# Adiabatic Transport of Cooper Pairs in Josephson Junction Circuits

**J. P. Pekola, J. M. Kivioja**
***Low Temperature Laboratory, Helsinki University of Technology, Finland***
**J. J. Toppari**
***University of Jyväskylä, Finland***
**A. O. Niskanen**
***VTT, Espoo, Finland***
**O.Buisson, F. Balestro, J. Claudon, F. Hekking**
***CNRS, Grenoble, France***

Kylmälaboratorio
Lågtemperaturlaboratoriet
Low Temperature Laboratory

# Contents

1. Adiabatic transport (phase biased circuits, environment, Landau-Zener limitation)
2. Experiments on a 3-junction Cooper pair pump
3. Flux assisted Cooper pair ”sluice”
4. Escape measurements by a hysteretic DC-SQUID

# Adiabatic transport of Cooper pairs

If the Hamiltonian of the system varies slowly enough, we may study its behaviour using the adiabatic approximation of quantum mechanics.

$\hat{H} = \hat{H}_C + \hat{H}_J$ varies due to gate (or flux) operations.

Charge pumped in a loop of control parameters is then

$$Q_P = 2\hbar \Im\mathrm{m} \left[ \sum_{n \neq m} \oint \frac{(\hat{I}_l)_{mn}}{E_m - E_n} \langle n | \partial_{\vec{q}} m \rangle \cdot d\vec{q} \right]$$

($m$ = 0 for transport in the ground state)

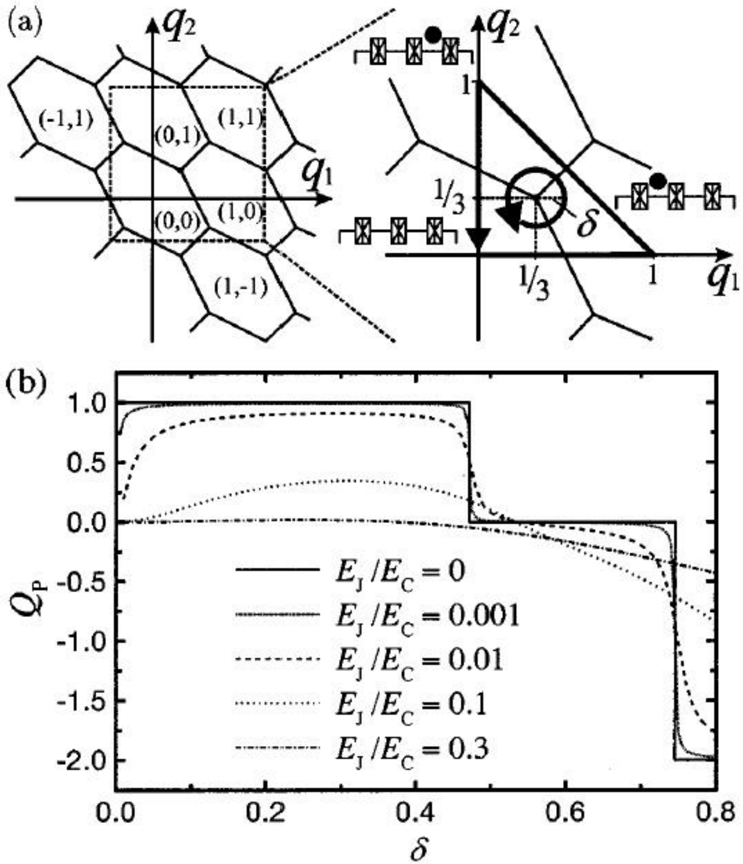
(J. P., J. Toppari, M. Aunola, M. Savolainen, D. Averin, PRB **60**, 9931 (1999).)

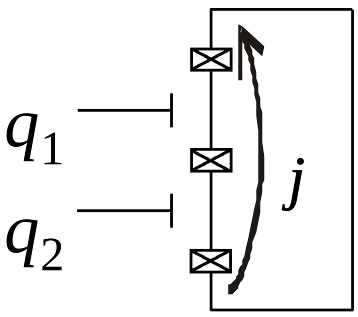
$Q_P$ is related to the Berry phase $\gamma$ associated with the adiabatic loop as

$$Q_{\mathrm{P}}/(2e) = -\frac{d\gamma}{d\varphi}$$

(M. Aunola and J. Toppari cond-mat/0303176)

# Perfectly phase-biased adiabatic CPP




$$Q_P = 2\hbar \Im\mathrm{m}\left[\sum_{n\neq m}\oint \frac{(\hat{I}_l)_{mn}}{E_m - E_n}\langle n|\partial_{\vec{q}} m\rangle \cdot d\vec{q}\right]$$

$$Q_{\mathrm{P}}/(2e) \simeq 1 - 9E_{\mathrm{J}}/E_{\mathrm{C}}\cos\varphi$$

**(triangular path)**

# Adiabatic transport vs. Landau-Zener band-crossing

$H = H_J + H_C$

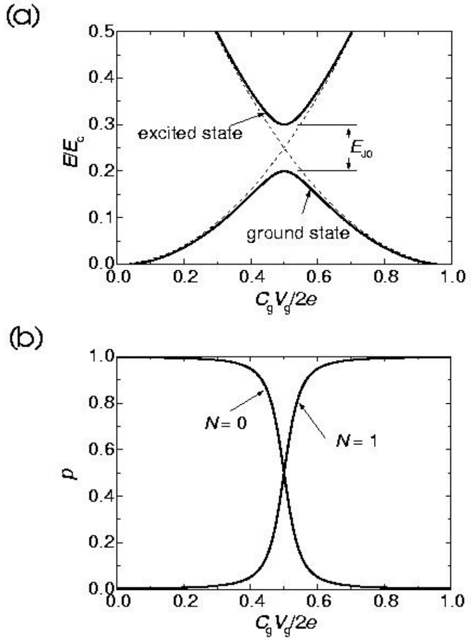
Adiabatic approximation is valid, if

$$|\langle 1|\dot{0}\rangle| << (E_1 - E_0)/\hbar$$

Band-crossing (LZ) becomes important, if this condition is violated.

For periodic gate operation this happens at

$$f_{LZ} \sim \frac{E_J^2}{\hbar E_C}$$



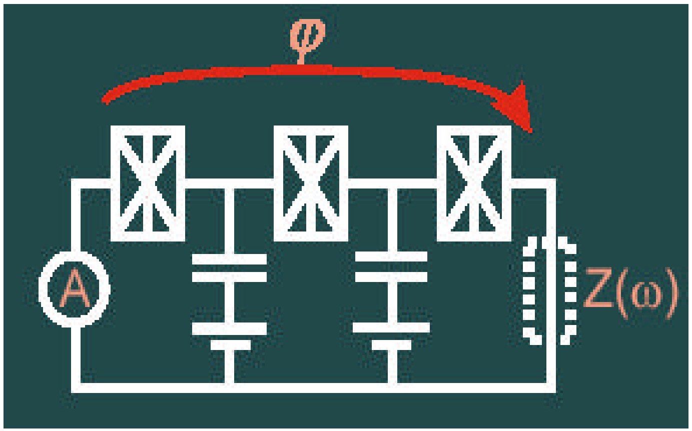
# Phase fluctuations due to environment

FDT:
$$J(t) = 2\int_0^\infty \frac{d\omega}{\omega}\frac{\mathrm{Re}\, Z_t(\omega)}{R_K}$$
$$\times\left\{\coth\left(\frac{\hbar\omega}{2k_BT}\right)[\cos(\omega t)-1]-i\,\sin(\omega t)\right\}$$

$$\Delta\varphi \equiv \varphi(t) - \varphi(0)$$
$$\langle(\Delta\varphi)^2\rangle = -2\,\mathrm{Re}\,J(t)$$

**J. P. and J. Toppari, PRB 64, 172509 (2001)**
**J. Martinis, S. Nam, J. Aumentado, K. Lang, C. Urbina (2003), discusses general noise-induced decoherence**

# Phase fluctuations due to environment



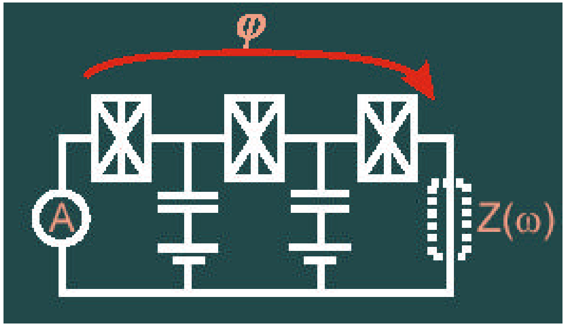

**Simple idea:**

**Determine the fluctuations of** $j$ **(in real time).**

**If** <(D j )$^2$> << **(**p**/2)$^2$, pumping is coherent**

**If** <(D j )$^2$> >> **(**p**/2)$^2$, pumping is incoherent**

**In the latter case:** $Q_{\mathrm{P}}/2e = 1 - 9E_{\mathrm{J}}/E_{\mathrm{C}}\langle\cos\varphi\rangle \to 1$




# Phase fluctuations: results in resistive environment

$T = 0$

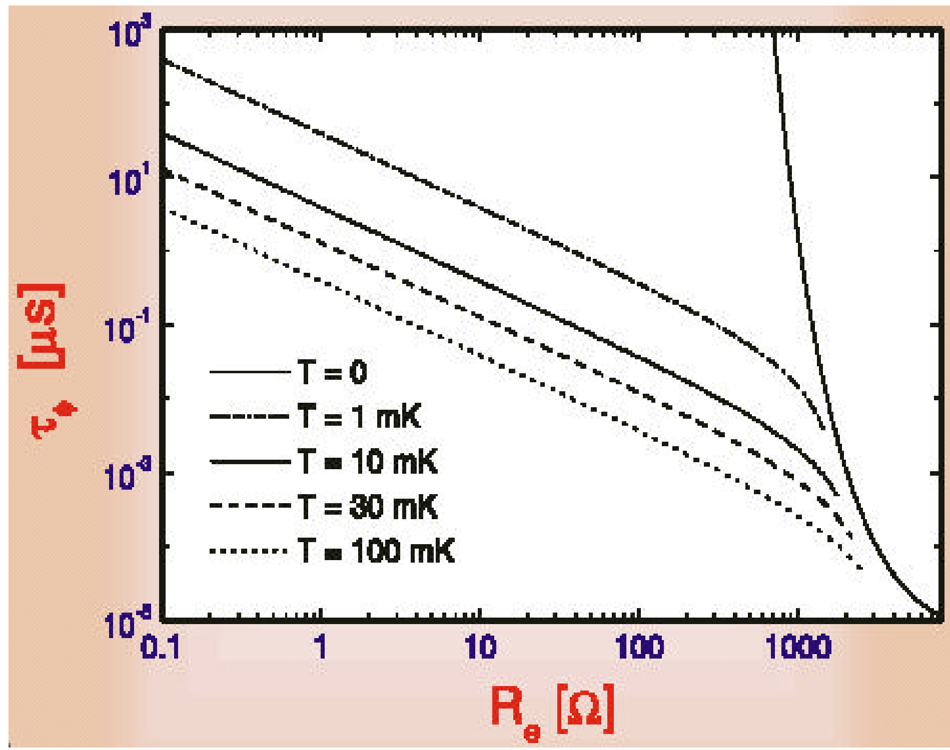
$$\langle(\Delta\varphi)^2\rangle = 4\frac{R_e}{R_K}[\ln(t/\tau)+\gamma]$$

$T > 0$

$$\langle(\Delta\varphi)^2\rangle \simeq 4\frac{R_e}{R_K}\left[\frac{\pi k_B T}{\hbar}t - \ln\left(\frac{2\pi k_B T\tau}{\hbar}\right)+\gamma\right]$$

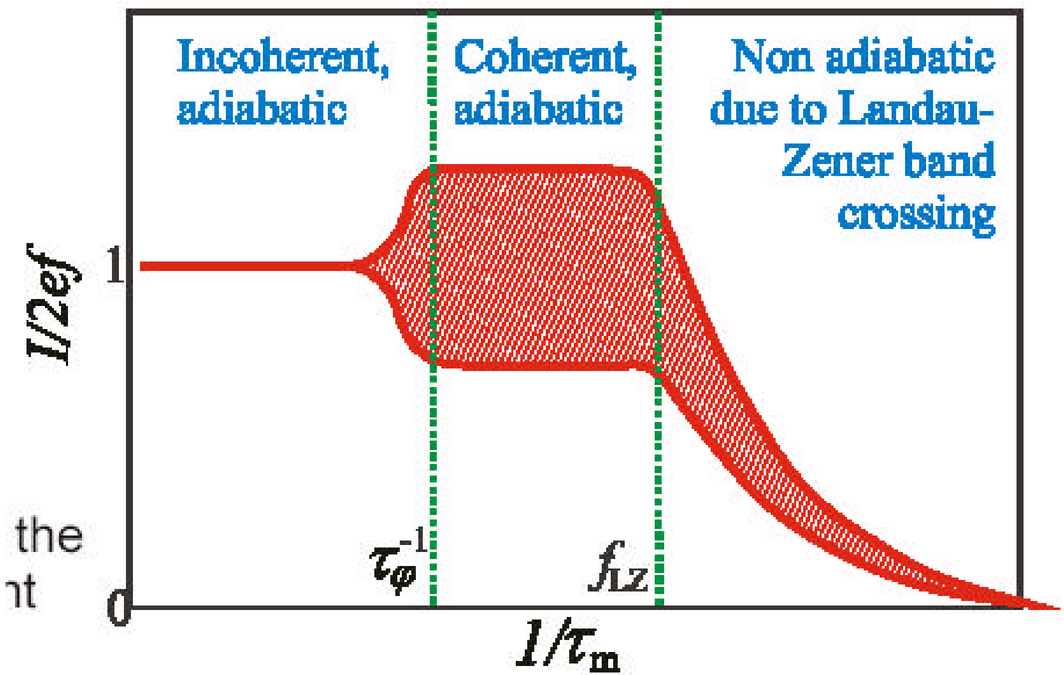
$$\tau_\varphi = \tau\exp\left(\frac{\pi^2}{16}\frac{R_K}{R_e}-\gamma\right) \qquad (T=0)$$

$$\tau_\varphi \simeq \frac{\pi}{16}\frac{\hbar}{k_B T}\frac{R_K}{R_e} \qquad (T>0)$$

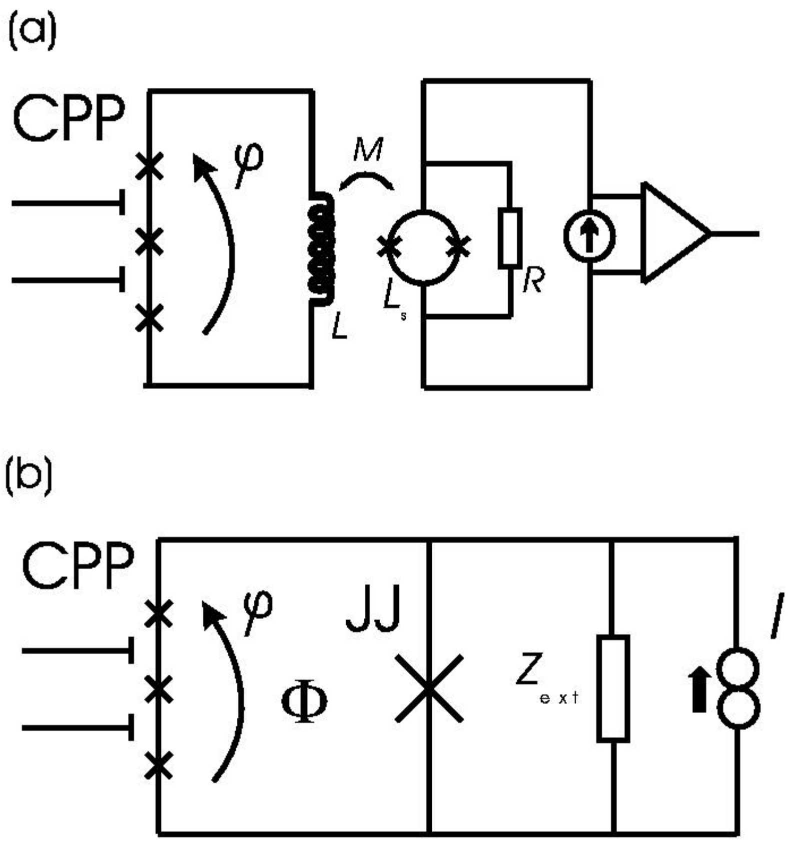



$\langle(\Delta\varphi)^2\rangle = (\pi/2)^2$ at $t = \tau_\varphi$

# Pumping regimes determined by Landau-Zener band-crossing and phase fluctuations



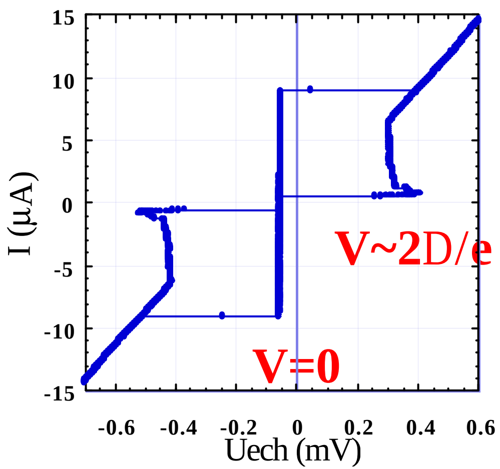
# SQUID amplifier or escape junction for inductive read-out




Current measurement by:

(a) SQUID amplifier

(b) Biased Josephson junction



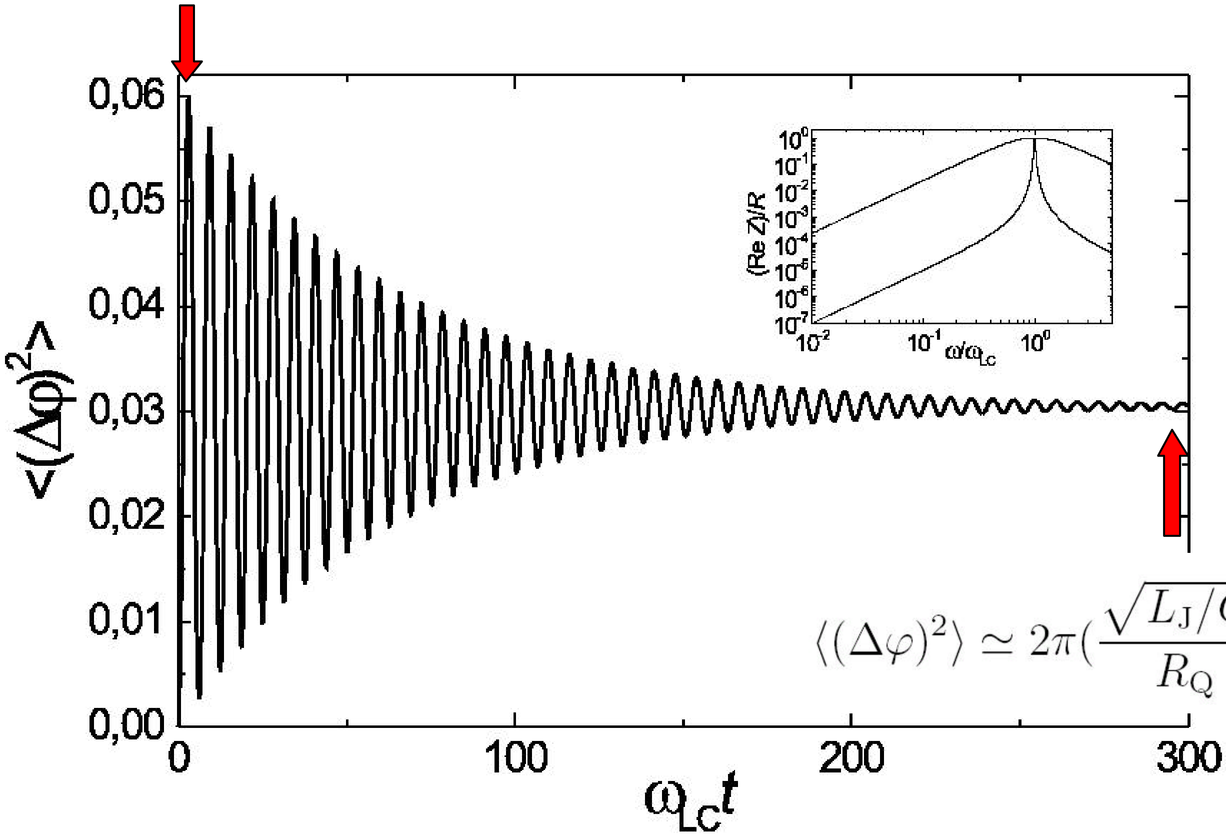

$$Q_{\mathrm{P}}/(2e) = 1 - [9E_{\mathrm{J}}/E_{\mathrm{C}} - (2\pi^2/3)LE_{\mathrm{J}}/\Phi_0^2]\cos\varphi$$

# Inductance limited phase fluctuations (underdamped case)

$$\sqrt{L_\mathrm{J}/C^*_\mathrm{J}}/R \ll 1$$

$$\langle(\Delta\varphi)^2\rangle \simeq 2\pi(\frac{\sqrt{L_\mathrm{J}/C^*_\mathrm{J}}}{R_\mathrm{Q}})\coth(\frac{\hbar\omega_\mathrm{LC}}{2k_\mathrm{B}T})[1-\cos(\omega_\mathrm{LC}t)]$$

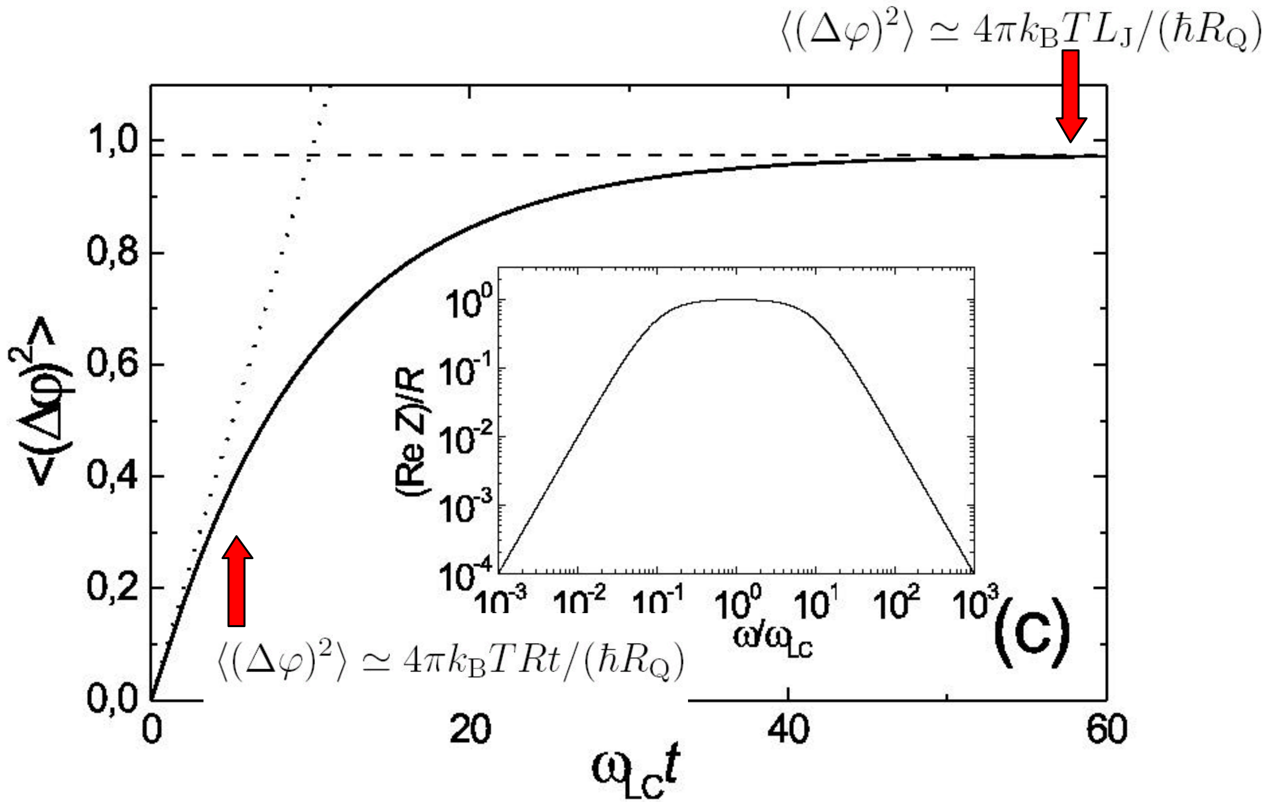
$$t_\mathrm{dec} = 2RC^*_\mathrm{J}$$



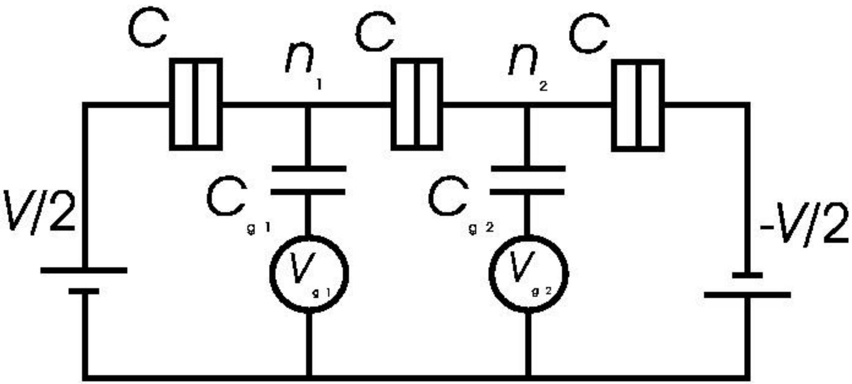

$$\langle(\Delta\varphi)^2\rangle \simeq 2\pi(\frac{\sqrt{L_\mathrm{J}/C^*_\mathrm{J}}}{R_\mathrm{Q}})\coth(\frac{\hbar\omega_\mathrm{LC}}{2k_\mathrm{B}T})$$

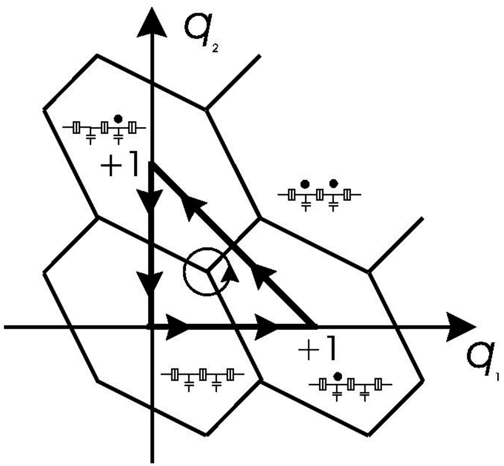
R. Fazio, F. Hekking, J.P., preprint (2003)

# Inductance limited phase fluctuations (overdamped case)



# Pumping of single electrons in a 3-junction pump




$$I \sim 1\mathrm{pA}$$

$$(f \sim 10\mathrm{MHz})$$

$$\delta I / I \sim 0.01$$

H. Pothier, P. Lafarge, C. Urbina, D. Esteve, M. Devoret, EPL 17, 249 (1992).

# Metrological charge pump

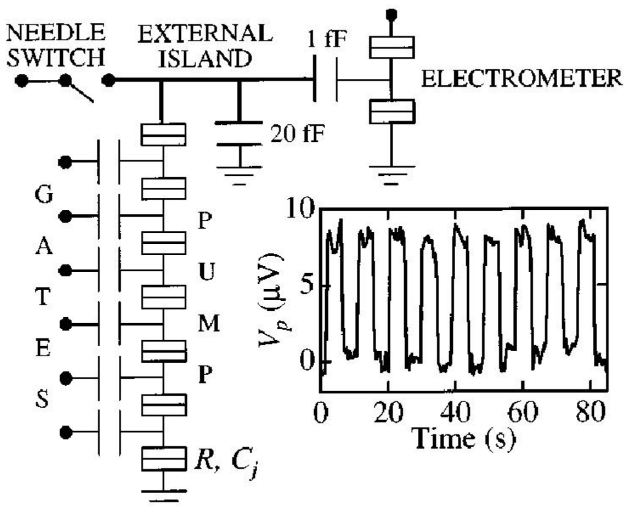
## Accuracy of electron counting using a 7-junction electron pump

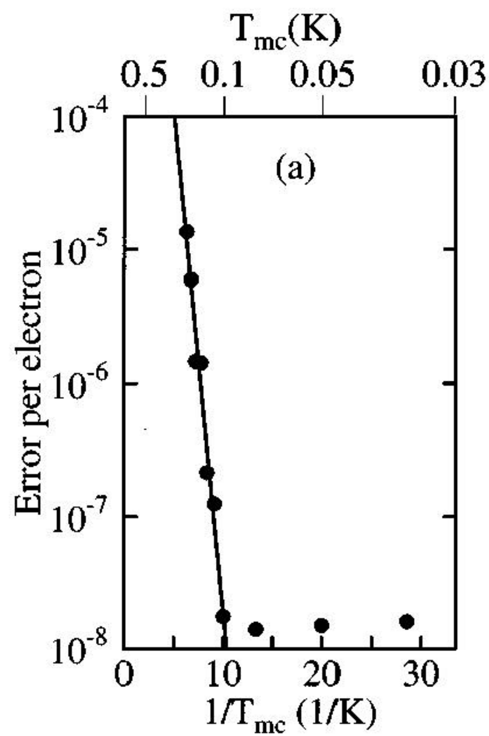
Mark W. Keller[a] and John M. Martinis
*National Institute of Standards and Technology, Boulder, Colorado 80303*

Neil M. Zimmerman
*National Institute of Standards and Technology, Gaithersburg, Maryland 20899*

Andrew H. Steinbach[b]
*National Institute of Standards and Technology, Boulder, Colorado 80303*

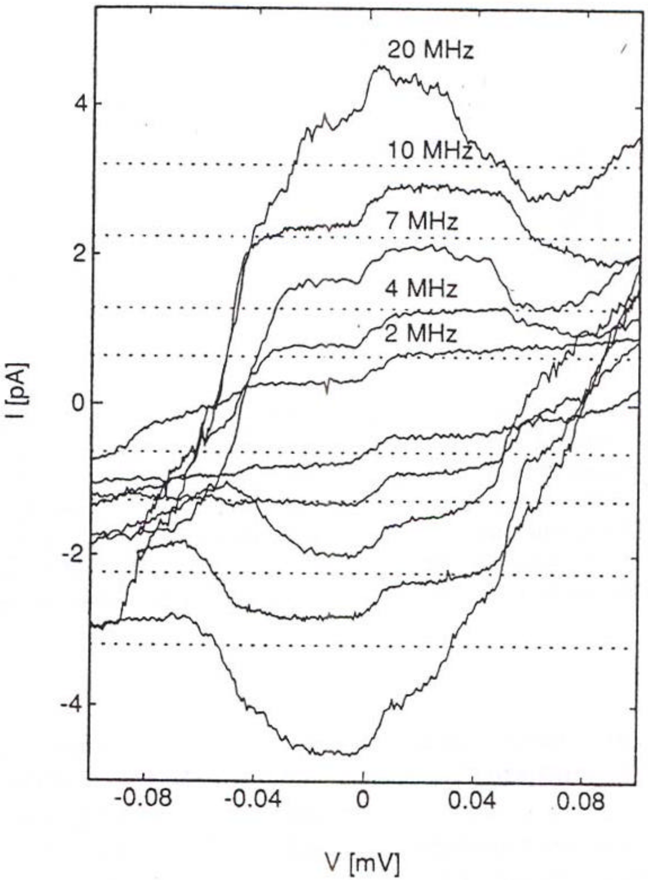
(Received 29 April 1996; accepted for publication 22 July 1996)

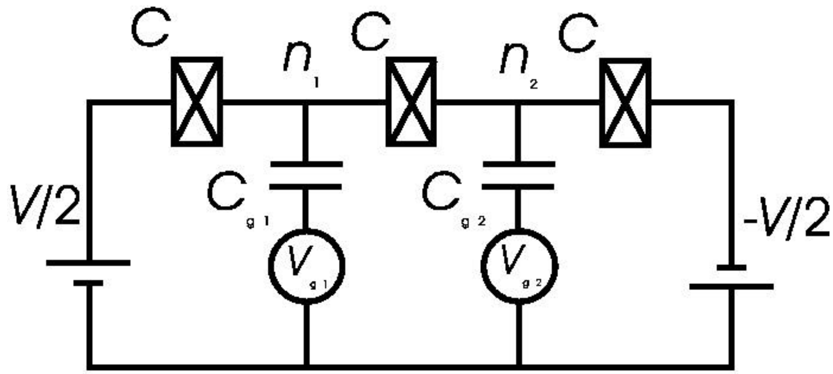



**Appl. Phys. Lett. 69, 1804 (1996).**

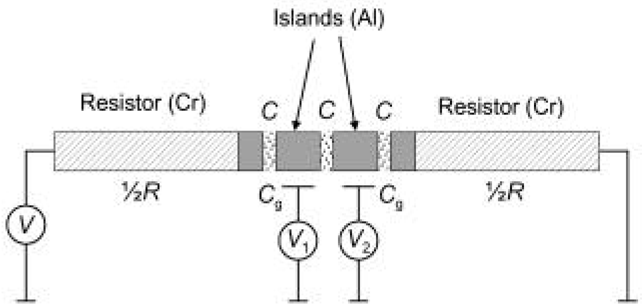
# The first 3 junction CPP

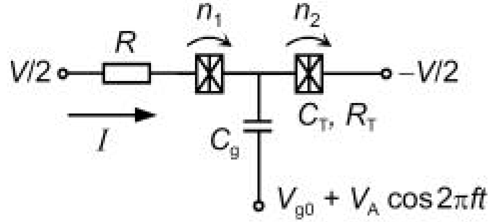


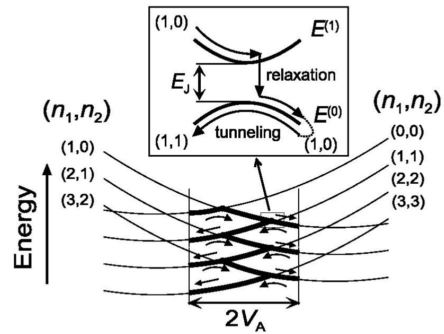



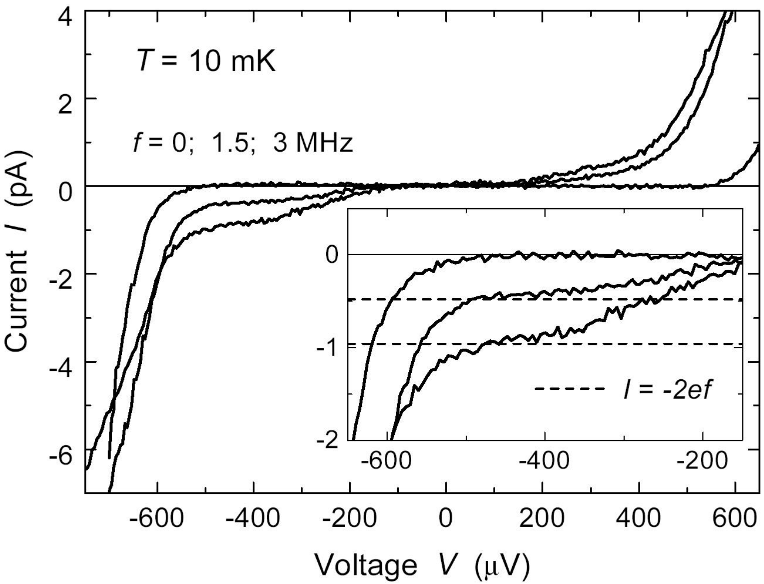

**L. Geerligs et al., Z. Phys. B: Condensed Matter 85, 349 (1991).**

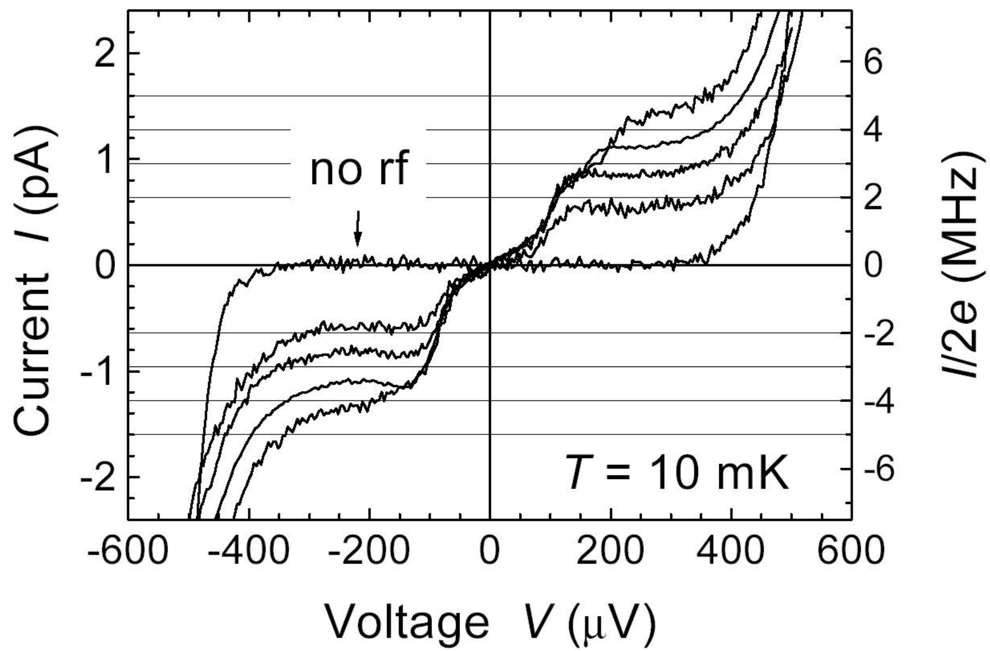
# R-pumps,
Zorin, Lotkhov, Bogoslovsky, Niemeyer (2000, 2001)



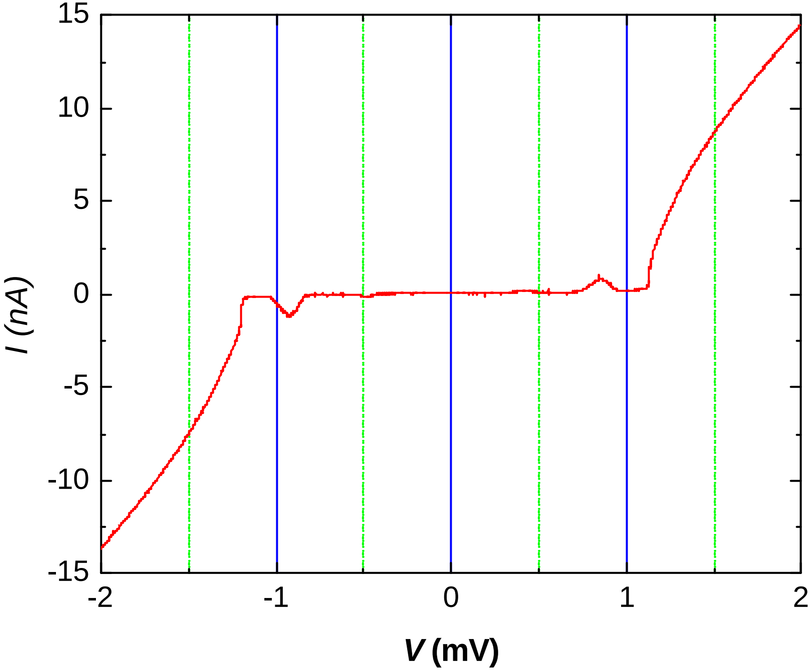



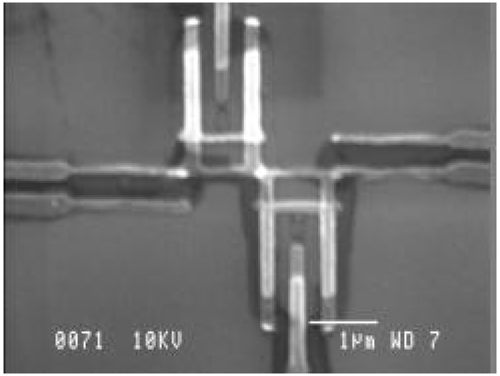



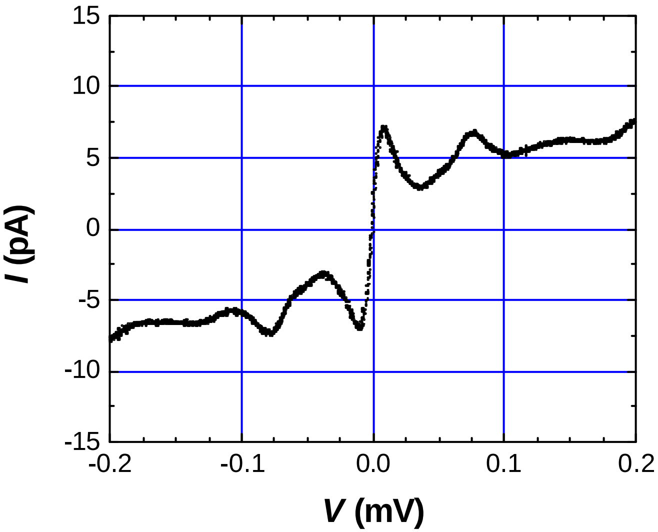



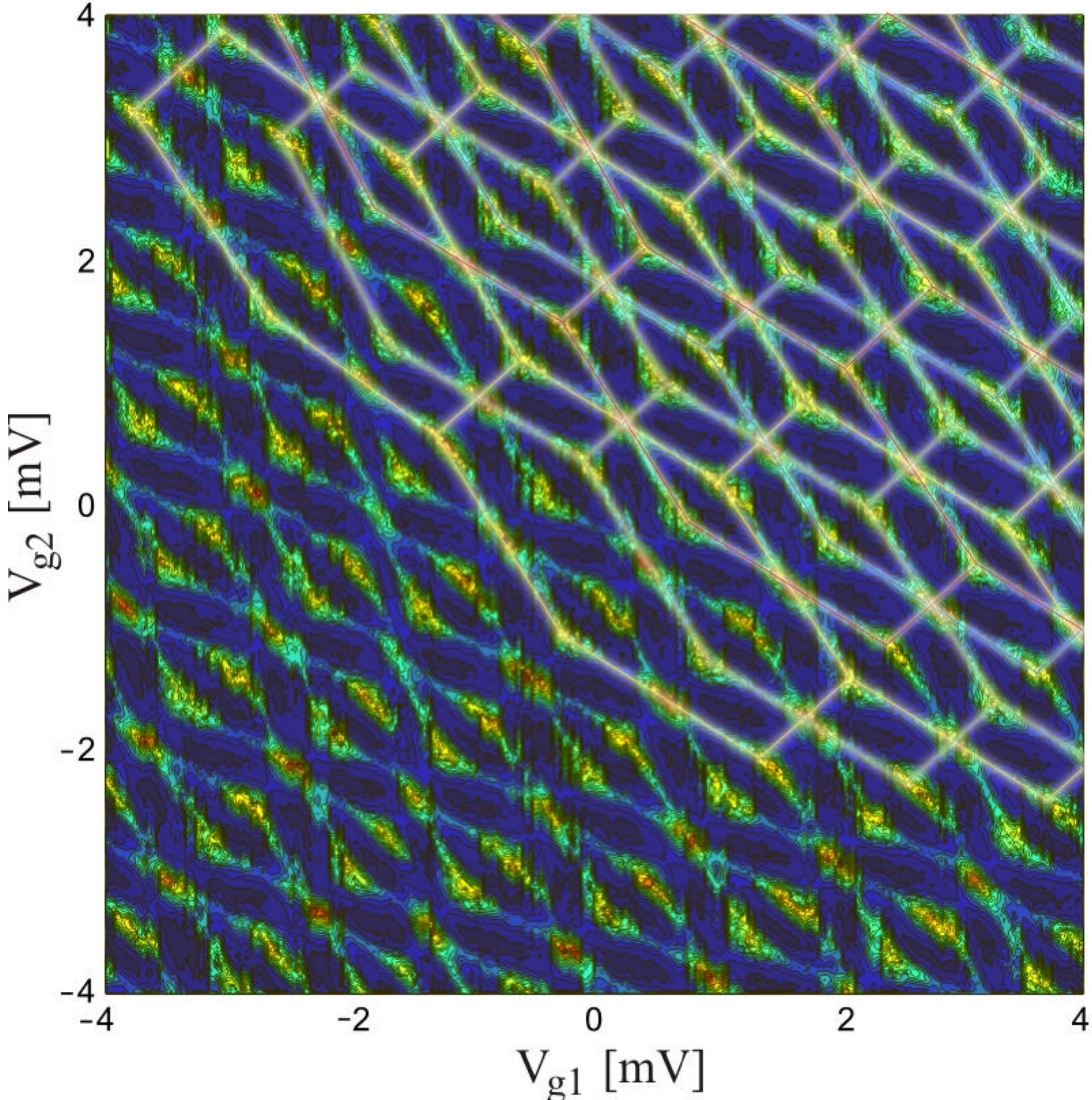
# Cooper pair pump



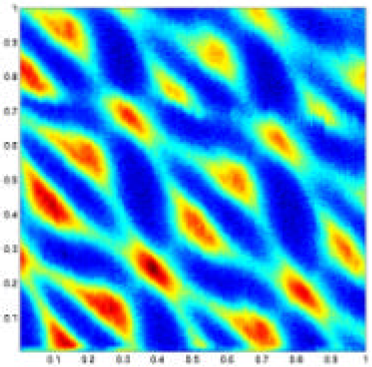



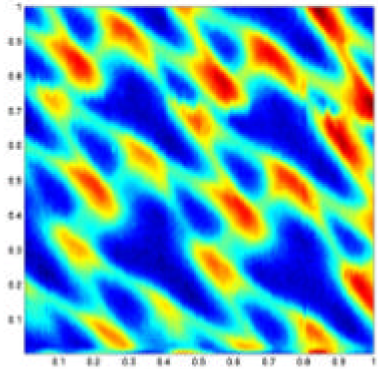
# Experimental determination of the gate dependences



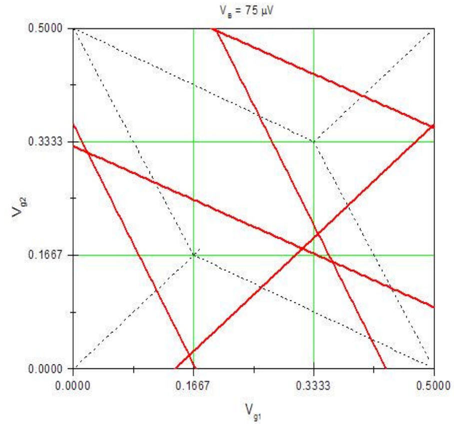
# Gate modulation at different bias points

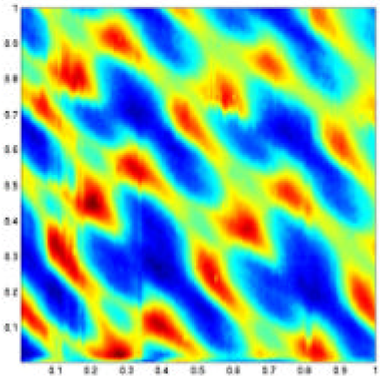


$V$ = 21 mV

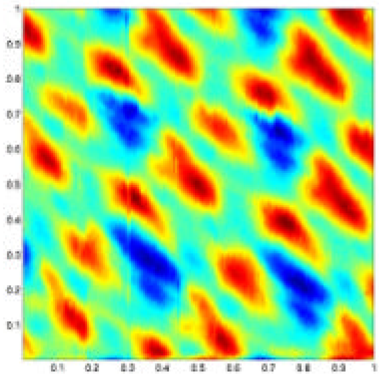


$V$ = 66 mV



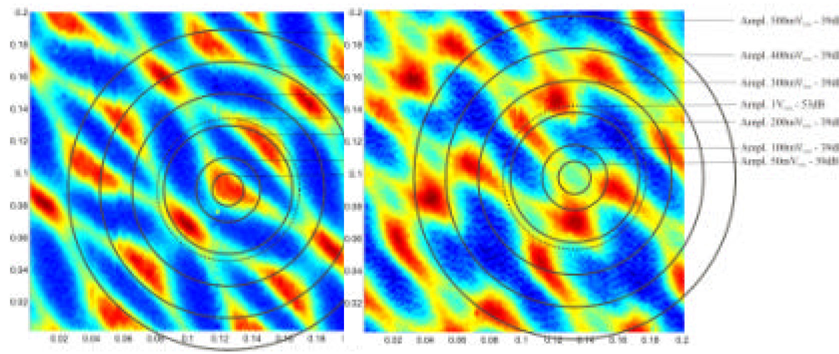



$V$ = 96 mV

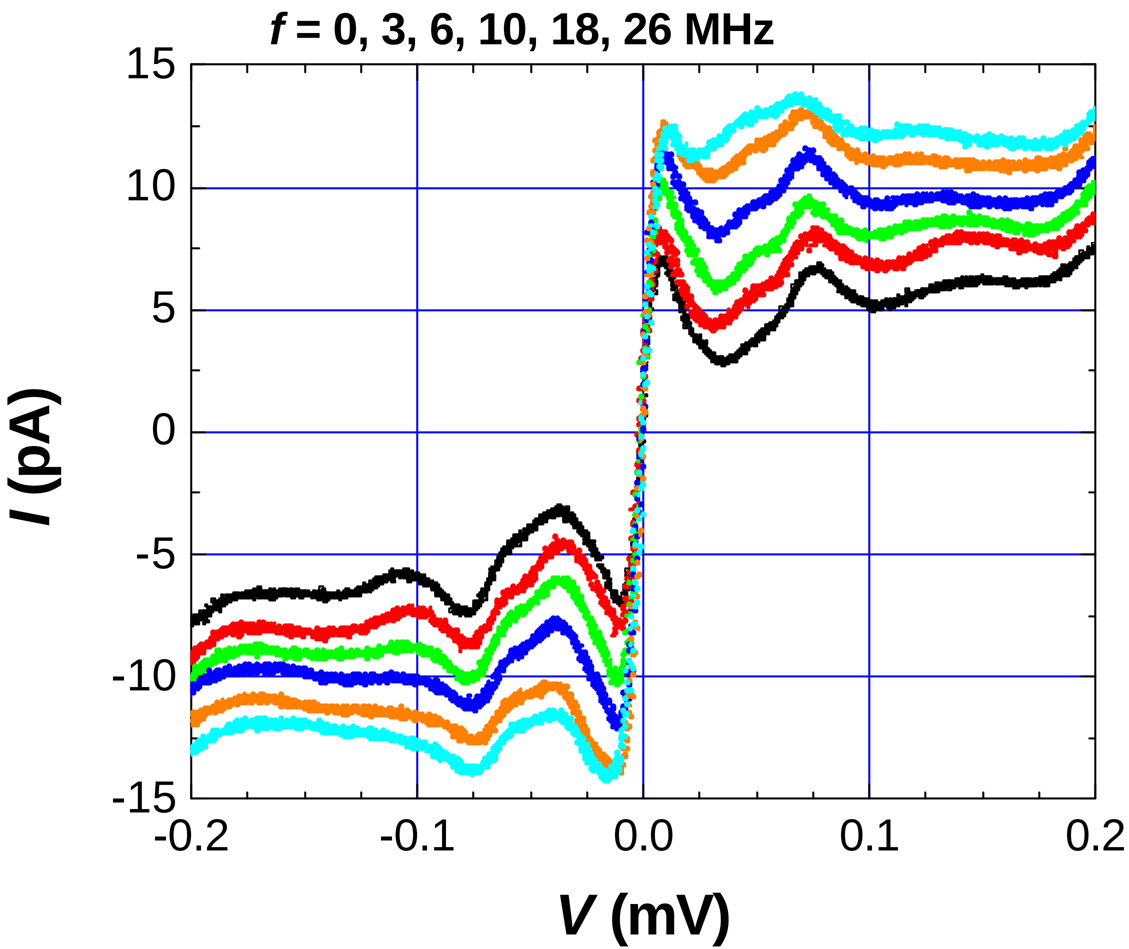


$V$ = 126 mV

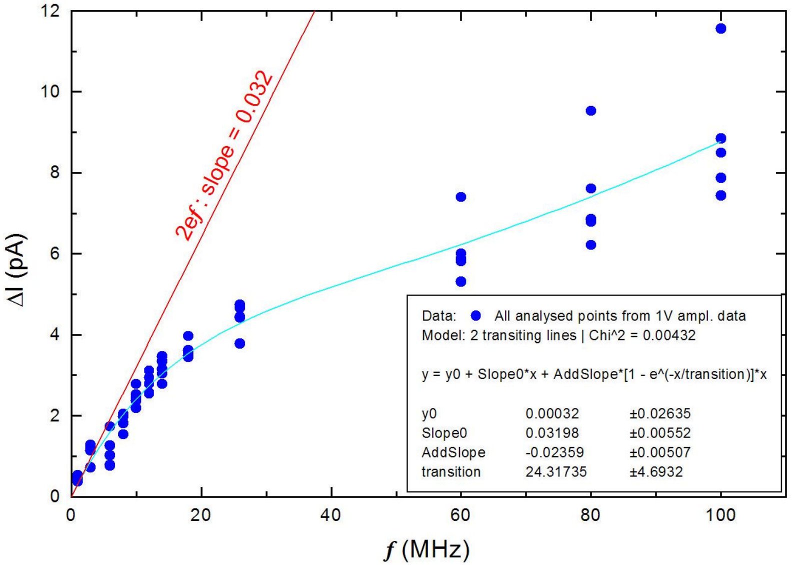
Pumping trajectories:

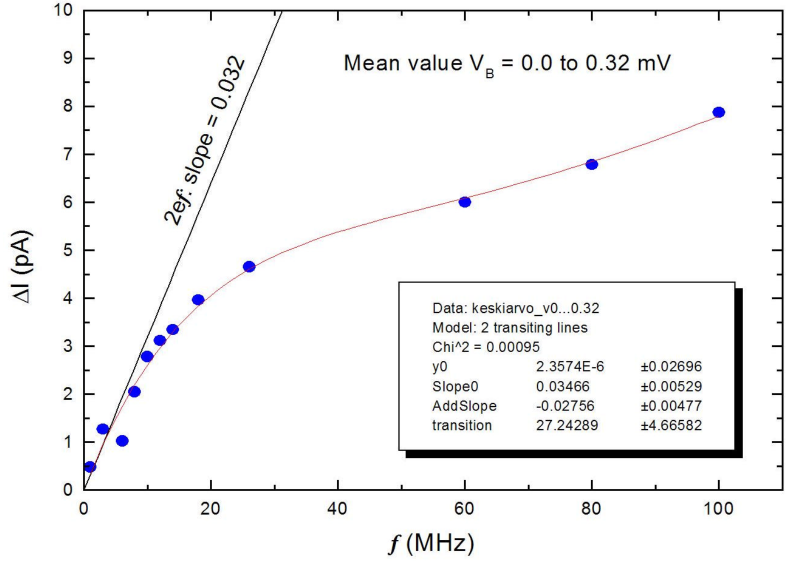


# *IV*-curves at various frequencies



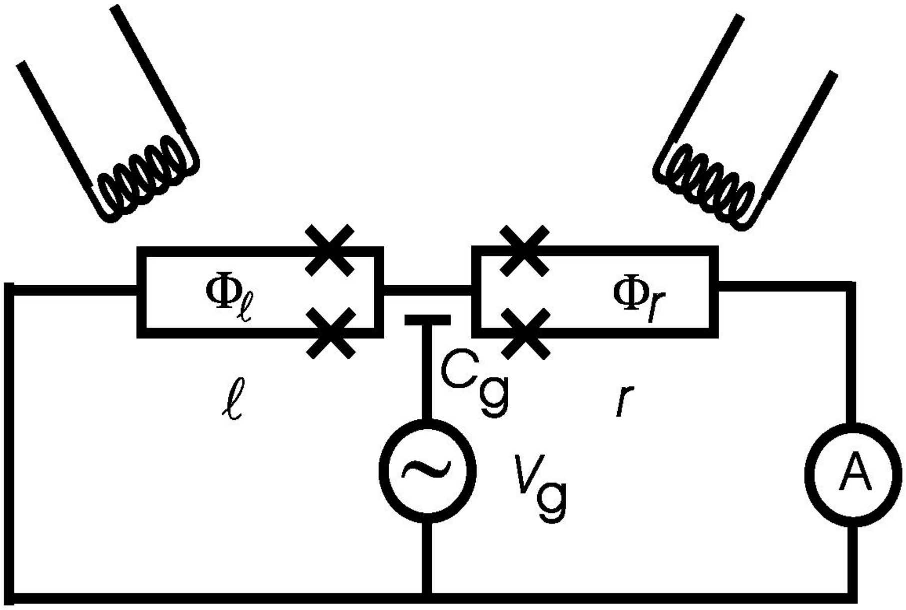
# Frequency dependence of ”plateaus”

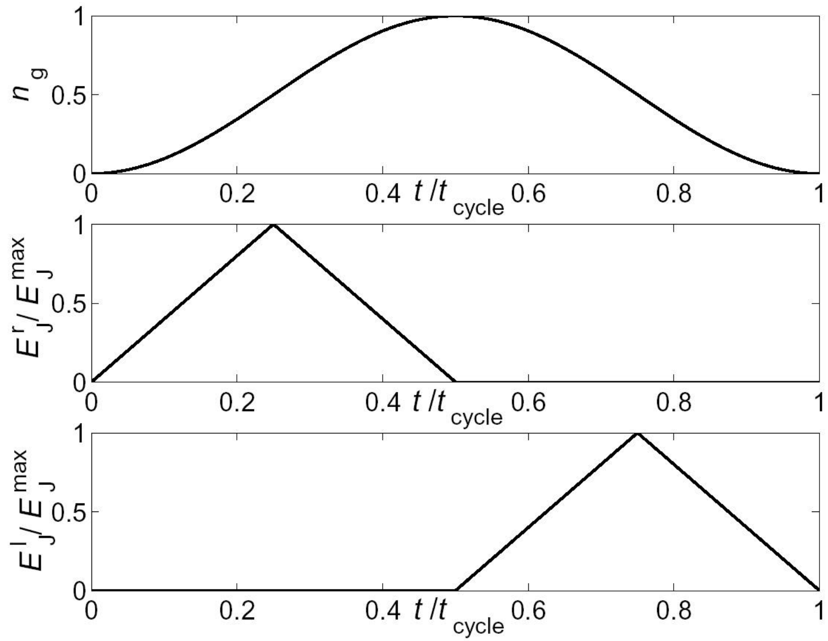



Fits to LZ-crossing model yield $f_{\mathrm{LZ}}$ = 20...25 MHz, we would predict 32 MHz

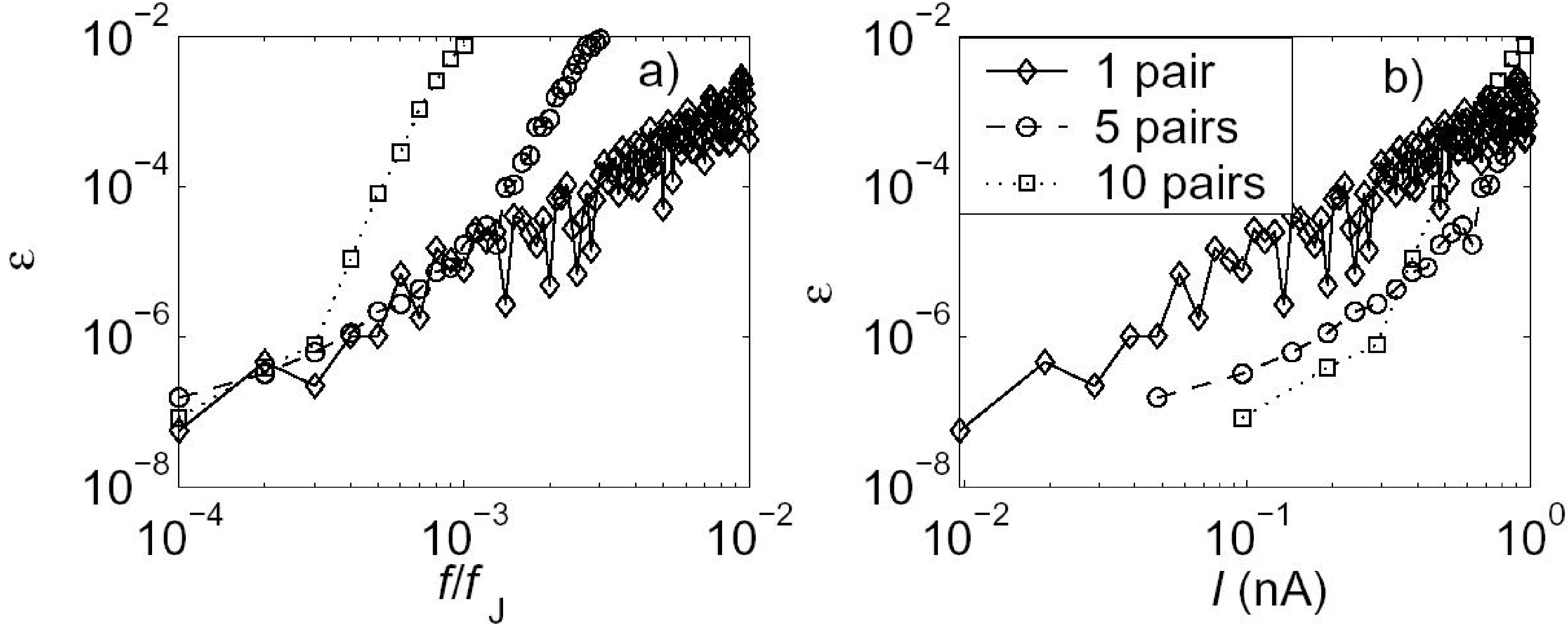
# Flux-assisted Cooper pair sluice




A. O. Niskanen, J. P. and H. Seppä, submitted 2003

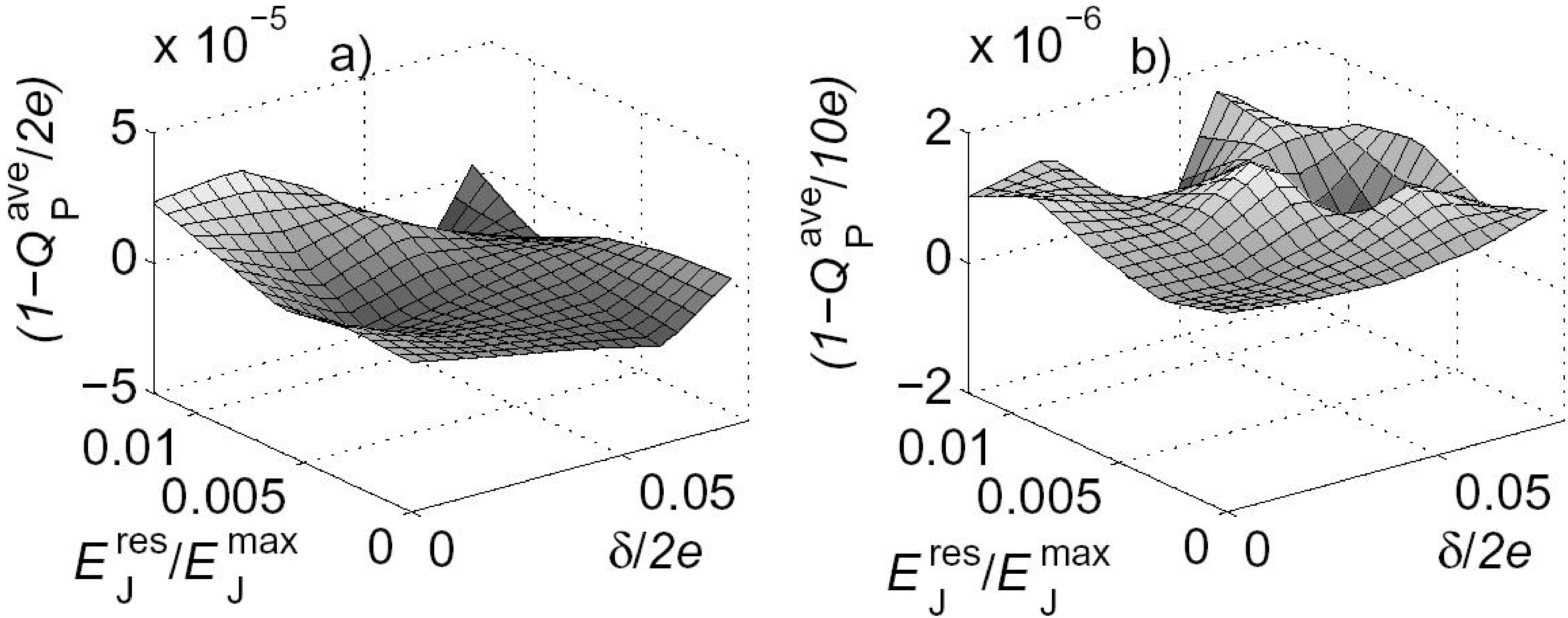
# Predicted accuracy of the (ideal) device




To study also the non-adiabaticity errors, we obtained these results by solving Schrödinger eq. and integrating in time, not by adiabatic approximation.

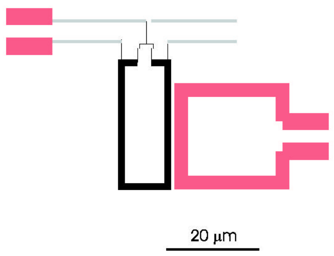
# Influence of residual Josephson coupling and offset charge

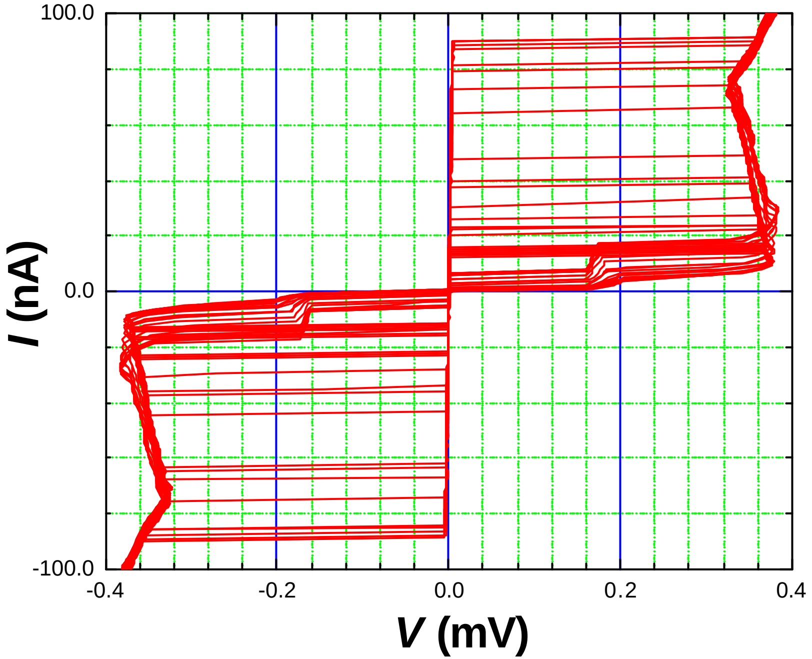
$$Q_\mathrm{P}/(2e) \simeq 1 - \frac{2\sqrt{(E_\mathrm{J}^\mathrm{max})^2 + E_\mathrm{C}^2}}{E_\mathrm{J}^\mathrm{max} E_\mathrm{C}} E_\mathrm{J}^\mathrm{res} \cos\varphi$$

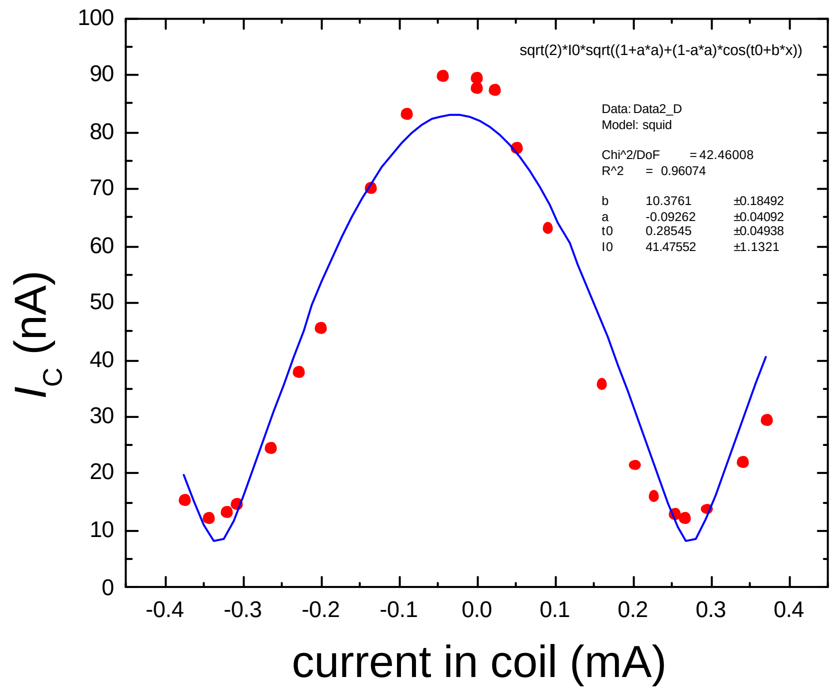



Moderate phase noise averages part of the $E_\mathrm{J}^\mathrm{res}$ -error out.

# First experimental tests of the idea: SQUIDs with small junctions



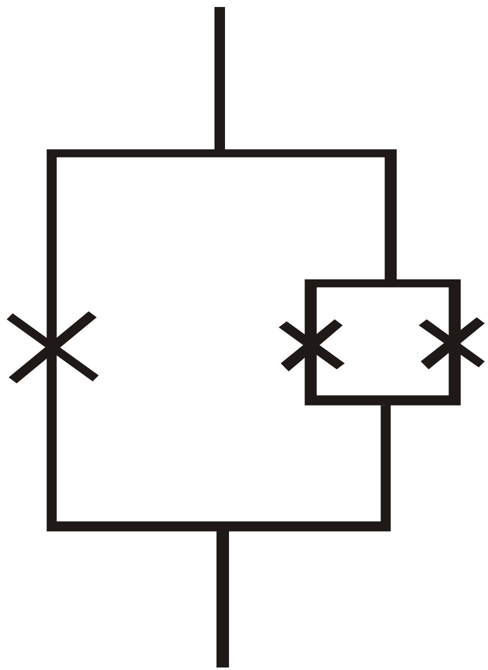




Ambegaokar-Baratoff prediction:

$I_{max} = 200$ nA

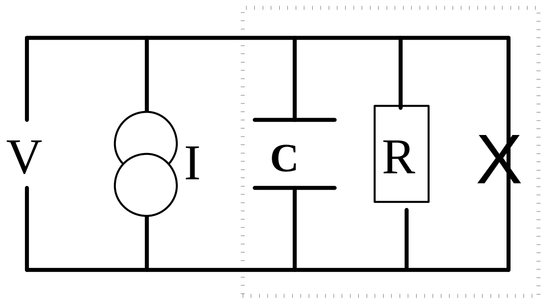
# Tunable SQUIDs?

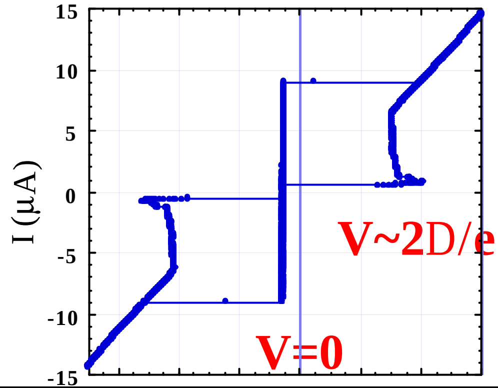


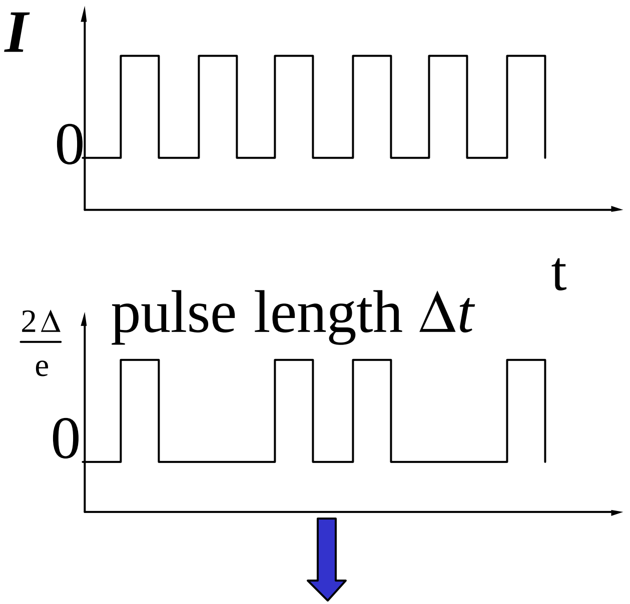
This is more difficult to control

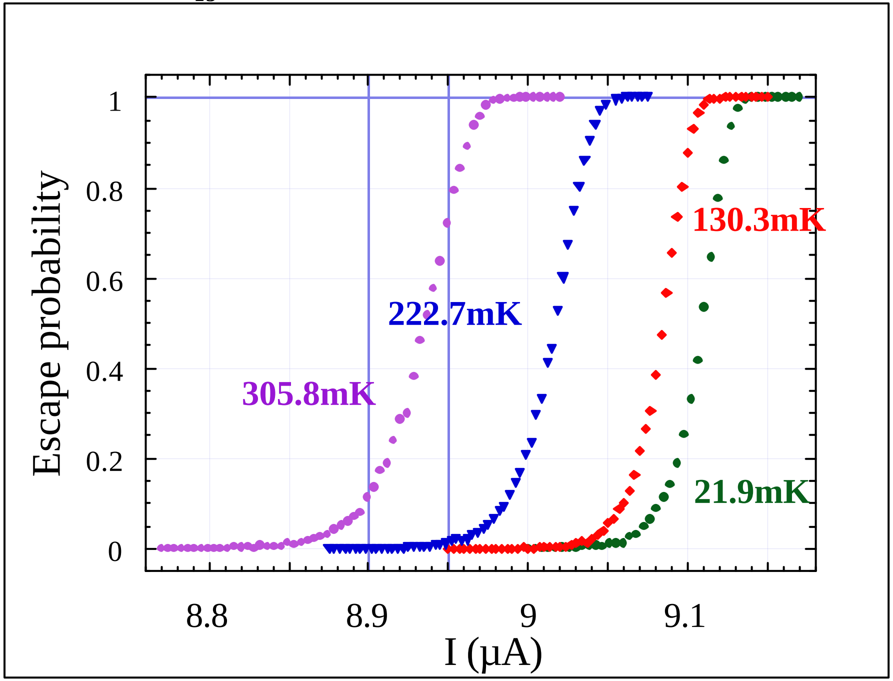
Apply the scheme of Romito, Plastina and Fazio?

# Escape measurements using trapezoidal current pulses



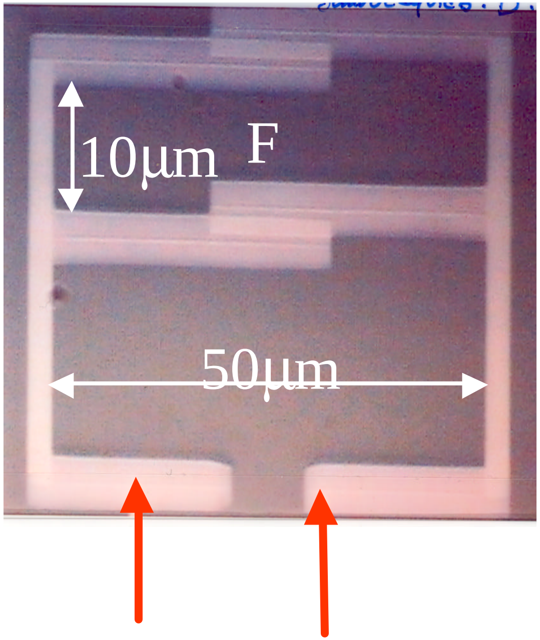



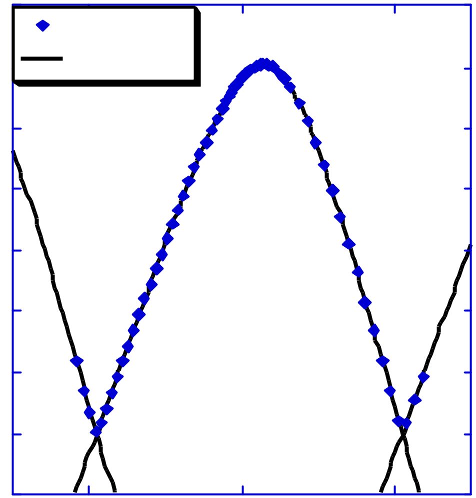




Escape probability for a current pulse

$$P(\mathrm{D}t, I) = 1 - \exp[-\mathrm{G}(I)\mathrm{D}t]$$

# MQT in a hysteretic DC-SQUID



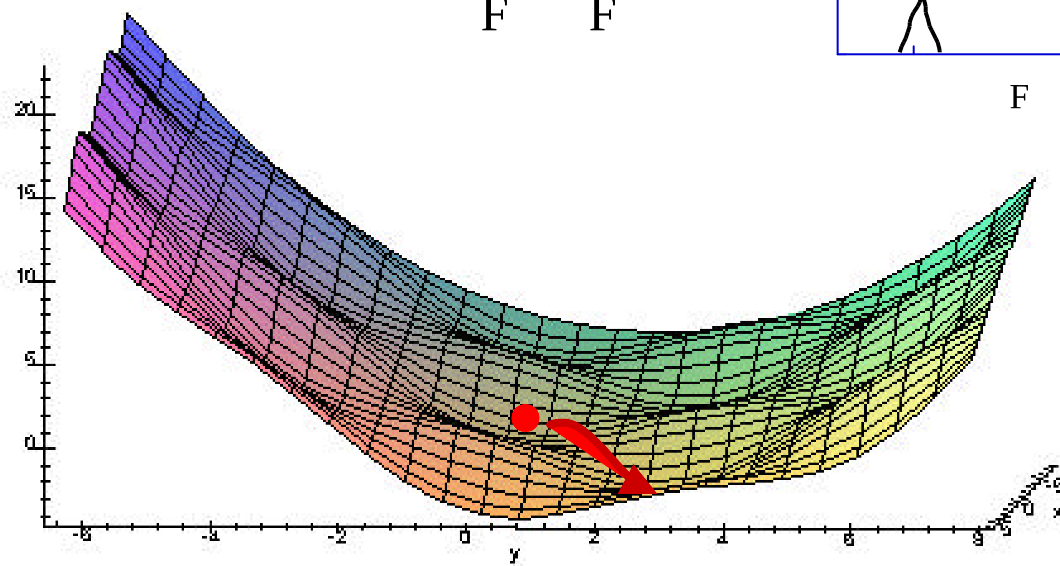
m

m

h

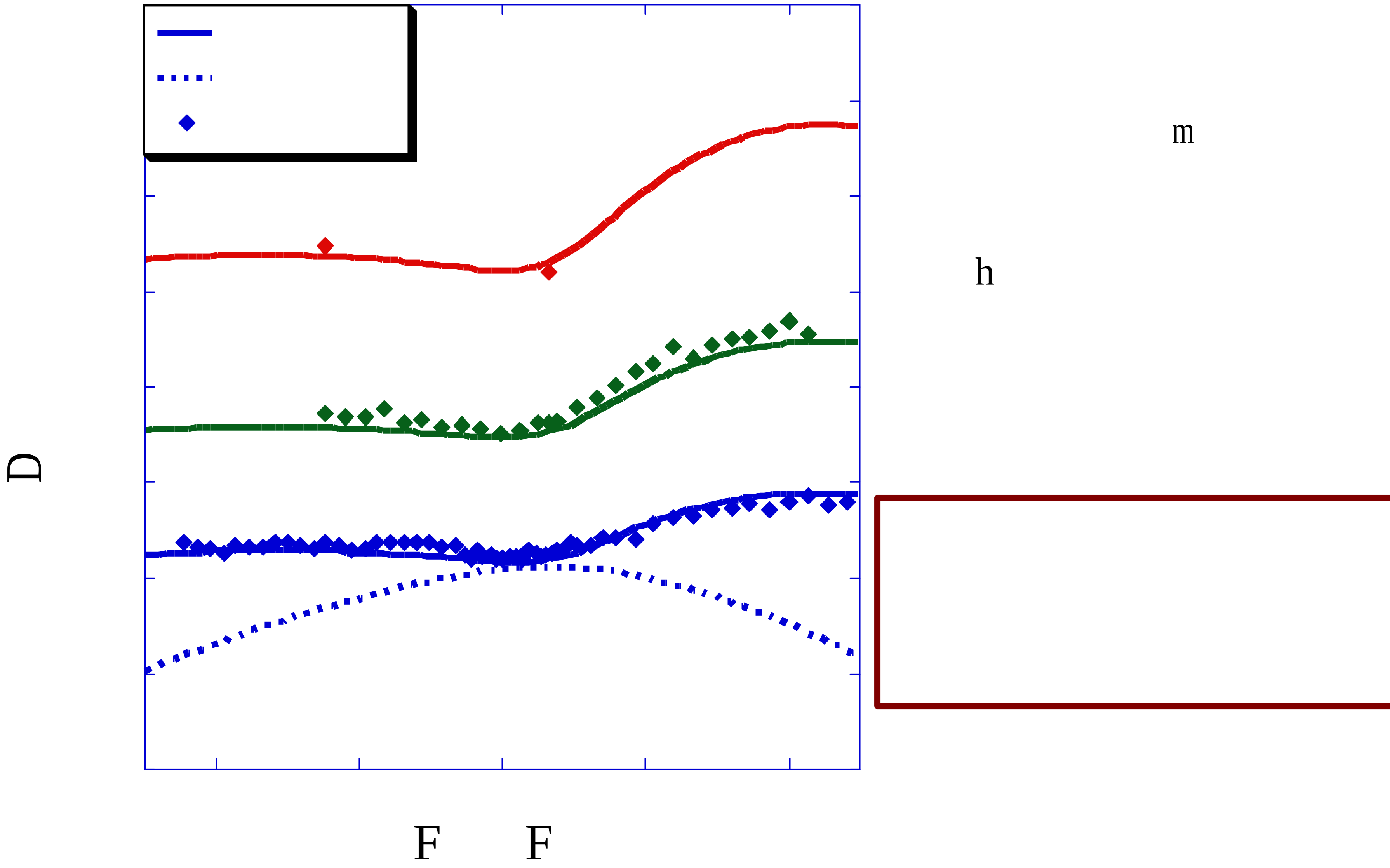
F F

F F



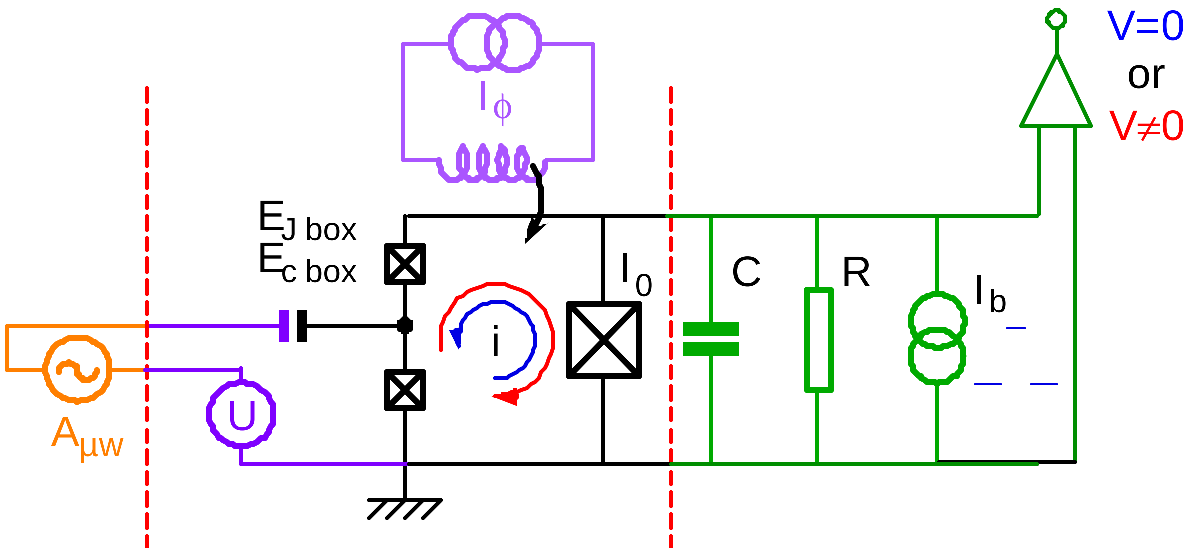

**Franck Balestro, PhD Thesis, UJF Grenoble (2003).**
**F. Balestro et al., preprint (2003).**

# Width of the escape histogram



# Quantronium

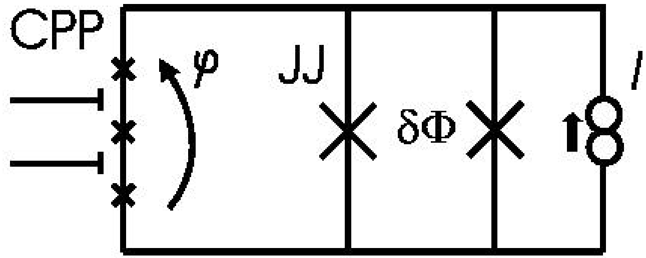
D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).

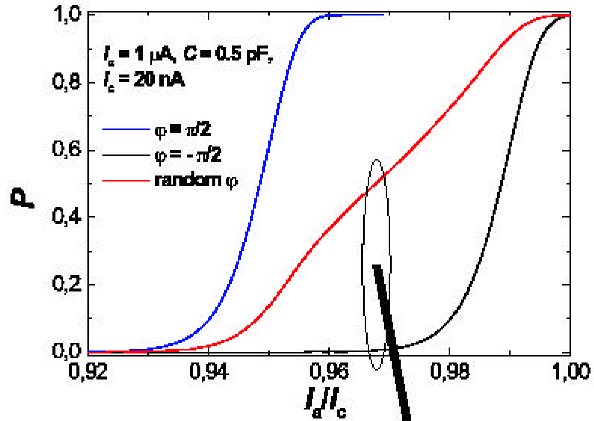


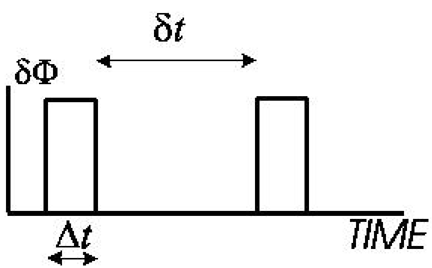

Possible to use an escape junction as a ’classical’ ammeter.

Replacing the JJ by a DC-SQUID allows fast measurements of the state of the quantum system (O. Buisson, F. Balestro, J.P., F. Hekking, PRL, to appear.)

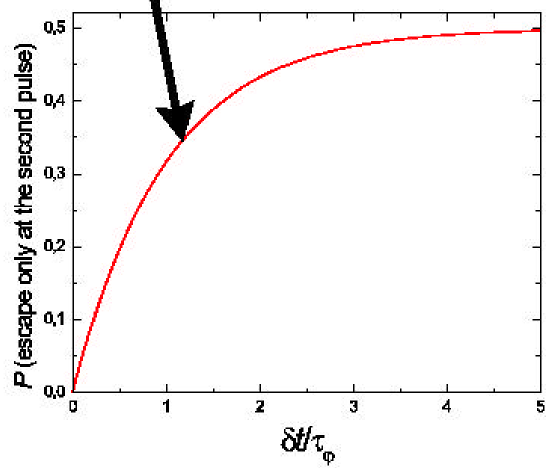
# Pulse measurements on DC-SQUIDs to extract phase fluctuations and coherent corrections to $Q_P$



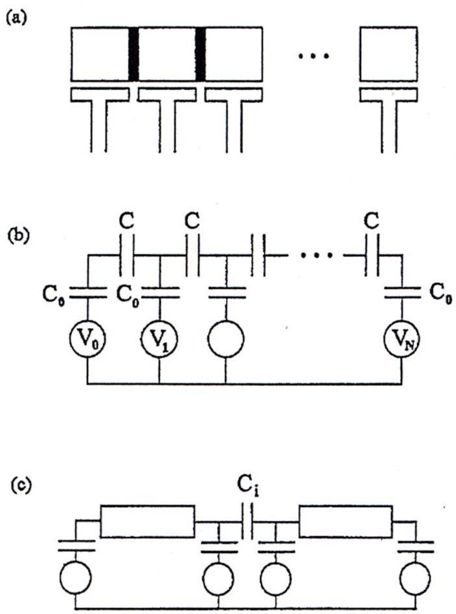

$\Delta t \sim 5$ ns



# Summary

1. In (conventional) Cooper pair pumps it is hard to control quasiparticle populations, supercurrent leakage, coherent corrections to quantized current and Landau-Zener transitions. Pumping cycles are complicated.
2. We propose and are testing a flux-assisted single island Cooper pair pump (”sluice”).
3. Inductive measurement scheme avoids phase fluctuations: experiments on a hysteretic DC-SQUID in macroscopic quantum tunnelling regime were presented. We plan to use this technique for ”non-invasive” measurements of coherent effects in Cooper pair circuits.

# Adiabatic manipulation of Cooper pairs in arrays of JJs




**Adiabatic quantum computing, D. Averin (1998).**

**Topologically stable Josephson qubits, Ioffe, Feigelman (2002).**

**Cooper pair manipulations in geometric qubits, Catania group (2000-).**

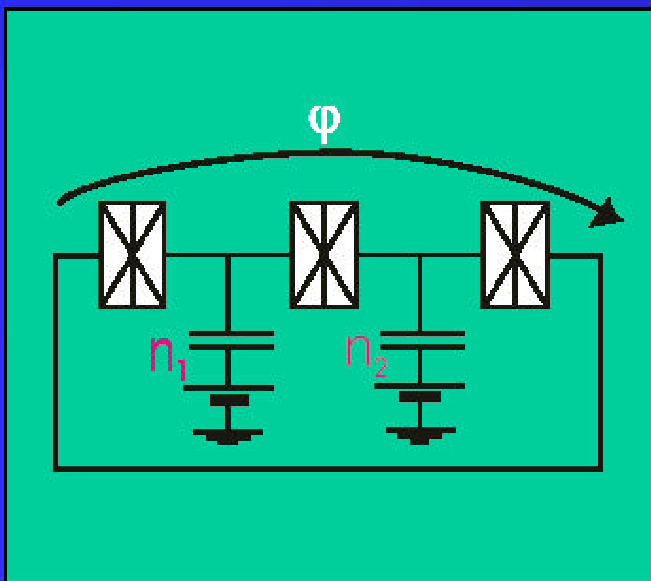
# Charging energy

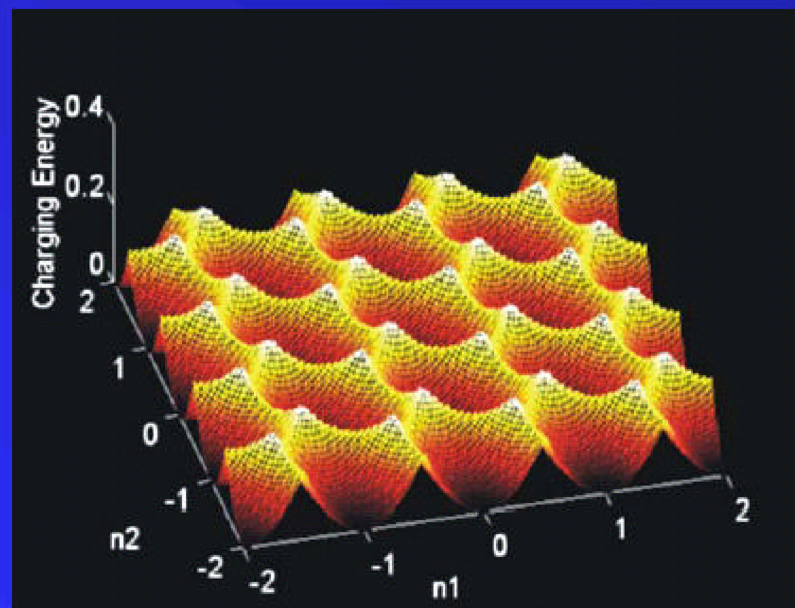
## Cooper pair pump

$$H_c = (2Ec/3)[(N_1-n_1)^2 + (N_2-n_2)^2 + (N_1-n_1)(N_2-n_2)]$$

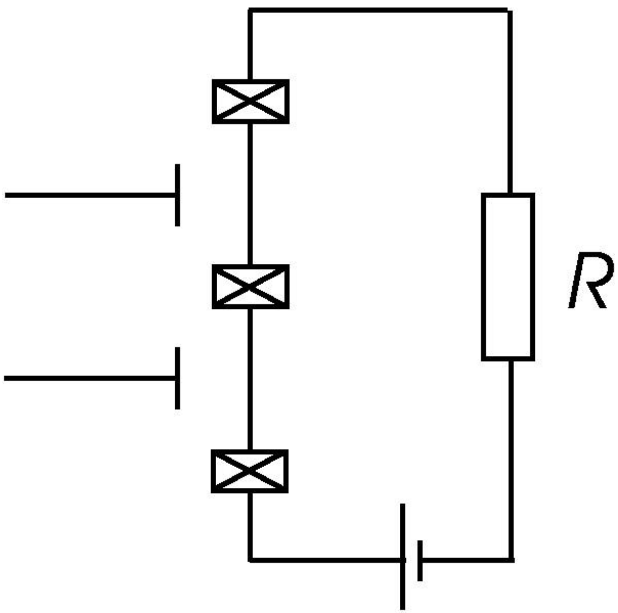
Ground state energy in $n_1, n_2$ -plane

$E_J/E_C << 1$

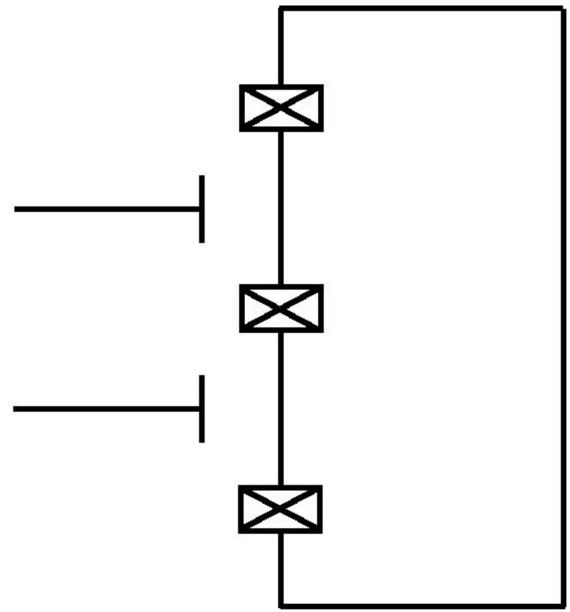


# Coherent vs incoherent Cooper pair pumping



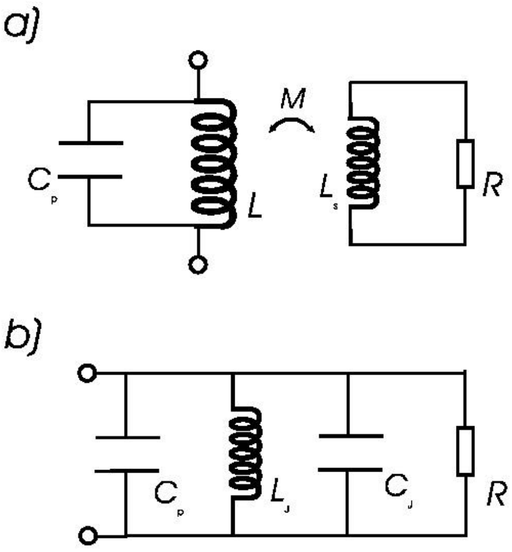

R-pump, $R >> R_Q$



Coherent pump

**S. Lotkhov, S. Bogoslovsky, A. Zorin, J. Niemeyer, APL 78, 946 (2001).**

# Circuit models of SQUID amplifier and escape measurement



# Parameters of the planned measurement

Pumping frequency <100 MHz, current <30 pA (aluminium)

Niobium would allow > 1 GHz and > 0.3 nA (?)

Measurement bandwidth > $\tau_\varphi^{-1}$ ~ 1 MHz