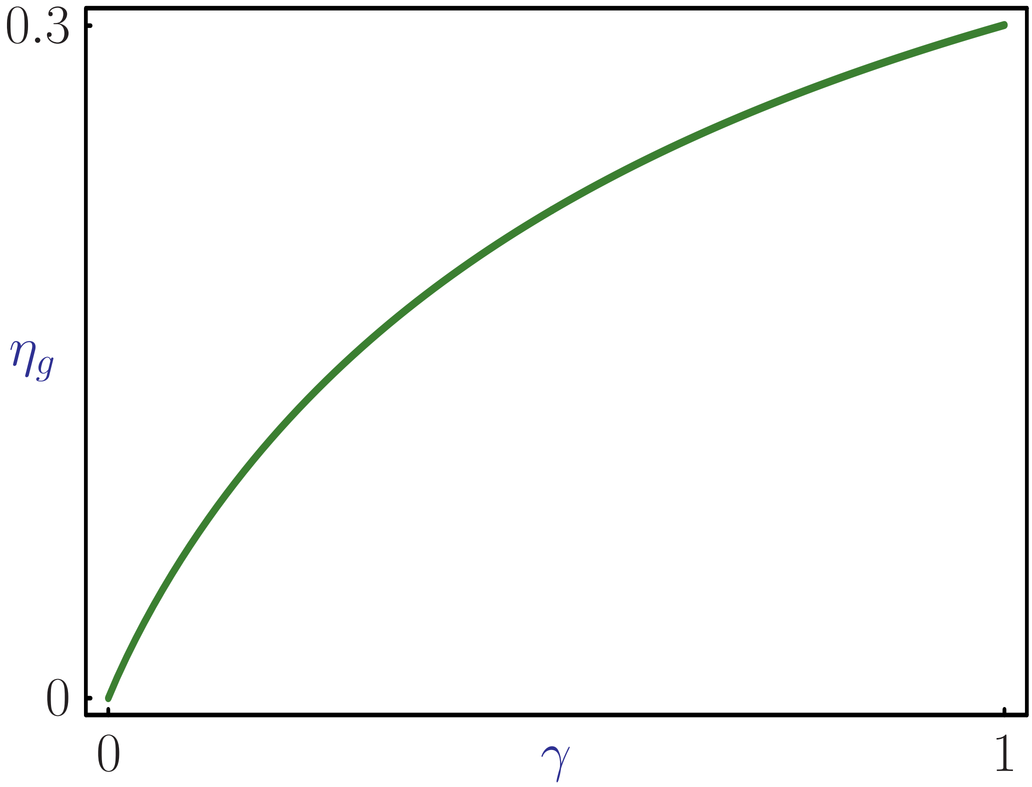
$$\eta = \frac{\gamma \tanh \psi}{1 + \gamma \cosh \psi}.$$

At $\gamma = 1$ (zero phase difference)

$$\eta_g = \left(\frac{1+\sqrt{5}}{2} \right)^{-5/2} \approx 0.300.$$

At $\gamma \ll 1$ (phase difference close to π)

$$\eta_g \approx \gamma - \frac{3}{2}\gamma^{5/3}.$$



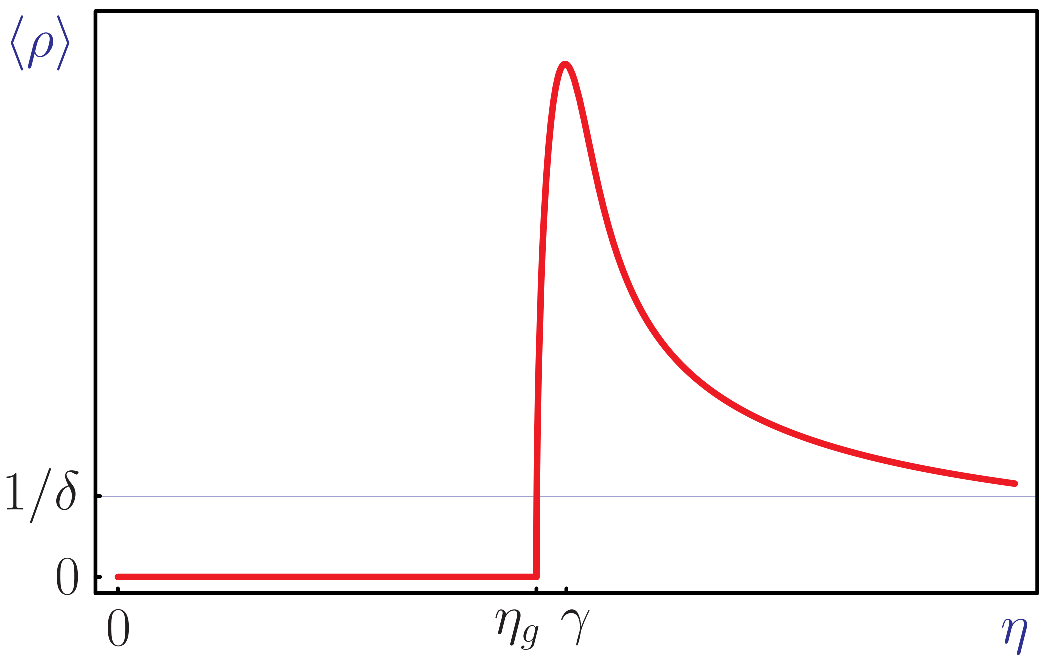
Density of states

At $\gamma \ll 1$ the angle ψ is large $(P = e^{\psi})$

$$\eta = \gamma\left(1 - \frac{2}{P^2} - \frac{\gamma P}{2}\right), \qquad \langle\rho\rangle = \frac{1}{\delta}\,\mathrm{Im}\,P.$$

Three ranges of the energy above the gap

- $\langle\rho\rangle \approx \sqrt{\frac{8}{3}}\frac{\sqrt{\eta-\eta_g}}{\gamma^{7/6}\delta}$, $\quad 0 < \eta - \eta_g \ll \gamma^{5/3}$
- $\langle\rho\rangle \approx \frac{1}{\delta}\sqrt{\frac{2\gamma}{\eta-\eta_g}}$, $\quad \gamma^{5/3} < \eta - \eta_g \ll \gamma$
- $\langle\rho\rangle \approx \frac{1}{\delta}$, $\quad \eta \gg \eta_g$



$\gamma = 0.01,\ \eta_g = 0.0093$

σ-model

$$\mathcal{S}[Q] = \frac{N}{2}\,\mathrm{STr}\left[i\eta\Lambda Q - \sum_{j=1,2}\log\left(1 - \frac{\Gamma}{2} + \frac{\Gamma}{2}\{Q^{(j)}, Q\}\right)\right].$$

Supermatrix $Q \in$ "PH" $\otimes$ "TR" $\otimes$ "FB".

Density of states

$$\langle\rho\rangle = \frac{1}{4\delta}\,\mathrm{Re}\int \mathrm{STr}(k\Lambda Q)e^{-\mathcal{S}[Q]}\mathcal{D}Q.$$

The integration runs over the manifold determined by:

- $Q^2 = 1$
- $\bar{Q} = Q$

Parametrization for the commuting part of Q

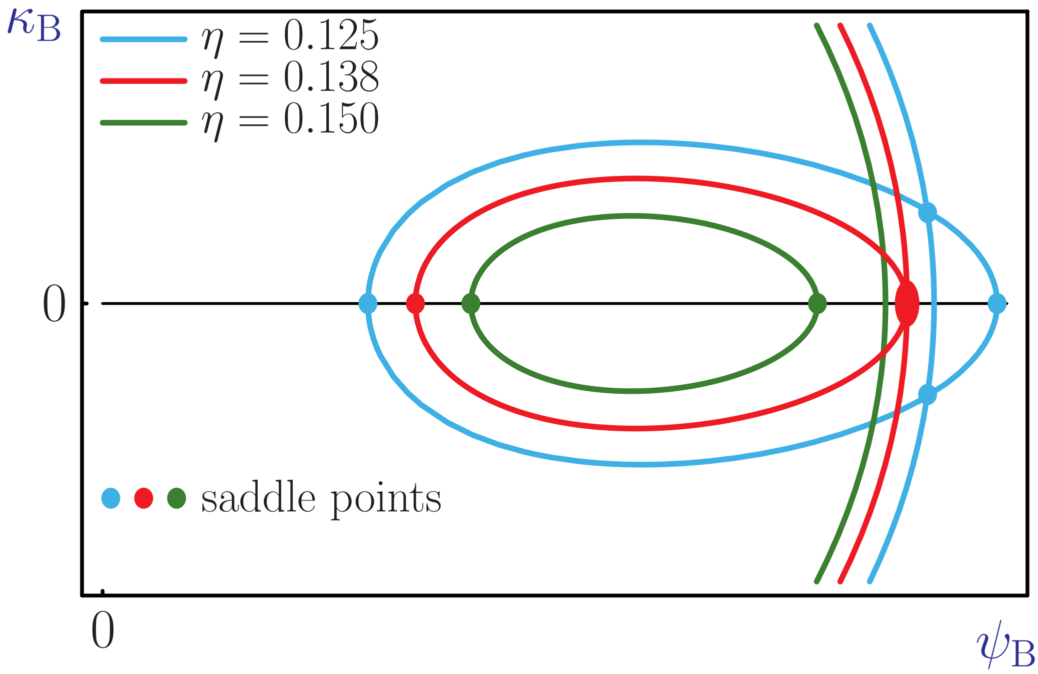
$$\begin{aligned} Q^{\mathrm{FF}} &= \hat{\sigma}_z\hat{\tau}_z\cos\theta_{\mathrm{F}} + \hat{\tau}_x\sin\theta_{\mathrm{F}}, \\ Q^{\mathrm{BB}} &= [\hat{\sigma}_z\cos k_{\mathrm{B}} + \hat{\tau}_z\sin k_{\mathrm{B}}(\hat{\sigma}_x\cos\chi_{\mathrm{B}} + \hat{\sigma}_y\sin\chi_{\mathrm{B}})] \\ &\quad\times[\hat{\tau}_z\cos\theta_{\mathrm{B}} + \hat{\sigma}_z\hat{\tau}_x\sin\theta_{\mathrm{B}}]. \end{aligned}$$

σ-model action $\mathcal{S} = \mathcal{S}^{\mathrm{FF}} - \mathcal{S}^{\mathrm{BB}}$,

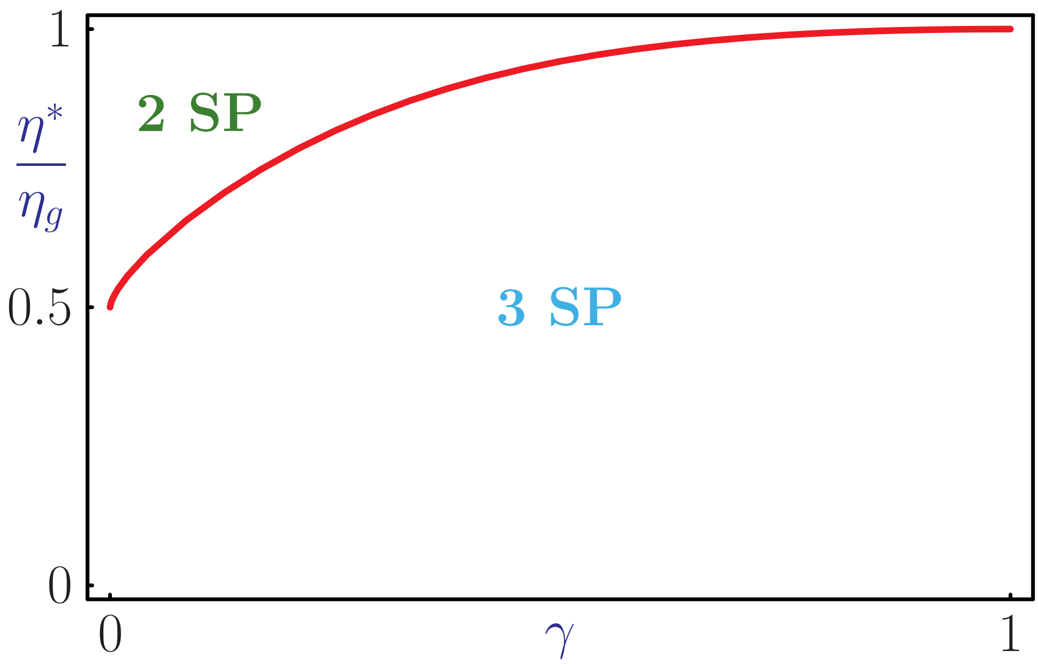
$$\begin{aligned} \mathcal{S}^{\mathrm{FF}} &= 2N\,[i\eta\cos\theta_{\mathrm{F}} - \log(1 + \gamma\sin\theta_{\mathrm{F}})], \\ \mathcal{S}^{\mathrm{BB}} &= 2N\,[i\eta\cos\theta_{\mathrm{B}}\cos k_{\mathrm{B}} - \log(\cos k_{\mathrm{B}} + \gamma\sin\theta_{\mathrm{B}})]. \end{aligned}$$

Saddle points

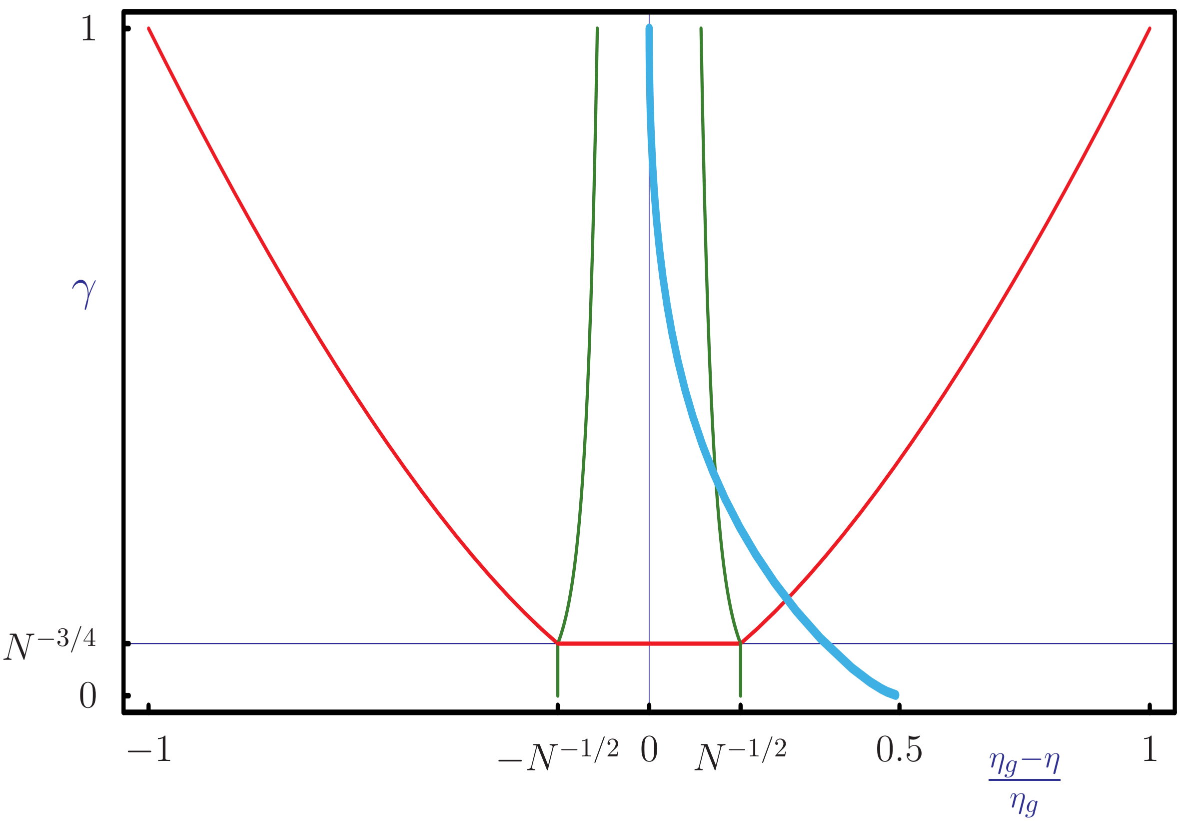
Saddle points in BB-sector ($\theta = \pi/2 + i\psi$, $k = i\kappa$):



Critical energy



"Phase diagram"



Universal result

Scaling:

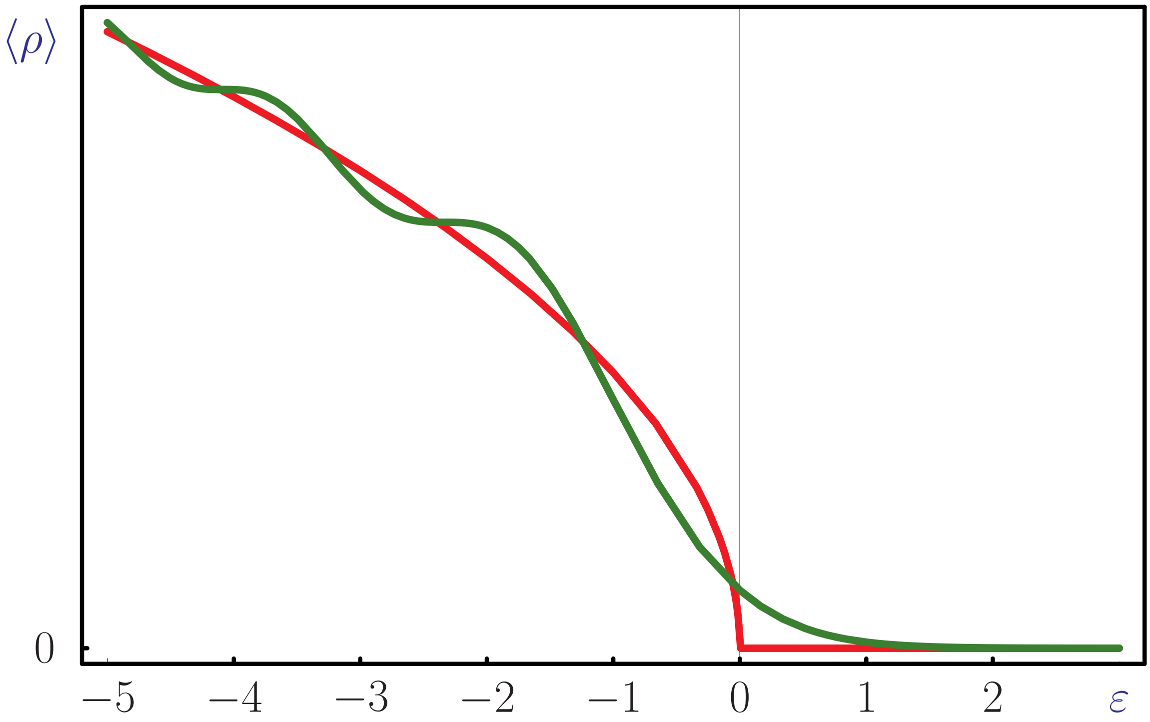
$$\Delta_g = \frac{(3N)^{1/3}\gamma^{7/9}}{2\pi}\delta, \qquad \varepsilon = \frac{E_g - E}{\Delta_g}.$$

The density of states

$$\langle\rho\rangle = \frac{1}{\Delta_g}\left(-\varepsilon\,\mathrm{Ai}^2(\varepsilon) + [\mathrm{Ai}'(\varepsilon)]^2\right).$$

Limiting cases:

- $\langle\rho\rangle = \dfrac{e^{-\frac{4}{3}\varepsilon^{3/2}}}{8\pi\Delta_g\varepsilon},$ $\qquad \varepsilon \gg 1$
- $\langle\rho\rangle = \dfrac{\sqrt{|\varepsilon|}}{\pi\Delta_g} - \dfrac{\cos\left(-\frac{4}{3}|\varepsilon|^{3/2}\right)}{4\pi\Delta_g\varepsilon},$ $\qquad \varepsilon \ll -1$



Nonuniversal result

Density of states as a function of $\varepsilon = \frac{\eta_g - \eta}{\eta_g}$

- $\frac{1}{\delta}\sqrt{\frac{2E_g}{E - E_g}}, \qquad -1 \ll \frac{E_g - E}{E_g} \ll -\gamma^{2/3},$

- $\frac{\exp\left[-\frac{N}{4}\left(\frac{E_g - E}{E_g}\right)^2\right]}{\delta N \gamma^{3/2}\left(2\frac{E_g - E}{E_g}\right)^{1/4}}, \qquad \gamma^{2/3} \ll \frac{E_g - E}{E_g} \ll 1$

Exact expression

Notations:

$$\bar{\gamma} = 2N^3\gamma^4$$

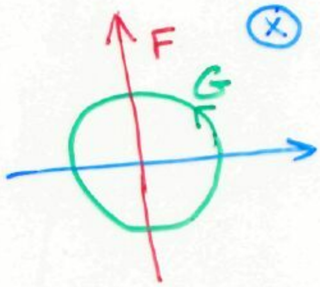
$$\bar{\varepsilon} = \frac{N^2\gamma^2}{2}\,\frac{E_g - E}{E_g}$$

Two special functions: $F(\bar{\gamma};\bar{\varepsilon})$ and $G(\bar{\gamma};\bar{\varepsilon})$ solutions of the equation

$$\pm\bar{\gamma}u''' + \bar{\varepsilon}u'' = u$$

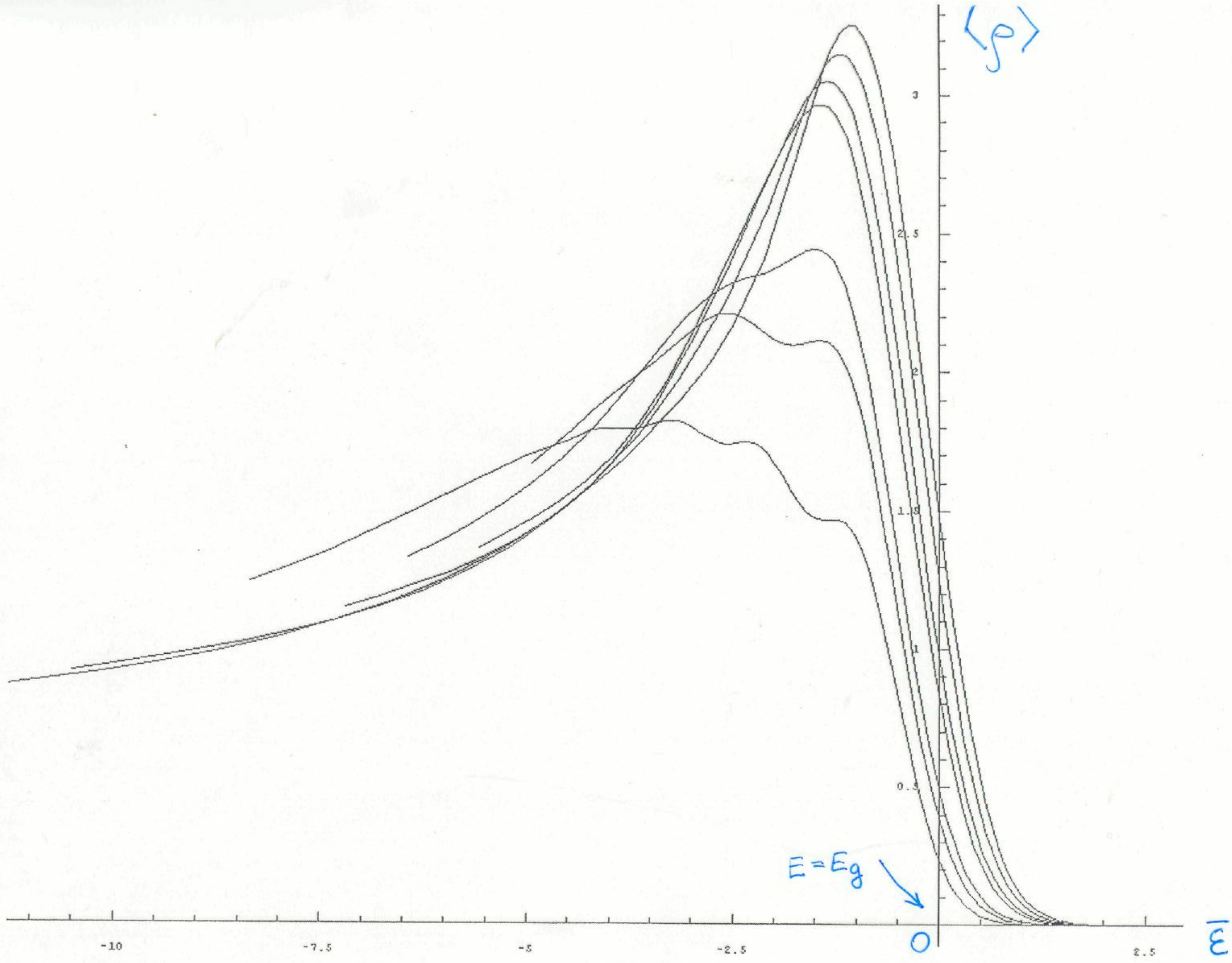
Integral representation:

$$\int dx\, \exp\left[x + \frac{\bar{\varepsilon}}{x} \pm \frac{\bar{\gamma}}{2x^2}\right]$$



Density of states:

$$\langle\rho\rangle = \frac{N\gamma}{4\pi\delta}\left[4F''G - 4F'G' + 4FG'' + F'G'' + F''G' - 4\bar{\varepsilon}F''G''\right]$$


⟨ρ⟩
3
2.5
2
1.5
1
0.5
E = Eg
0
-10
-7.5
-5
-2.5
2.5
ε̄