Contribution of geometrical resonances to the Andreev process in double-barrier FISIF and SIFIS structures

Z. Radović
M. Božović
Z. Pajović
Department of Physics
University of Belgrade

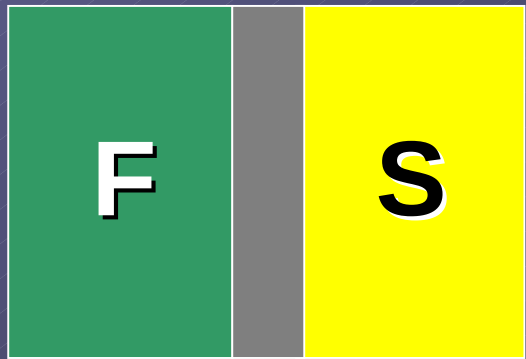
N. Lazarides
N. Flytzanis
Department of Physics
University of Crete, Heraklion

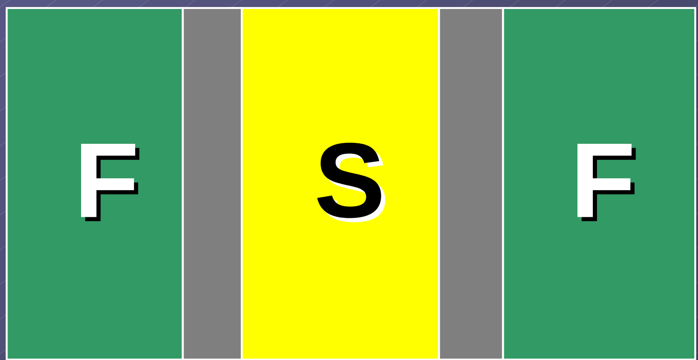
Outline

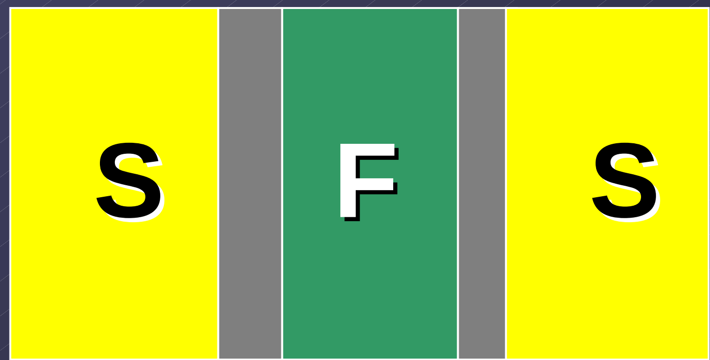
- FIS vs. FISIF (SIFIS) junctions – microscopic theory
- Incoherent transport and spin accumulation
- Coherent transport in clean FISIF junctions:
  - Scattering problem
  - Differential conductances (charge and spin)
- Coherent transport in clean SIFIS junctions :
  - Scattering problem
  - dc Josephson current

Why FS vs. FSF (SFS) junctions?

- Tunneling spectroscopy of superconductors by spin-polarized currents.
- Interplay of ferromagnetism and superconductivity.
- Ballistic transport – interference effects
- LCs in quantum computers ?

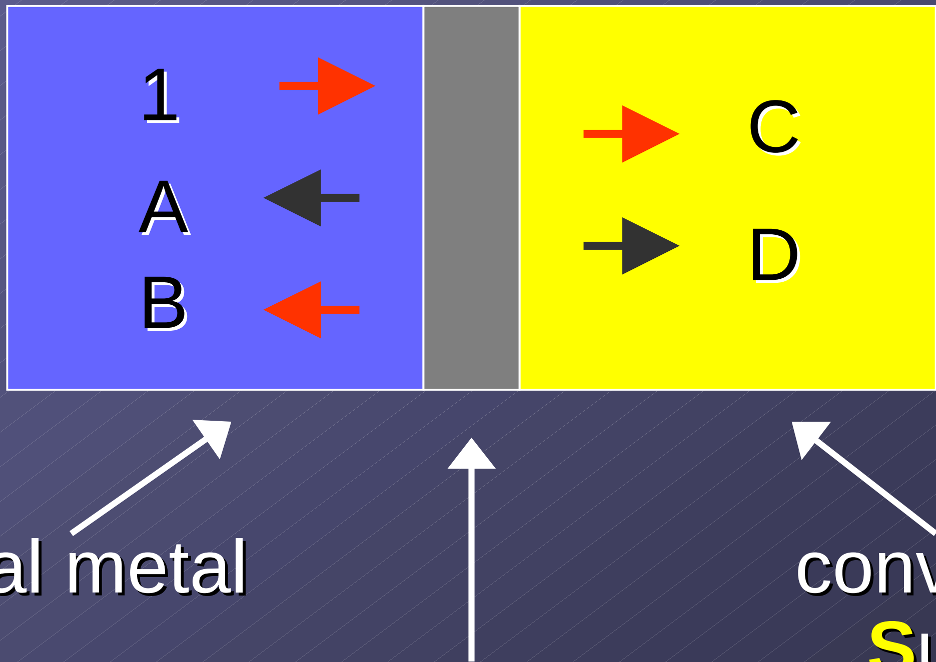






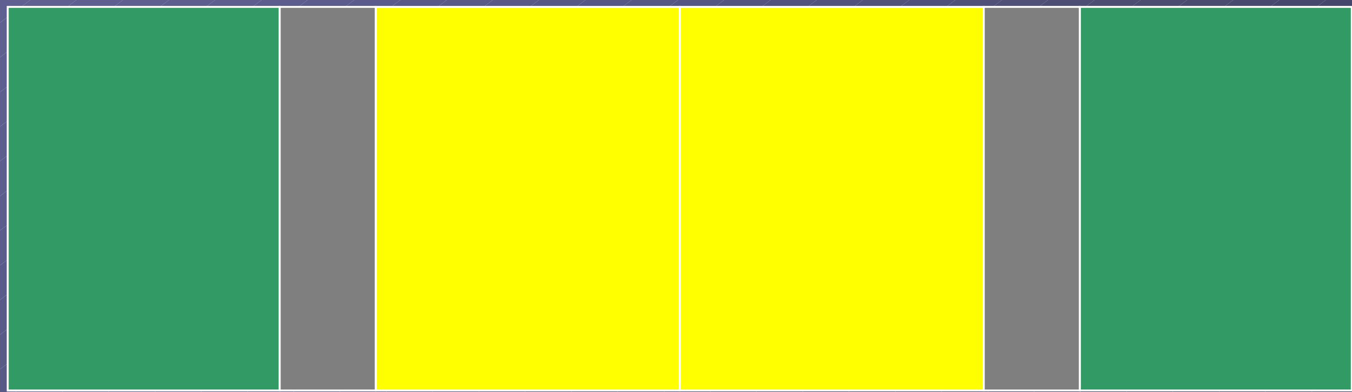
The BTK model

G. E. Blonder, M. Tinkham, and T. M. Klapwijk,
Phys. Rev. B **25**, 4515 (1982).



Incoherent transport

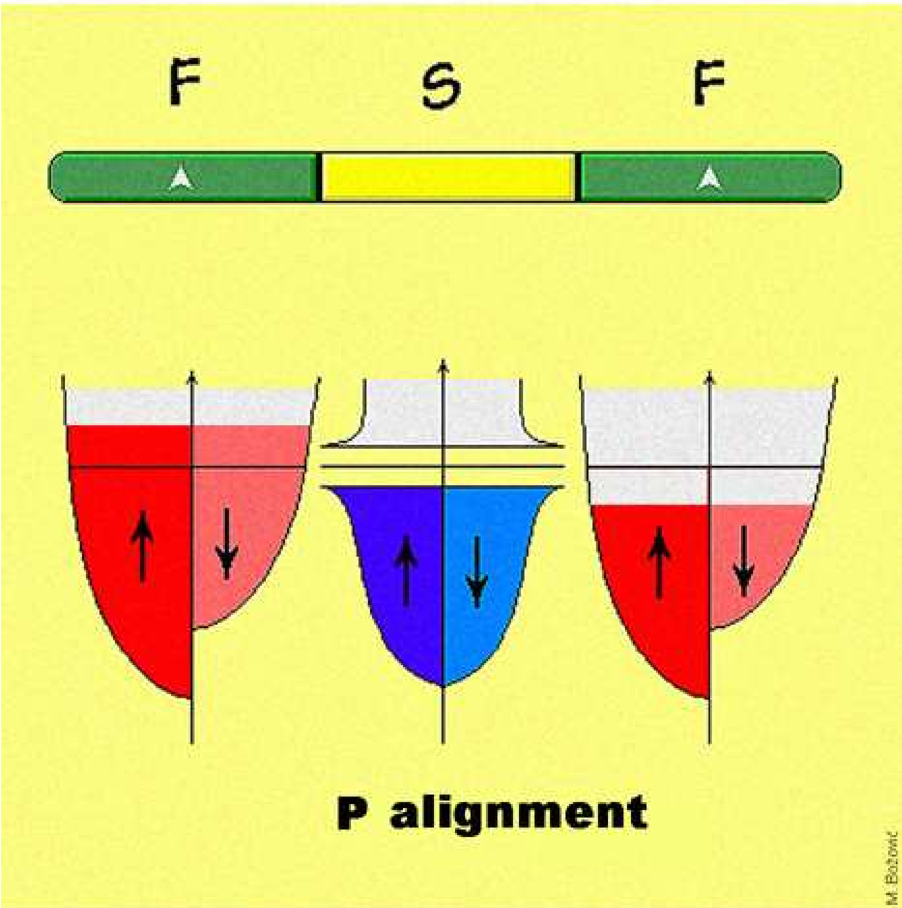
Z. Zheng *et al*., Phys. Rev. B **62**, 14 326 (2000).

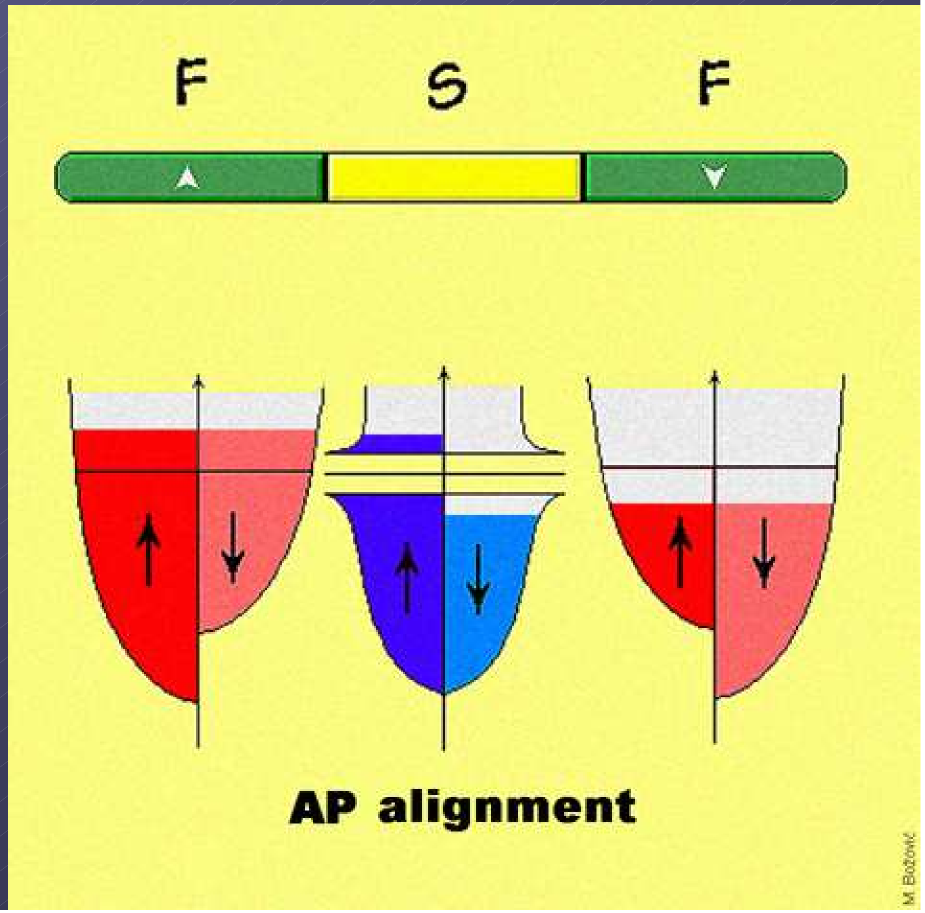


F **S** **+** **S** **F**

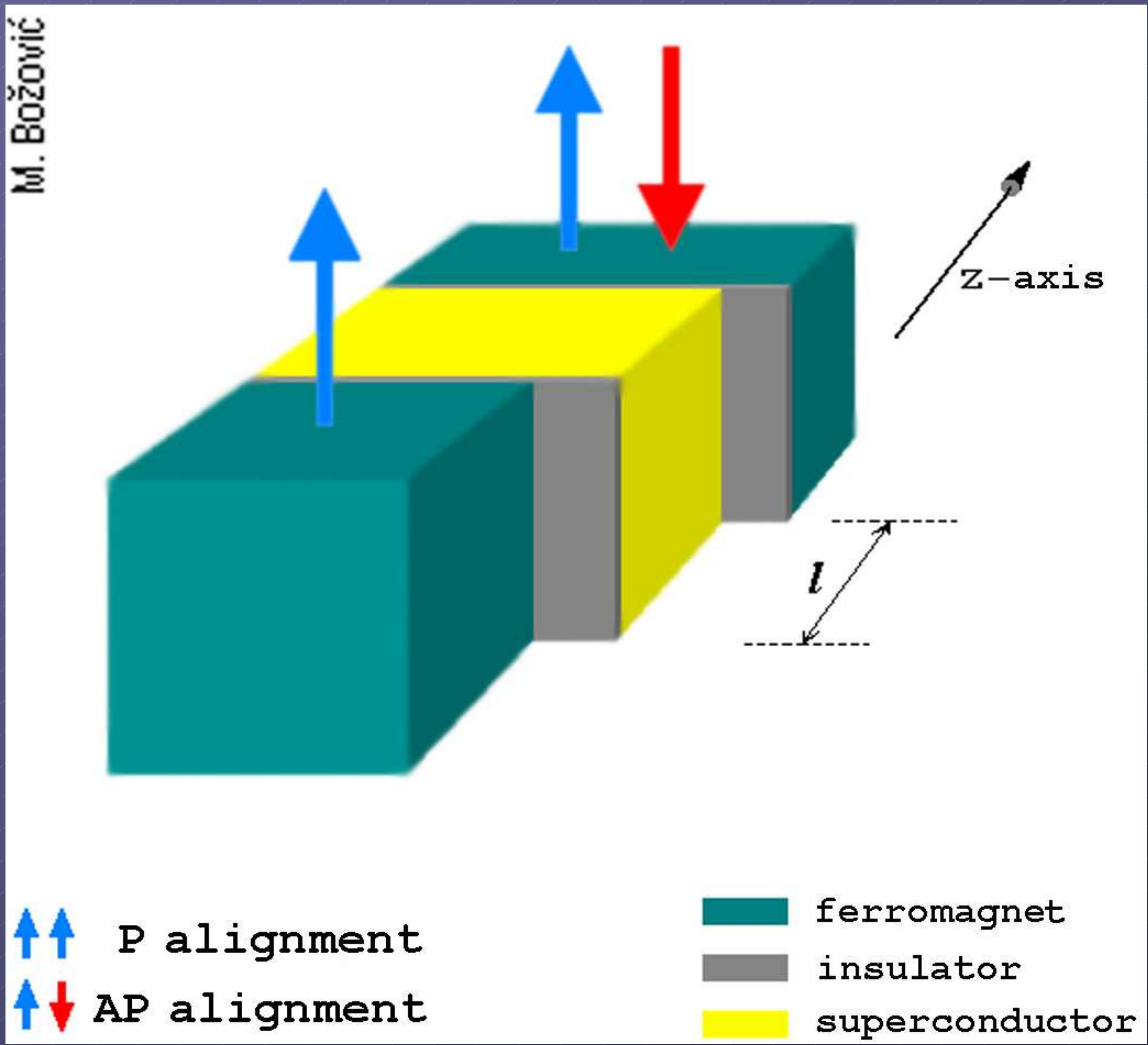
Incoherent transport

S. Takahashi, I. Imamura, and S. Maekawa,
Phys. Rev. Lett. **82**, 3911 (1999).





The Model (FISIF)



Scattering Problem

$$\begin{pmatrix} H_0(\mathbf{r}) - \rho_\sigma h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H_0(\mathbf{r}) + \rho_{\bar{\sigma}} h(\mathbf{r}) \end{pmatrix} \Psi_\sigma(\mathbf{r}) = E\Psi_\sigma(\mathbf{r})$$

$$\Psi_\sigma(\mathbf{r}) \equiv \begin{pmatrix} u_\sigma(\mathbf{r}) \\ v_{\bar{\sigma}}(\mathbf{r}) \end{pmatrix} = \exp(i\mathbf{k}_{\parallel,\sigma} \cdot \mathbf{r})\psi(z)$$

Exchange energy $h(\mathbf{r})/E_F^{(F)} = X[\Theta(-z) \pm \Theta(z-l)]$ $\rho_{\uparrow,\downarrow} = \pm 1$

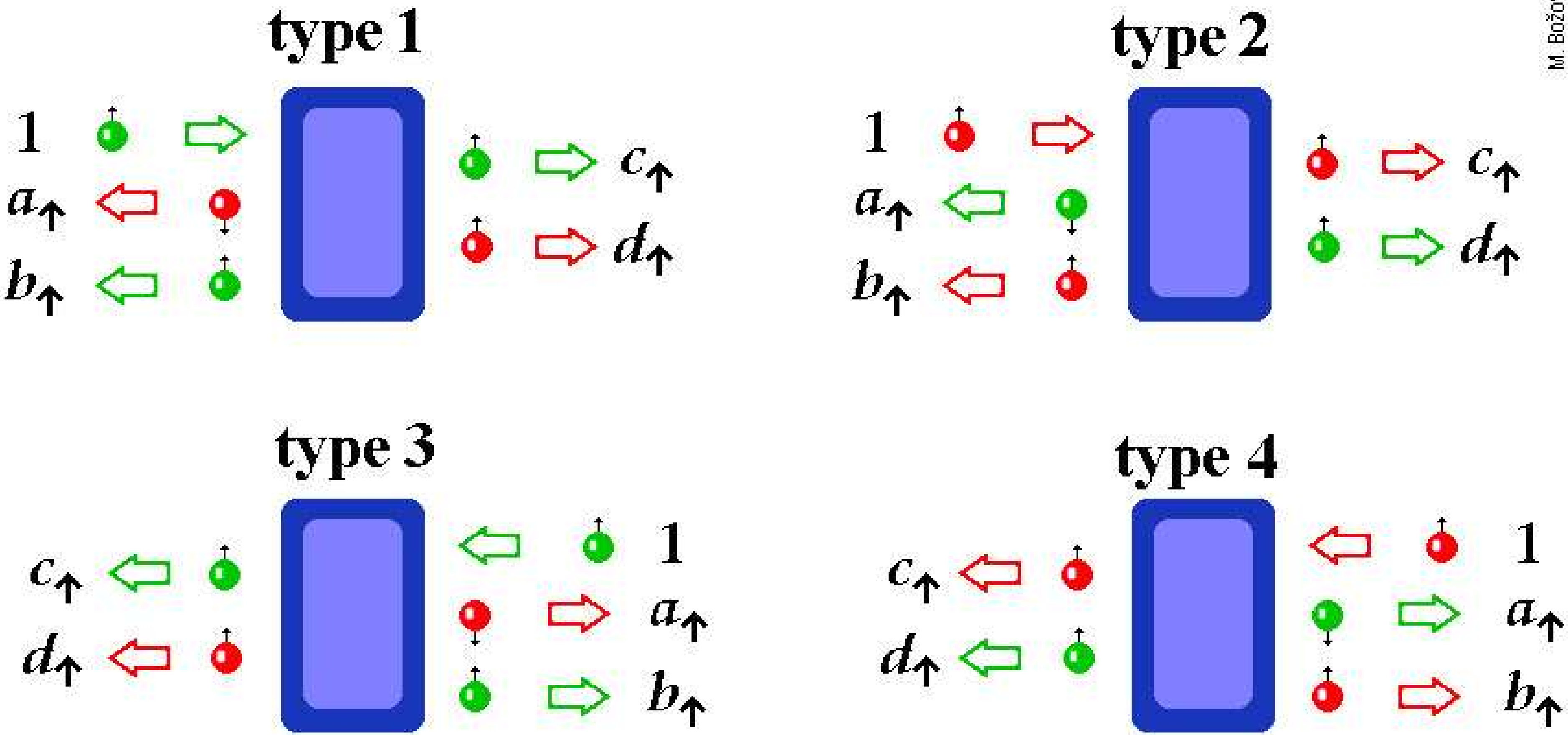
Stepwise pair potential $\Delta(\mathbf{r}) = \Delta\Theta(z)\Theta(l-z)$

Interface potential $\hat{W}[\delta(z) + \delta(l-z)]$ $Z = 2m\hat{W}/\hbar^2 k_F^{(S)}$

FWVM parameter $\kappa = k_F^{(F)}/k_F^{(S)}$

Scattering Problem

A. Furusaki and M. Tsukada,
Solid State Commun. **78**, 299 (1991).



M. Božović

Solutions

$$\psi_1(z)=\begin{cases}[\exp(ik_\sigma^+ z)+b_\sigma(E,\theta)\exp(-ik_\sigma^- z)]\begin{pmatrix}1\\0\end{pmatrix}+a_\sigma(E,\theta)\exp(ik_\sigma^- z)\begin{pmatrix}0\\1\end{pmatrix}, & z<0\\[c_1(E,\theta)\exp(iq_\sigma^+ z)+c_2(E,\theta)\exp(-iq_\sigma^+ z)]\begin{pmatrix}\bar{u}\\\bar{v}\end{pmatrix} & \\+[c_3(E,\theta)\exp(iq_\sigma^- z)+c_4(E,\theta)\exp(-iq_\sigma^- z)]\begin{pmatrix}\bar{v}\\\bar{u}\end{pmatrix}, & 0<z<l\\c_\sigma(E,\theta)\exp(ik_{\sigma[\bar{\sigma}]}^+ z)\begin{pmatrix}1\\0\end{pmatrix}+d_\sigma(E,\theta)\exp(-ik_{\bar{\sigma}[\sigma]}^+ z)\begin{pmatrix}0\\1\end{pmatrix}, & z>l\end{cases}$$

$$\bar{u}=\sqrt{(1+\Omega/E)/2} \qquad \bar{v}=\sqrt{(1-\Omega/E)/2}$$

$$\Omega=\sqrt{E^2-\Delta^2}$$

Wave vector components

Perpendicular component in the ferromagnets

$$k_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2)\left(E_F^{(F)} + \rho_\sigma h_0 \pm E\right) - \mathbf{k}_{\parallel,\sigma}^2}$$

Perpendicular component in the superconductor

$$q_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2)\left(E_F^{(S)} \pm \Omega\right) - \mathbf{k}_{\parallel,\sigma}^2}$$

Conserved parallel component

$$\left|\mathbf{k}_{\parallel,\sigma}\right| = \sqrt{(2m/\hbar^2)\left(E_F^{(F)} + \rho_\sigma h_0 + E\right)} \sin\theta$$

Wave vector components

Neglecting $E/E_F^{(F)} << 1$ and $\Delta/E_F^{(S)} << 1$

except in the exponents $\zeta_{\pm} = l\left(q_\sigma^{+} \pm q_\sigma^{-}\right)$

the approximated wave-vector components, in units of $k_F^{(S)}$ are

$$\tilde{k}_\sigma = \lambda_\sigma \cos\theta$$

$$\tilde{q}_\sigma = \sqrt{1-\tilde{\mathbf{k}}_{\parallel,\sigma}^2}$$

$$\left|\tilde{\mathbf{k}}_{\parallel,\sigma}\right| = \lambda_\sigma \sin\theta$$

$$\lambda_\sigma = \kappa\sqrt{1+\rho_\sigma X}$$

Boundary conditions

$$\begin{aligned}
\psi_\sigma(z)\big|_{z=0_-} &= \psi_\sigma(z)\big|_{z=0_+}, \\
\frac{d\psi_\sigma(z)}{dz}\bigg|_{z=0_-} &= \frac{d\psi_\sigma(z)}{dz}\bigg|_{z=0_+} - \frac{2m\hat{W}}{\hbar^2}\psi_\sigma(0), \\
\psi_\sigma(z)\big|_{z=l_-} &= \psi_\sigma(z)\big|_{z=l_+}, \\
\frac{d\psi_\sigma(z)}{dz}\bigg|_{z=l_-} &= \frac{d\psi_\sigma(z)}{dz}\bigg|_{z=l_+} - \frac{2m\hat{W}}{\hbar^2}\psi_\sigma(l).
\end{aligned}$$

$$
\begin{aligned}
a_\sigma(E,\theta) &= \frac{4(\tilde{k}_\sigma/\tilde{q}_\sigma)\Delta\sin(\zeta_-/2)}{\Gamma}\left[\mathcal{A}_+^R E\sin(\zeta_-/2) + i\mathcal{B}_+^R\Omega\cos(\zeta_-/2)\right], \\
b_\sigma(E,\theta) &= \frac{1}{\Gamma}\Big[\mathcal{A}_+^R\mathcal{C}_+\Delta^2 - \left(\mathcal{A}_+^R\mathcal{C}_+E^2 + \mathcal{B}_+^R\mathcal{D}_+\Omega^2\right)\cos(\zeta_-) \\
&+ \left(\mathcal{A}_-^R\mathcal{C}_- + \mathcal{B}_-^R\mathcal{D}_-\right)\Omega^2\cos(\zeta_+) + i\left(\mathcal{B}_+^R\mathcal{C}_+ + \mathcal{A}_+^R\mathcal{D}_+\right)E\Omega\sin(\zeta_-) \\
&- i\left(\mathcal{B}_-^R\mathcal{C}_- + \mathcal{A}_-^R\mathcal{D}_-\right)\Omega^2\sin(\zeta_+)\Big], \\
c_\sigma(E,\theta) &= \frac{4(\tilde{k}_\sigma/\tilde{q}_\sigma)\Omega e^{-i\tilde{k}_{\bar{\sigma}}l}}{\Gamma}\times \\
&\times\Big\{i\left[\mathcal{F}_+\cos(\zeta_+/2) + i\mathcal{E}_+\sin(\zeta_+/2)\right]E\sin(\zeta_-/2) \\
&- \left[\mathcal{E}_+\cos(\zeta_+/2) + i\mathcal{F}_+\sin(\zeta_+/2)\right]\Omega\cos(\zeta_-/2)\Big\}, \\
d_\sigma(E,\theta) &= \frac{4(\tilde{k}_\sigma/\tilde{q}_\sigma)\Delta\Omega e^{i\tilde{k}_\sigma l}}{\Gamma}\times \\
&\times i\left[\mathcal{F}_-\cos(\zeta_+/2) + i\mathcal{E}_-\sin(\zeta_+/2)\right]\sin(\zeta_-/2),
\end{aligned}
$$

$$
\begin{aligned}
\Gamma = \mathcal{A}_+^L\mathcal{A}_+^R\Delta^2 &- \left(\mathcal{A}_+^L\mathcal{A}_+^R E^2 + \mathcal{B}_+^L\mathcal{B}_+^R\Omega^2\right)\cos(\zeta_-) + \left(\mathcal{A}_-^L\mathcal{A}_-^R + \mathcal{B}_-^L\mathcal{B}_-^R\right)\Omega^2\cos(\zeta_+) \\
&+ i\left(\mathcal{A}_+^L\mathcal{B}_+^R + \mathcal{B}_+^L\mathcal{A}_+^R\right)E\Omega\sin(\zeta_-) - i\left(\mathcal{A}_-^L\mathcal{B}_-^R + \mathcal{B}_-^L\mathcal{A}_-^R\right)\Omega^2\sin(\zeta_+).
\end{aligned}
$$

$$\mathcal{A}_{\pm}^{L(R)} = K_1^{L(R)} \pm K_2^{L(R)}, \qquad \mathcal{B}_{\pm}^{L(R)} = 1 \pm K_1^{L(R)} K_2^{L(R)}, \qquad \mathcal{C}_{\pm} = {K_1^L}^* \mp K_2^L,$$

$$\mathcal{D}_{\pm} = -(1 \mp {K_1^L}^* K_2^L), \qquad \mathcal{E}_{\pm} = K_2^L \pm K_2^R, \qquad \mathcal{F}_{\pm} = 1 \pm K_2^L K_2^R,$$

$$K_1^L = \frac{\tilde{k}_\sigma + iZ}{\tilde{q}_\sigma}, \qquad K_2^L = \frac{\tilde{k}_{\bar{\sigma}} - iZ}{\tilde{q}_\sigma},$$

$$K_1^R = \frac{\tilde{k}_{\sigma[\bar{\sigma}]} + iZ}{\tilde{q}_\sigma}, \qquad K_2^R = \frac{\tilde{k}_{\bar{\sigma}[\sigma]} - iZ}{\tilde{q}_\sigma},$$

Scattering probabilities

$$A_\sigma(E,\theta) + B_\sigma(E,\theta) + C_\sigma(E,\theta) + D_\sigma(E,\theta) = 1,$$

$$\begin{aligned}
A_\sigma(E,\theta) &= \mathrm{Re}\left(\frac{\tilde{k}_{\bar{\sigma}}}{\tilde{k}_\sigma}\right) |a_\sigma(E,\theta)|^2, \\
B_\sigma(E,\theta) &= |b_\sigma(E,\theta)|^2, \\
C_\sigma(E,\theta) &= \mathrm{Re}\left(\frac{\tilde{k}_{\sigma[\bar{\sigma}]}}{\tilde{k}_\sigma}\right) |c_\sigma(E,\theta)|^2, \\
D_\sigma(E,\theta) &= \mathrm{Re}\left(\frac{\tilde{k}_{\bar{\sigma}[\sigma]}}{\tilde{k}_\sigma}\right) |d_\sigma(E,\theta)|^2,
\end{aligned}$$

Two limits

- Metallic limit ($Z = 0$)
  Andreev reflection vanishes at geometrical resonances:

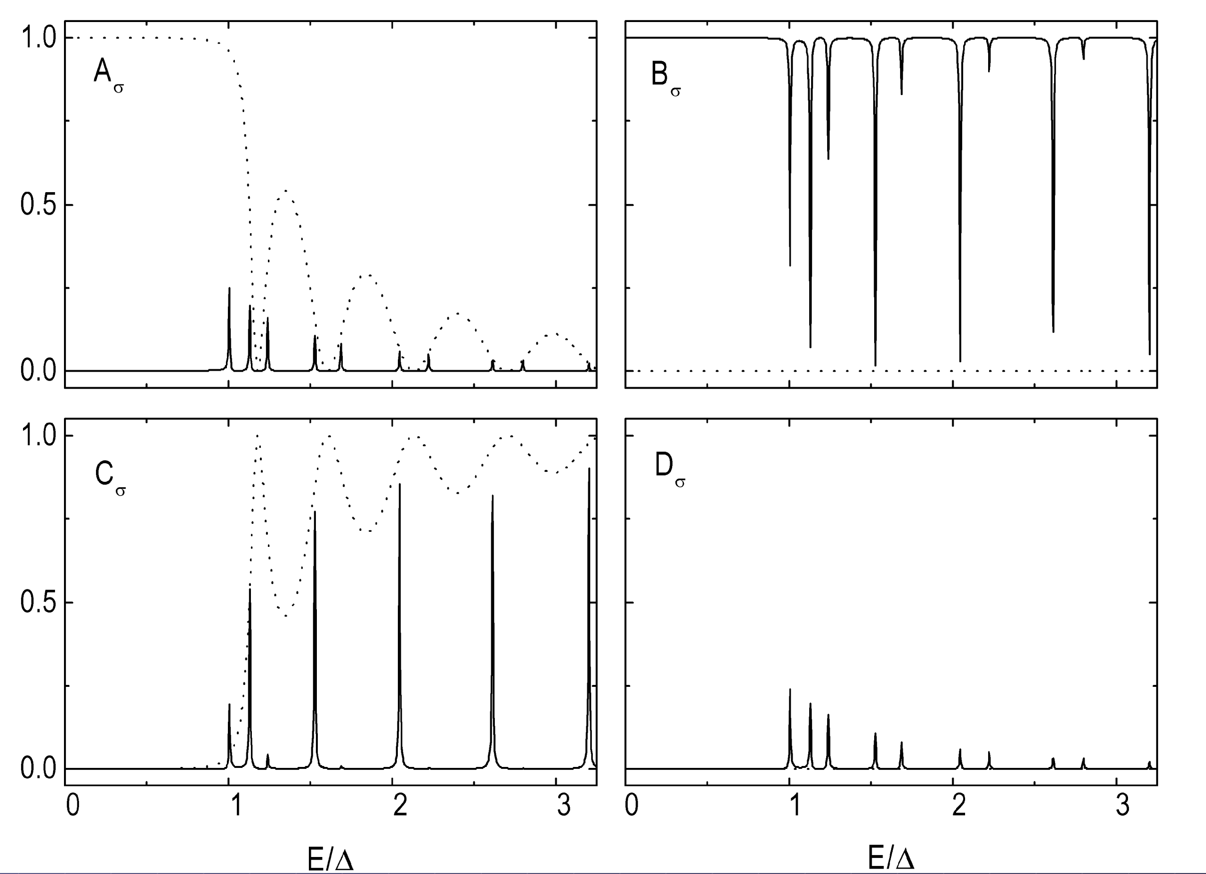
$$A_\sigma = D_\sigma = 0 \quad \text{when} \quad l(q_\sigma^+ - q_\sigma^-) = 2n\pi$$

$$q_\sigma^\pm = \sqrt{(2m/\hbar^2)[E_F^{(S)} \pm \Omega] - \mathbf{k}_{\parallel,\sigma}^2} \qquad \Omega = \sqrt{E^2 - \Delta^2}$$

- Tunnel limit ($Z \to \infty$)
  Transport through the bound states:

$$lq_\sigma^+ = n_1\pi \qquad lq_\sigma^- = n_2\pi \qquad n_1 - n_2 = 2n$$

Metallic vs. Tunnel NSN junction



$$X = 0$$

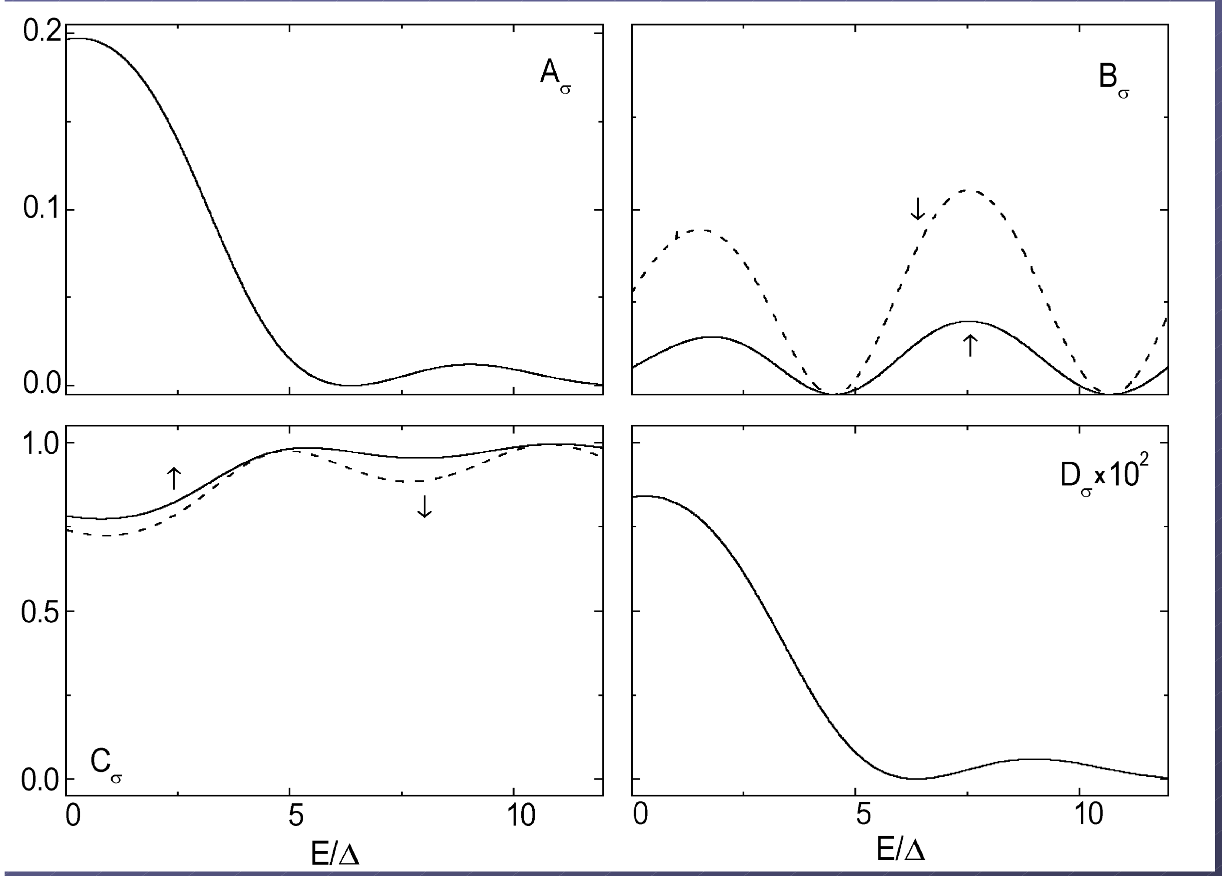
$$lk_F^{(S)} = 10^4 \ [l/\xi_0 \approx 10]$$

$$\theta = 0$$

$$\kappa = 1, \ \Delta / E_F^{(S)} = 10^{-3}$$

M. Božović and Z. Radović in *Supercond. and Rel. Ox.: Phys. and nanoeng. V, Proc. of SPIE, vol. 4811* (Seattle, 2002), p. 216.

FSF double junction (P alignment)



$Z = 0,\ X = 0.5$

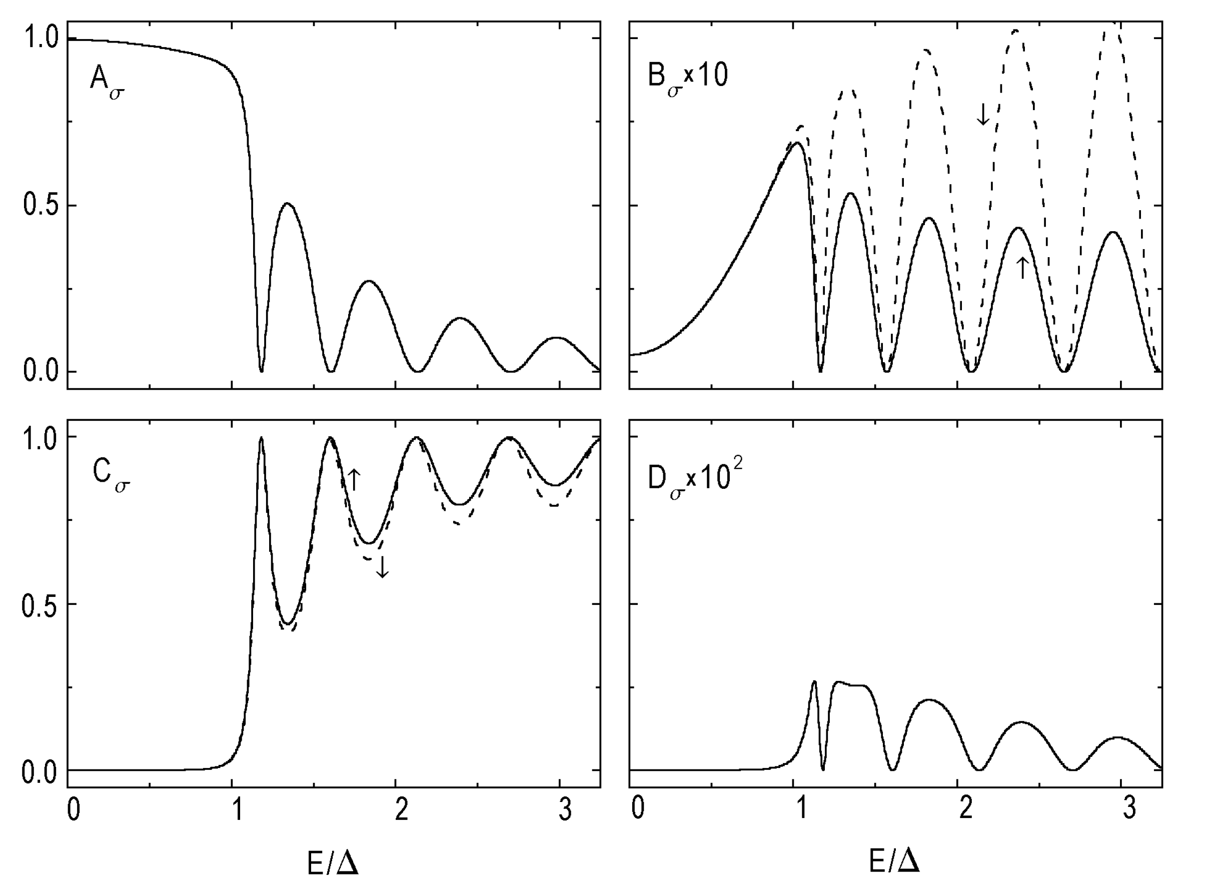
$lk_F^{(S)} = 10^3\ [l/\xi_0 \approx 1]$

$\theta = 0$

$\kappa = 1,\ \Delta / E_F^{(S)} = 10^{-3}$

M. Božović and Z. Radović,
Phys. Rev. B **66**, 134524 (2002)

FSF double junction (P alignment)



$Z=0,\ X=0.5$

$lk_F^{(S)}=10^4\ [l/\xi_0\approx 10]$

$\theta=0$

$\kappa=1,\ \Delta/E_F^{(S)}=10^{-3}$

M. Božović and Z. Radović,
Phys. Rev. B **66**, 134524 (2002)

Current

$$j_q(V) = \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3\mathbf{k}}{(2\pi)^3} e\mathbf{v}_\sigma \cdot \hat{\mathbf{z}}\ \delta f(\mathbf{k},V)$$

charge

$$I_q(V) = \frac{1}{e} \int_{-\infty}^{\infty} dE \left[f_0(E - eV/2) - f_0(E + eV/2) \right] G_q(E)$$

spin

$$I_s(V) = \frac{1}{e} \int_{-\infty}^{\infty} dE \left[f_0(E - eV/2) - f_0(E + eV/2) \right] G_s(E)$$

Differential conductances (at T=0)

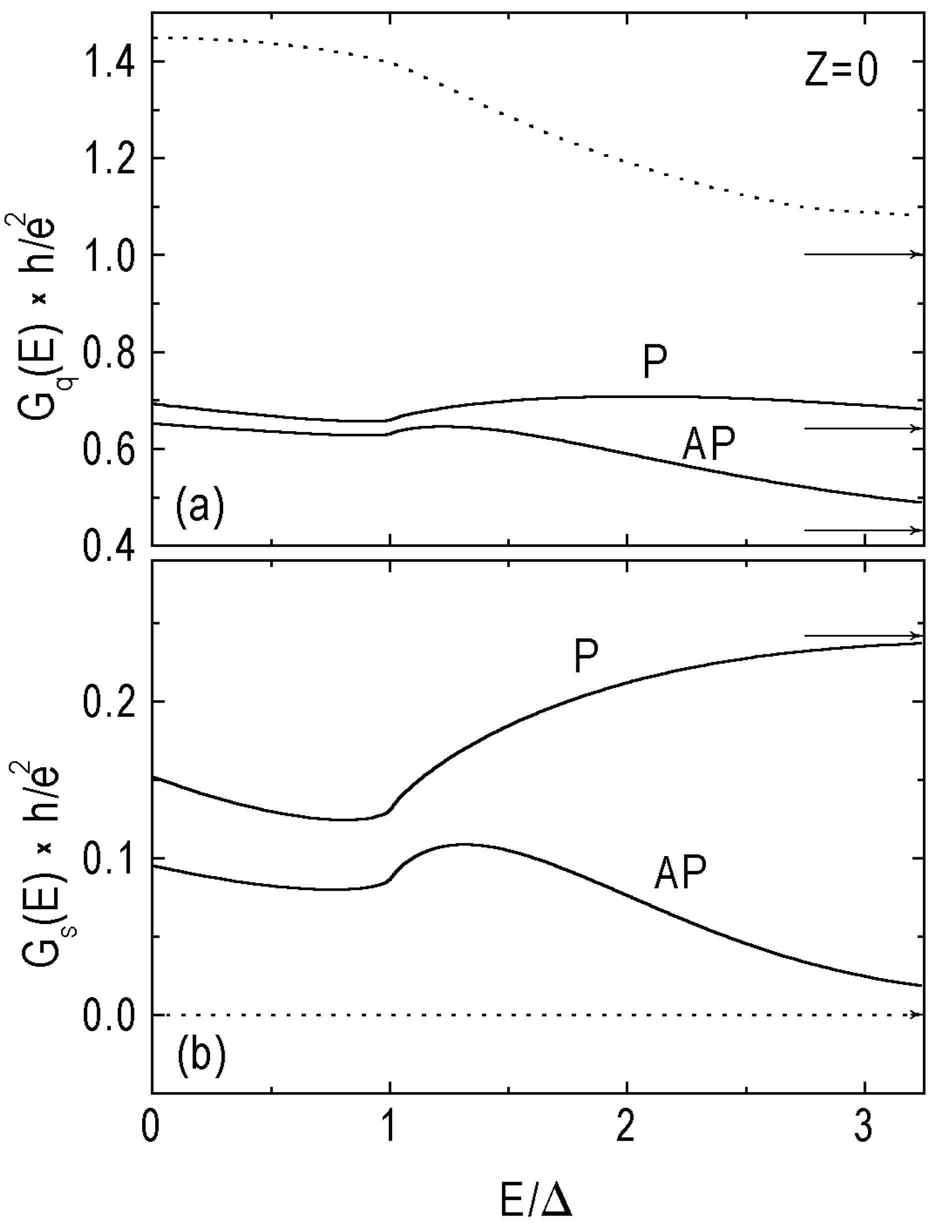
charge

$$G_q(E) = \frac{e^2}{2h} \sum_{\sigma} \lambda_{\sigma}^2 \int_0^{\theta_{c1,\sigma}} d\theta \sin\theta \cos\theta \, [1 + A_{\sigma}(E,\theta) - B_{\sigma}(E,\theta)]$$

spin

$$G_s(E) = \frac{e^2}{2h} \sum_{\sigma} \rho_{\sigma} \lambda_{\sigma}^2 \int_0^{\theta_{c1,\sigma}} d\theta \sin\theta \cos\theta \, [1 - A_{\sigma}(E,\theta) - B_{\sigma}(E,\theta)]$$

$$\lambda_{\sigma} = \kappa \sqrt{1 + \rho_{\sigma} X} \qquad \theta_{c1,\uparrow} = \arcsin(1/\lambda_{\uparrow}) \qquad \theta_{c1,\downarrow} = \pi / 2$$



FSF

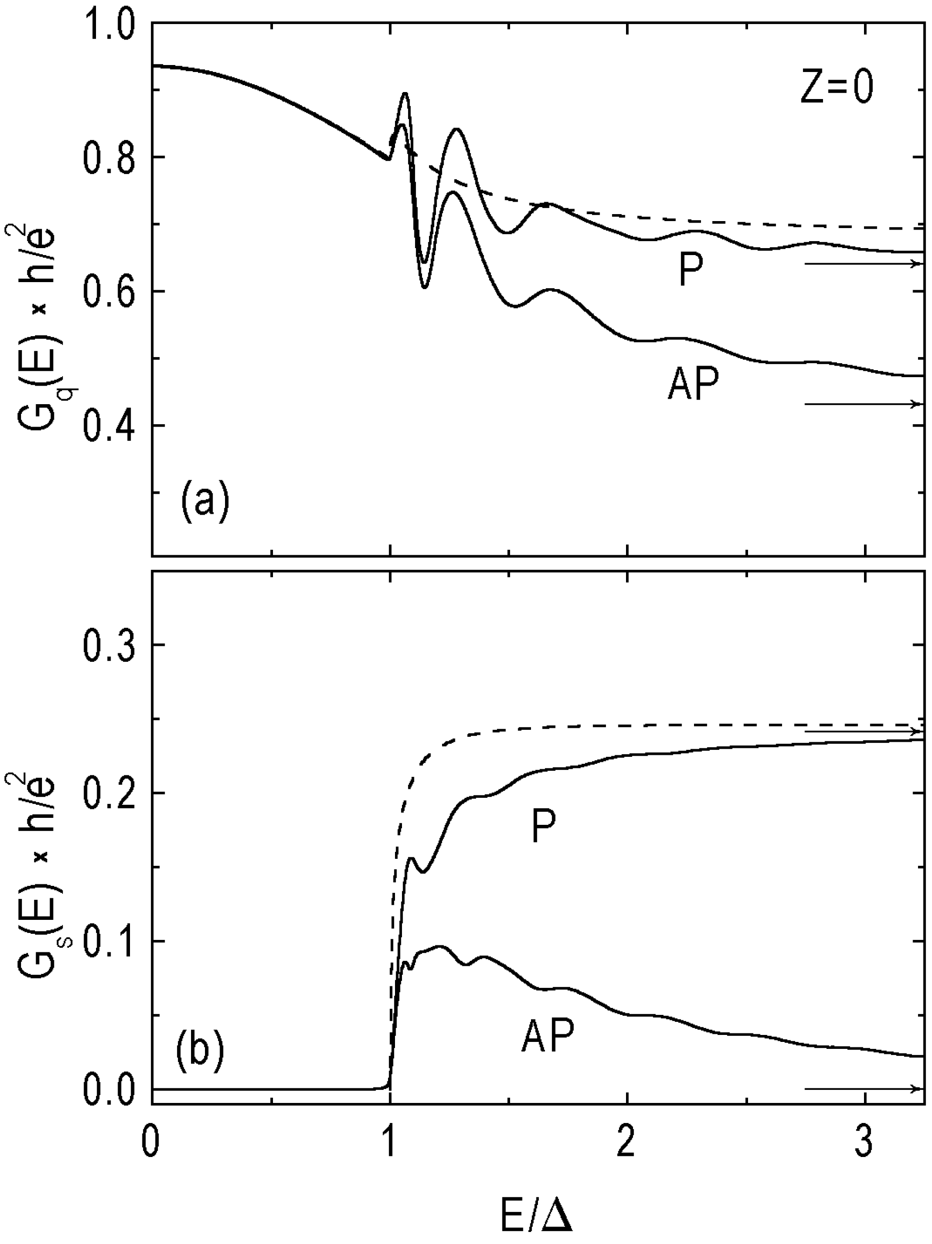
double junction (thin S film)

$Z = 0,\ X = 0.5$

$lk_F^{(S)} = 10^4\ [l/\xi_0 \approx 1]$

$\kappa = 1,\ \Delta / E_F^{(S)} = 10^{-3}$

M. Božović and Z. Radović, Phys. Rev. B **66**, 134524 (2002)



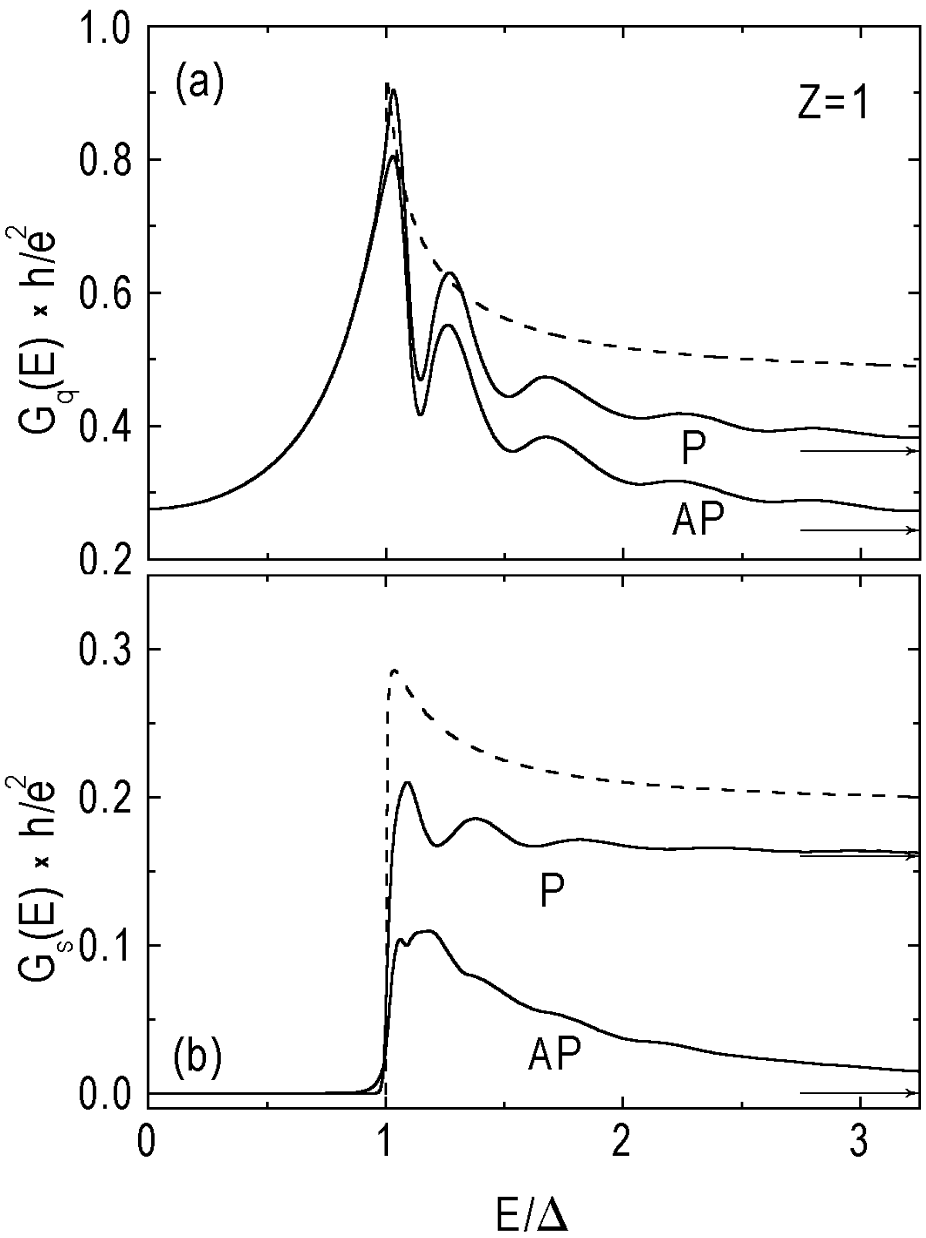
FSF double junction (thick S film)

$Z = 0,\ X = 0.5$

$lk_F^{(S)} = 10^4\ [l/\xi_0 \approx 10]$

$\kappa = 1,\ \Delta / E_F^{(S)} = 10^{-3}$

M. Božović and Z. Radović, Phys. Rev. B **66**, 134524 (2002)



FSF double junction (thick S film)

$Z = 1$, $X = 0.5$

$lk_F^{(S)} = 10^4$ $[l/\xi_0 \approx 10]$

$\kappa = 1$, $\Delta / E_F^{(S)} = 10^{-3}$

M. Božović and Z. Radović, Phys. Rev. B **66**, 134524 (2002)

Transparent NSN double junction

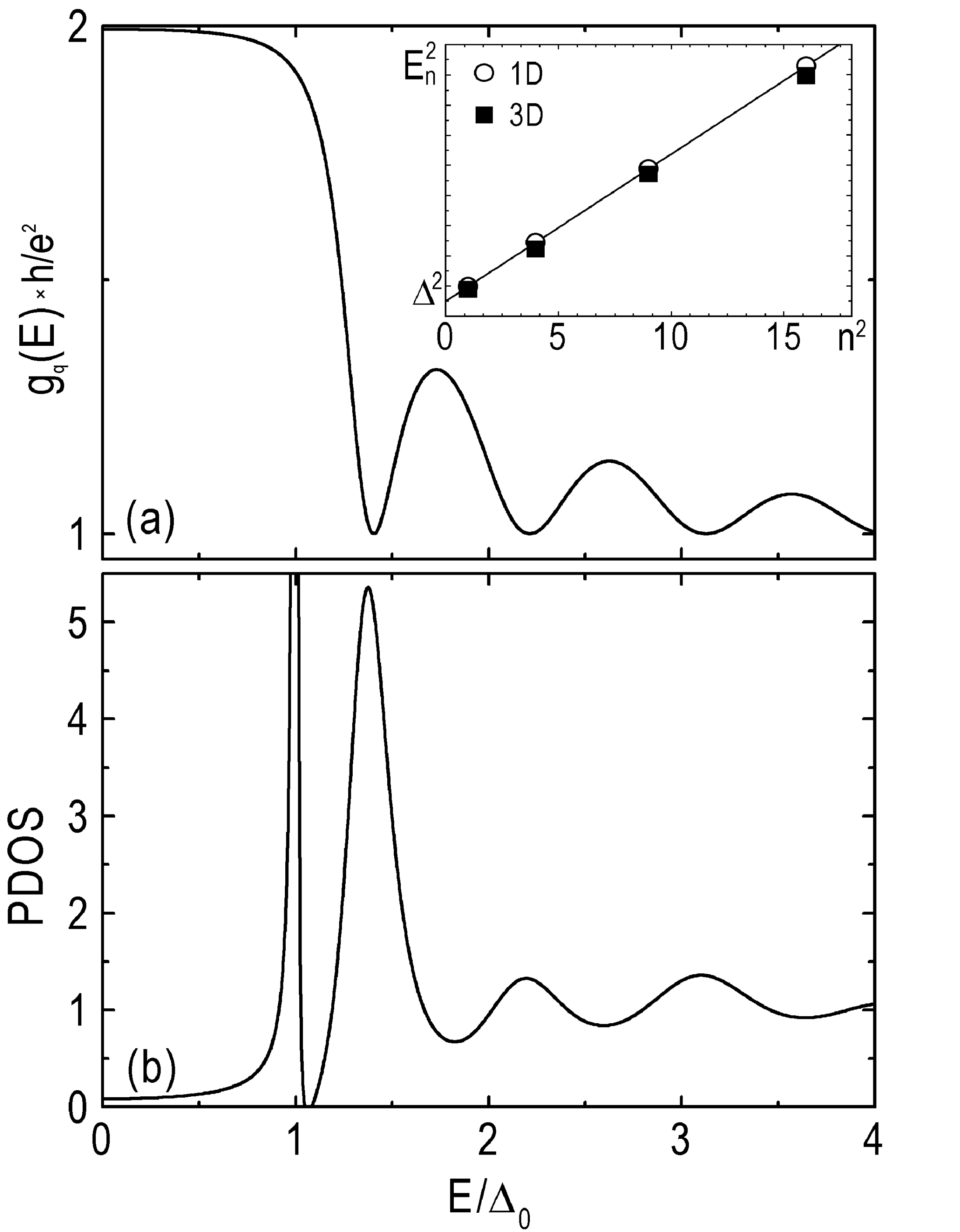
$$A_\sigma(E,\theta) = \left| \frac{\Delta \sin(\zeta_- / 2)}{E \sin(\zeta_- / 2) + i\Omega \cos(\zeta_- / 2)} \right|^2$$

$$\tilde{N}(z,\theta,E) = \frac{1}{\Gamma(E)\cos\theta} \times$$
$$\mathrm{Re}\left\{ 2E^2(E^2+\Omega^2) + 2E^2\Delta^2\cos\zeta \right.$$
$$+ \left[\cos(\zeta(z/d-1)) + \cos(\zeta z/d)\right]$$
$$\times \left[\Delta^4 - \Delta^2(E^2+\Omega^2)\cos\zeta\right]$$
$$\left. +2E^2\Delta^2[\sin(\zeta(z/d-1)) - \sin(\zeta z/d)]\sin\zeta \right\}$$

$$\Gamma(E) = [(E^2+\Omega^2)\cos\zeta - \Delta^2]^2 + 4E^2\Omega^2\sin^2\zeta$$

$$\frac{N(z,E)}{N(0)} = \int_0^{\pi/2} d\theta \sin\theta\cos\theta\, \tilde{N}(z,\theta,E)$$

M. Božović, Z. Pajović, and Z. Radović, Physica C, in press (2003).



NSN double junction (1D, thick S film)

$$\theta = 0$$

$$Z = 0$$

$$X = 0$$

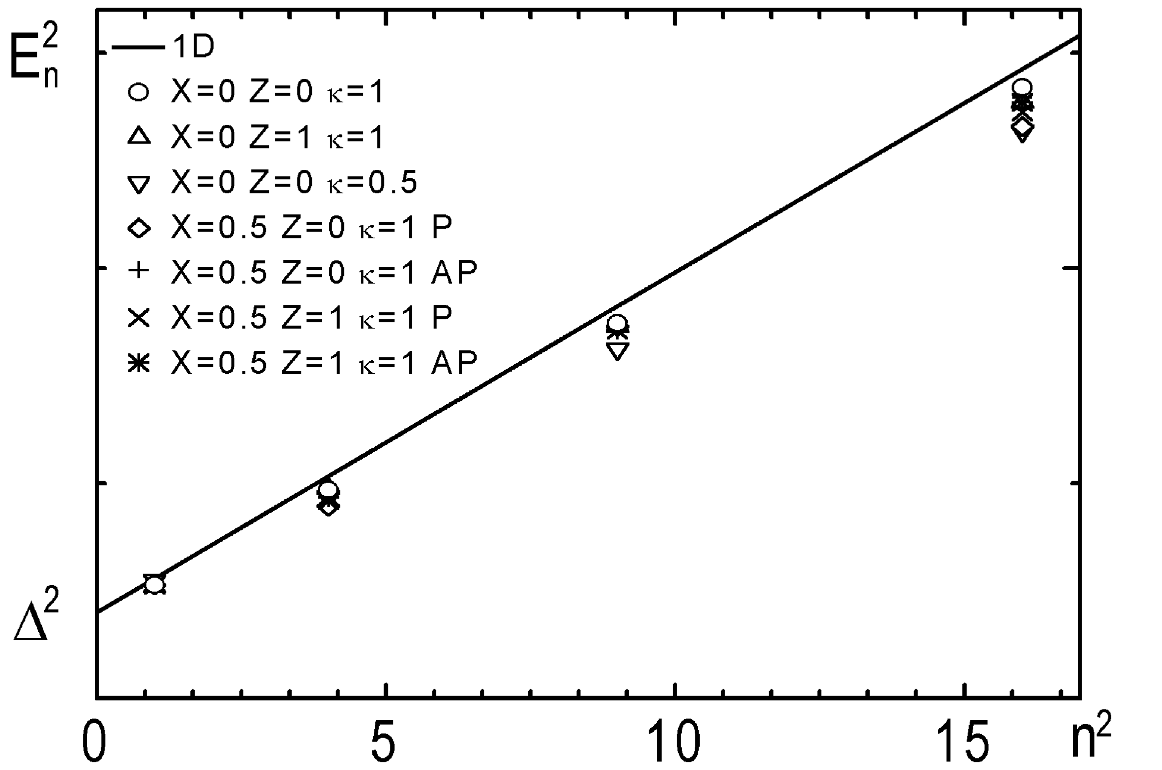
$$l / \xi_0 \approx 10$$

$$\kappa = 1,\ \Delta_0 / E_F^{(S)} = 10^{-3}$$

M. Božović, Z. Pajović, and Z. Radović, Physica C, in press (2003).

How to infer Δ and v_F in the superconductor?

Conductance minima satisfy: $$E_n^2 = \Delta^2 + \left(\frac{\pi \hbar v_F^{(S)}}{l} \right)^2 n^2$$

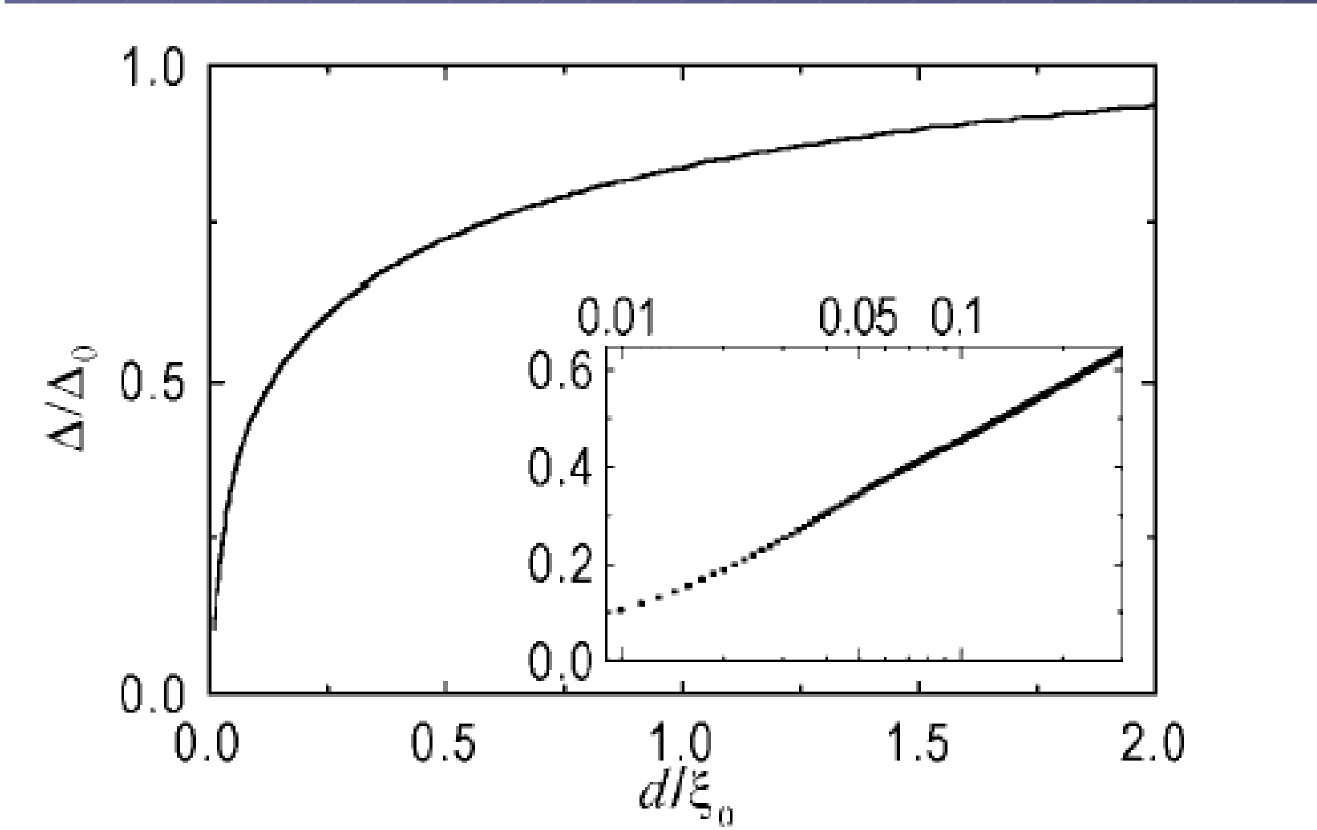


Ballistic spectroscopy

O. Nesher and G. Koren, Phys. Rev. B **60**, 9287 (1999).

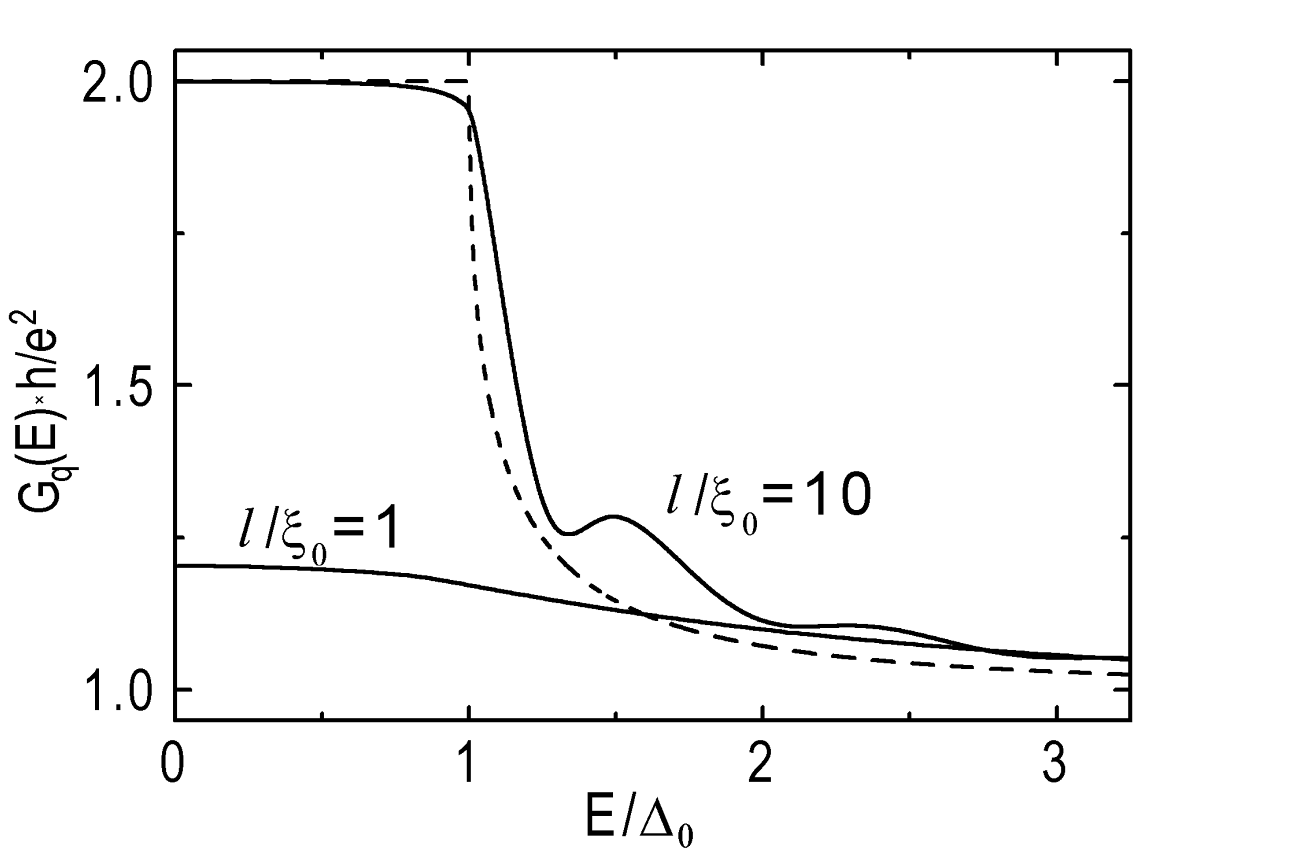
M. Božović and Z. Radović in *Supercond. and Rel. Ox.: Phys. and nanoeng. V, Proc. of SPIE, vol. 4811* (Seattle, 2002), p. 216.

Transparent NSN double junction: self-consistent pair potential in thin films



M. Božović, Z. Pajović, and Z. Radović,
Physica C, in press (2003).

Transparent NSN double junction: the conductance spectra



3D

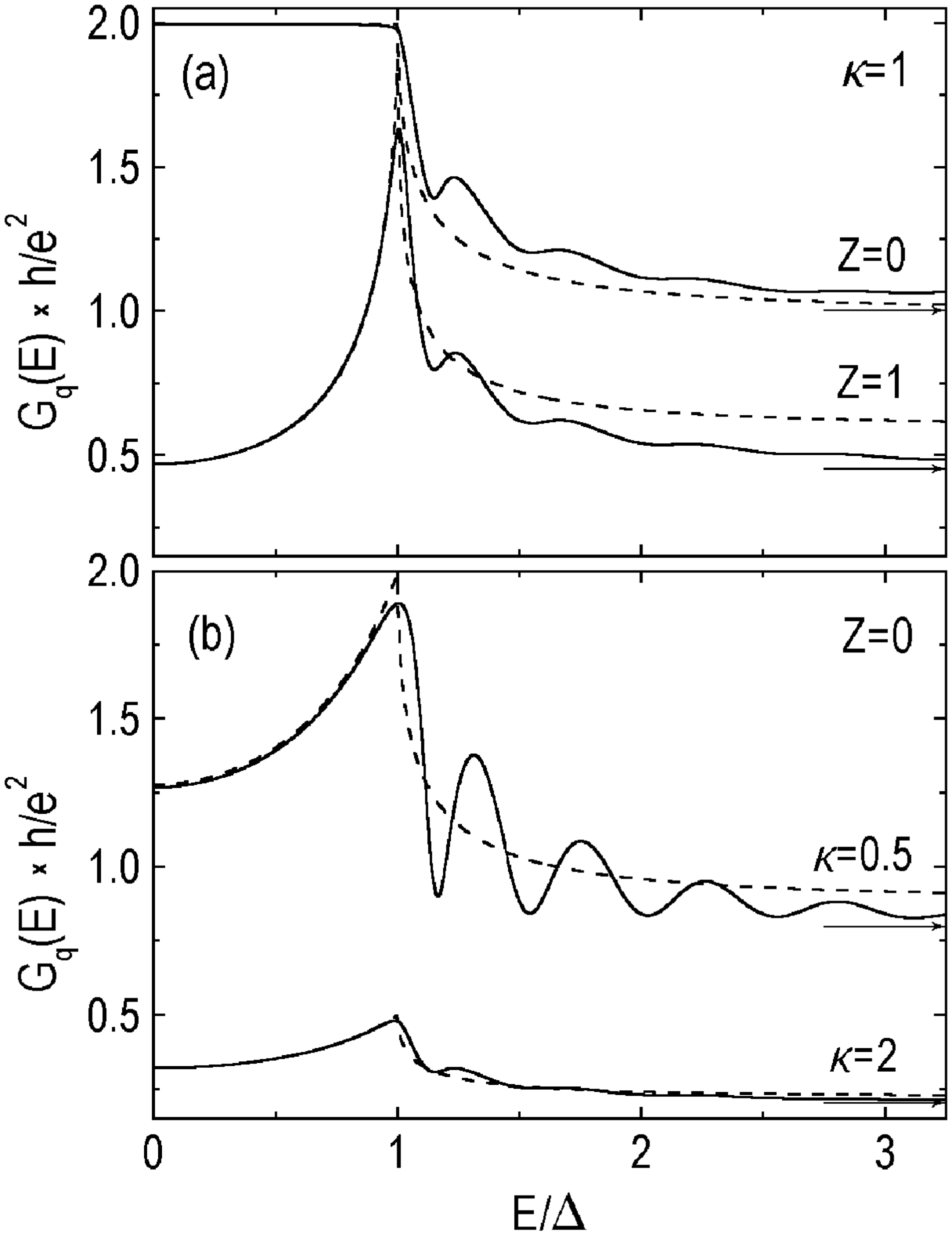
$Z=0$

$X=0$

$\kappa=1$

$\Delta/E_F^{(S)}=10^{-3}$

M. Božović, Z. Pajović, and Z. Radović, Physica C, in press (2003).



NSN double junction (thick S film)

Influence of κ and *Z*

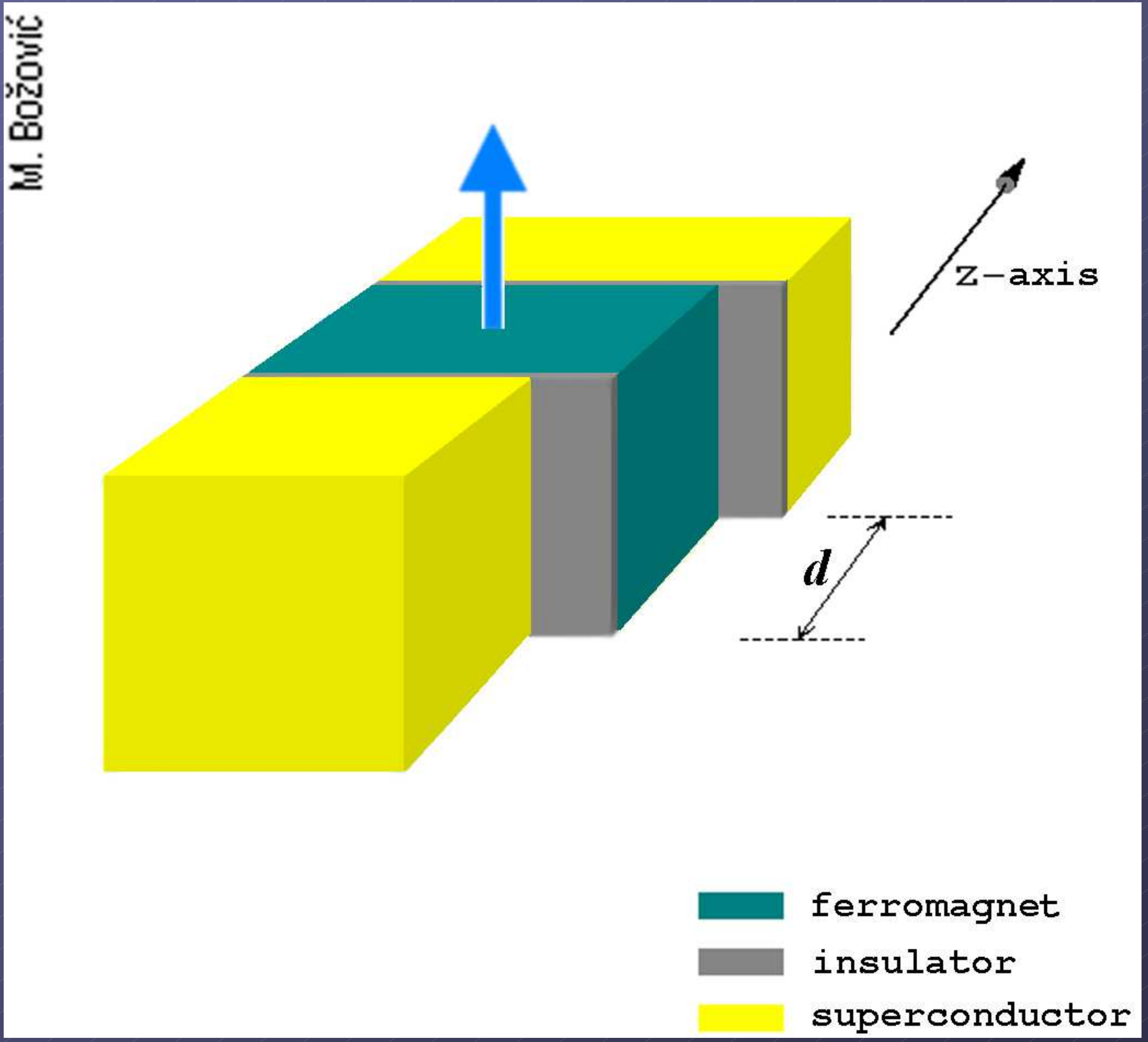
$$X = 0$$

$$lk_F^{(S)} = 10^4 \; [l/\xi_0 \approx 10]$$

$$\Delta / E_F^{(S)} = 10^{-3}$$

M. Božović and Z. Radović in *Supercond. and Rel. Ox.: Phys. and nanoeng. V, Proc. of SPIE, vol. 4811* (Seattle, 2002), p. 216.

The Model (SIFIS)



Scattering Problem

$$\begin{pmatrix} H_0(\mathbf{r}) - \rho_\sigma h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H_0(\mathbf{r}) + \rho_{\bar{\sigma}} h(\mathbf{r}) \end{pmatrix} \Psi_\sigma(\mathbf{r}) = E\Psi_\sigma(\mathbf{r})$$

$$\Psi_\sigma(\mathbf{r}) \equiv \begin{pmatrix} u_\sigma(\mathbf{r}) \\ v_{\bar{\sigma}}(\mathbf{r}) \end{pmatrix} = \exp(i\mathbf{k}_\parallel \cdot \mathbf{r})\psi_\sigma(z)$$

Exchange energy $h(\mathbf{r})/E_F^{(F)} = X\Theta(z)\Theta(l-z)$ $\rho_{\uparrow,\downarrow} = \pm 1$

Stepwise pair potential $\Delta(\mathbf{r}) = \Delta[\Theta(-z) \pm \Theta(z-l)]$

Interface potential $\hat{W}[\delta(z) + \delta(l-z)]$ $Z = 2m\hat{W}/\hbar^2 k_F^{(S)}$

FWVM parameter $\kappa = k_F^{(F)}/k_F^{(S)}$ $Z_\theta = Z/\cos\theta$

Solutions

$$\psi_1(z)=\begin{cases}[\exp(iq^+z)+b_1(E,\theta)\exp(-iq^+z)]\begin{pmatrix}\bar{u}e^{i\phi_L/2}\\ \bar{v}e^{-i\phi_L/2}\end{pmatrix}+a_1(E,\theta)\exp(iq^-z)\begin{pmatrix}\bar{v}e^{i\phi_L/2}\\ \bar{u}e^{-i\phi_L/2}\end{pmatrix}, & z<0\\ [C_1(E,\theta)\exp(ik_\sigma^+z)+C_2(E,\theta)\exp(-ik_\sigma^+z)]\begin{pmatrix}1\\0\end{pmatrix} & \\ +[C_3(E,\theta)\exp(ik_{\bar{\sigma}}^-z)+C_4(E,\theta)\exp(-ik_{\bar{\sigma}}^-z)]\begin{pmatrix}0\\1\end{pmatrix}, & 0<z<d\\ c_1(E,\theta)\exp(iq^+z)\begin{pmatrix}\bar{u}e^{i\phi_R/2}\\ \bar{v}e^{-i\phi_R/2}\end{pmatrix}+d_1(E,\theta)\exp(-iq^-z)\begin{pmatrix}\bar{v}e^{i\phi_R/2}\\ \bar{u}e^{-i\phi_R/2}\end{pmatrix}, & z>d\end{cases}$$

$$\bar{u}=\sqrt{(1+\Omega/E)/2} \qquad \bar{v}=\sqrt{(1-\Omega/E)/2}$$

$$\Omega=\sqrt{E^2-\Delta^2}$$

Wave vector components

Perpendicular component in the ferromagnet

$$k_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2)\left(E_F^{(F)} + \rho_\sigma h_0 \pm E\right) - \mathbf{k}_{\|}^2}$$

Perpendicular component in the superconductors

$$q_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2)\left(E_F^{(S)} \pm \Omega\right) - \mathbf{k}_{\|}^2}$$

Conserved parallel component

$$\left|\mathbf{k}_{\|}\right| = \sqrt{(2m/\hbar^2)\left(E_F^{(S)} + \Omega\right)} \sin\theta$$

Wave vector components

Neglecting $\Omega / E_F^{(S)} << 1$ and $\Delta / E_F^{(S)} << 1$

except in the exponents $\zeta_\sigma^\pm = d\left(k_\sigma^+ \pm k_{\bar{\sigma}}^-\right)$

the reduced wave-vector components,
in units of $k_F^{(S)} \cos\theta$ are

$$\beta_\sigma = \frac{\sqrt{\lambda_\sigma^2 - \sin^2\theta}}{\cos\theta}$$

where

$$\lambda_\sigma = \kappa\sqrt{1 + \rho_\sigma X}$$

The Josephson current

$$I = \frac{4\pi k_B T \Delta^2}{eR} \int_0^{\pi/2} d\theta \sin\theta \cos\theta \sum_{\omega_n, \sigma} \frac{\beta_\sigma \beta_{\bar{\sigma}} \sin\phi}{G_n}$$

$$G_n = 8\Delta^2 \beta_\sigma \beta_{\bar{\sigma}} \cos\phi - \mathcal{G}_1^- \cos(\zeta_\sigma^-) + \mathcal{G}_1^+ \cos(\zeta_\sigma^+) + i\mathcal{G}_2^- \sin(\zeta_\sigma^-) - i\mathcal{G}_2^+ \sin(\zeta_\sigma^+)$$

$$\begin{aligned}
\mathcal{G}_1^\pm &= -\left\{\omega_n \left(\beta_\sigma \mp \beta_{\bar{\sigma}}\right) + \Omega_n \left[1 + Z_\theta^2 - iZ_\theta \left(\beta_\sigma \pm \beta_{\bar{\sigma}}\right) \mp \beta_\sigma \beta_{\bar{\sigma}}\right]\right\}^2 \\
&\quad - \left\{\omega_n \left(\beta_\sigma \mp \beta_{\bar{\sigma}}\right) - \Omega_n \left[1 + Z_\theta^2 + iZ_\theta \left(\beta_\sigma \pm \beta_{\bar{\sigma}}\right) \mp \beta_\sigma \beta_{\bar{\sigma}}\right]\right\}^2 \\
\mathcal{G}_2^\pm &= 4i\Omega_n \left(1 + Z_\theta^2 \mp \beta_\sigma \beta_{\bar{\sigma}}\right) \left[i\omega_n \left(\beta_\sigma \mp \beta_{\bar{\sigma}}\right) + \Omega_n Z_\theta \left(\beta_\sigma \pm \beta_{\bar{\sigma}}\right)\right]
\end{aligned}$$

$$\Omega_n = \sqrt{\omega_n^2 + \Delta^2} \qquad \omega_n = \pi k_B T (2n+1)$$

The normal resistance

$$\frac{R}{R_N} = \int_0^{\pi/2} d\theta \sin\theta \cos\theta \sum_\sigma \left(1 - |b_N|^2\right)$$

$$b_N = \frac{2 Z_\theta \beta_\sigma \cos(dk_\sigma^+) + \left(1 + Z_\theta^2 - \beta_\sigma^2\right) \sin(dk_\sigma^+)}{2i(1 + iZ_\theta)\beta_\sigma \cos(dk_\sigma^+) + (1 + 2iZ_\theta - Z_\theta^2 + \beta_\sigma^2) \sin(dk_\sigma^+)}$$

$$R = 2\pi^2 \hbar / S e^2 {k_F^{(F)}}^2$$

Generalization of the Furusaki-Tsukada formula …

A. Furusaki and M. Tsukada, Phys. Rev. B **43**, 10164 (1991).

… to the ballistic double-barrier SNS junction

Z. Radović, N. Lazarides, and N. Flytzanis, Phys. Rev. B, in press (2003)

SNS junction (*X*=0)

$$I = \frac{\pi k_B T \Delta^2}{eR} \int_0^{\pi/2} d\theta \sin\theta \cos\theta \sum_{\omega_n} \frac{\sin\phi}{\Gamma_n}$$

$$\Gamma_n = \Delta^2 \cos\phi + (K^2\Omega_n^2 + \omega_n^2)\cosh\left(\frac{2\omega_n d}{\hbar v_N}\right) + 2K\omega_n\Omega_n \sinh\left(\frac{2\omega_n d}{\hbar v_N}\right)$$
$$- (K^2 - 1 - 2Z_\theta^2)\Omega_n^2 \cos(2k_N d) + 2Z_\theta\left(K^2 - 1 - Z_\theta^2\right)^{1/2}\Omega_n^2 \sin(2k_N d)$$

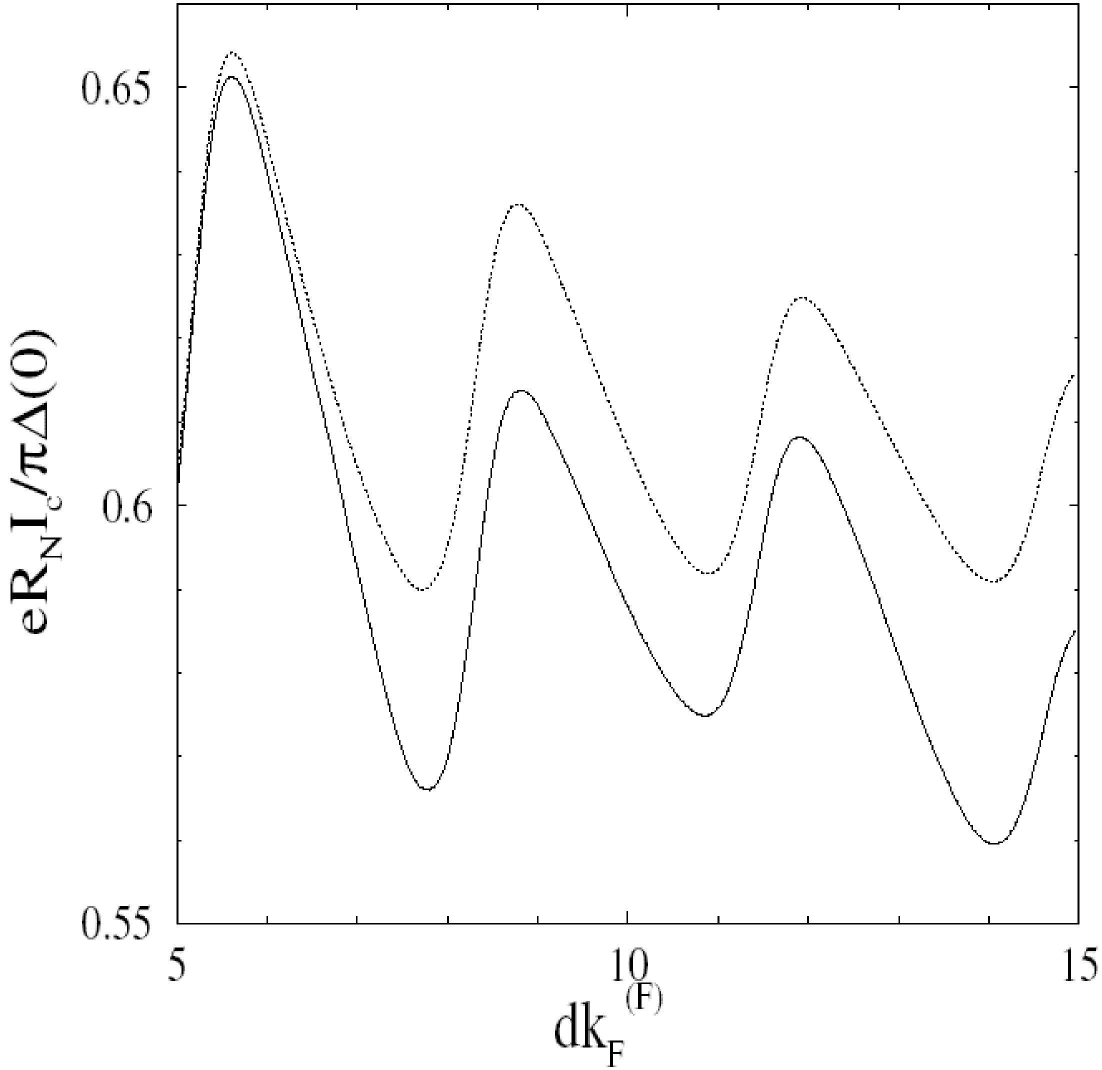
$$K = \frac{1}{2}\left(\beta + \frac{1 + Z_\theta^2}{\beta}\right)$$

$$\beta = \frac{k_N}{k_S} \qquad v_N = \frac{\hbar k_N}{m}$$

$$k_N = \sqrt{k_F^{(N)2} - \mathbf{k}_\parallel^2} \qquad k_S = \sqrt{k_F^{(S)2} - \mathbf{k}_\parallel^2}$$

$$\left|\mathbf{k}_\parallel\right| = k_F^{(S)} \sin\theta$$

SFS double junction: the maximum current I_c



$T / T_c = 0.1$

$Z = 1$

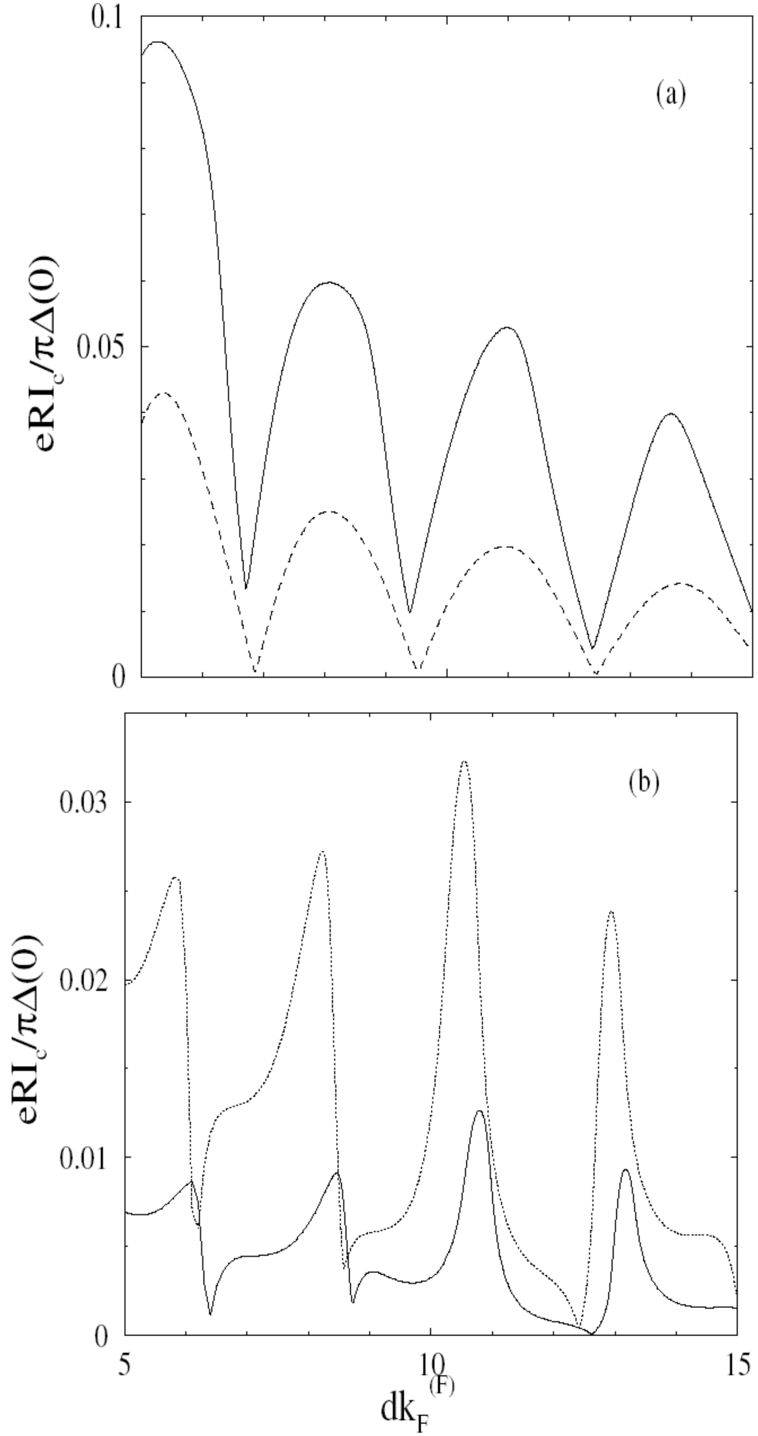
$X = 0.01$ (solid)

$X = 0$ (dotted)

$\kappa = 1$

$\Delta / E_F^{(S)} = 10^{-3}$

Z. Radović, N. Lazarides, and N. Flytzanis, Phys. Rev. B, in press (2003)



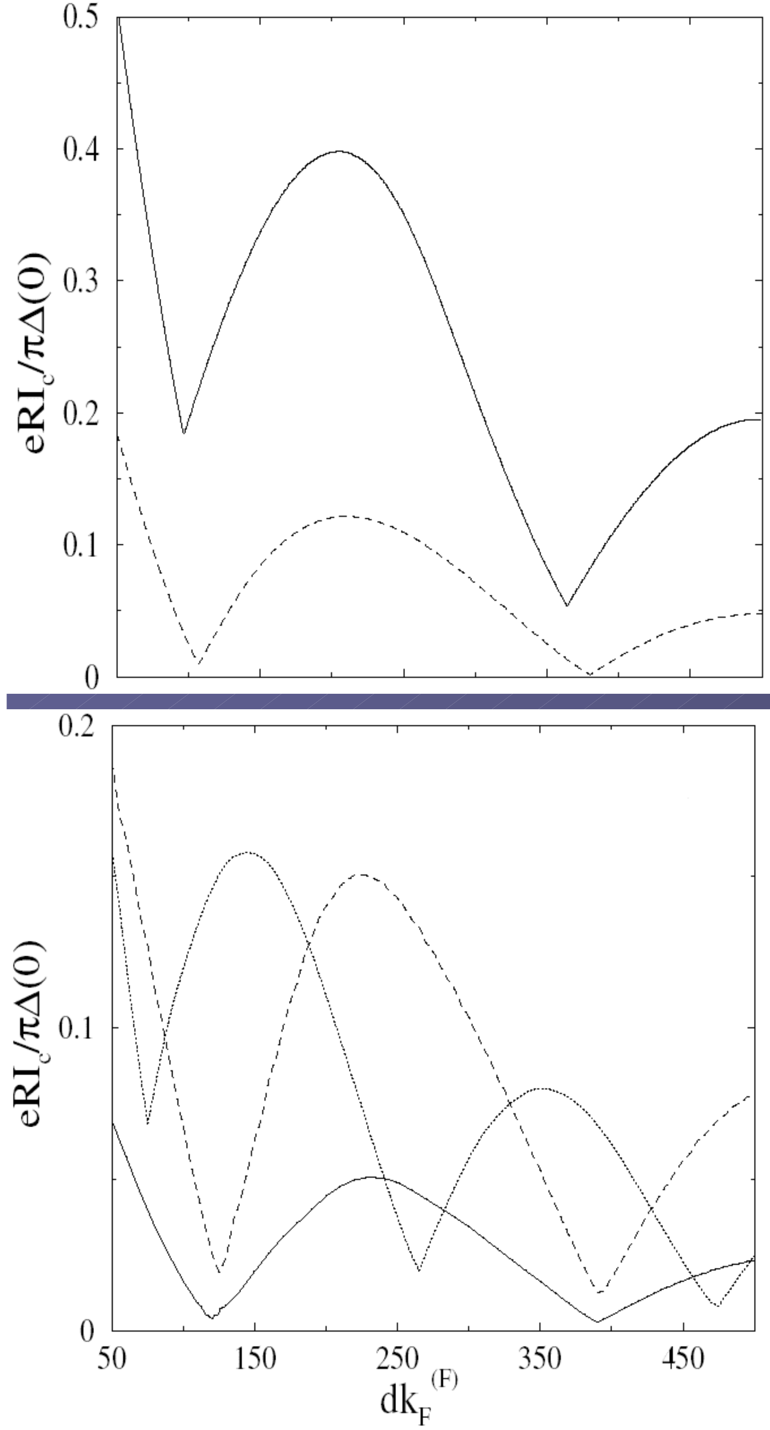
Strong ferromagnet

$$X = 0.9$$

$$\Delta / E_F^{(S)} = 10^{-3}$$

Top panel:	Bottom panel:
$Z = 0$	$Z = 1$
$\kappa = 1$	$T / T_c = 0.1$
$T / T_c = 0.1$ (solid)	$\kappa = 0.7$ (solid)
$T / T_c = 0.7$ (dotted)	$\kappa = 1$ (dotted)

Z. Radović, N. Lazarides, and N. Flytzanis, Phys. Rev. B, in press (2003)



Weak ferromagnet

$$X = 0.01$$

$$\Delta / E_F^{(S)} = 10^{-3}$$

Top panel:

$Z = 0$

$\kappa = 1$

$T / T_c = 0.1$ (solid)

$T / T_c = 0.7$ (dashed)

Bottom panel:

$T / T_c = 0.1$

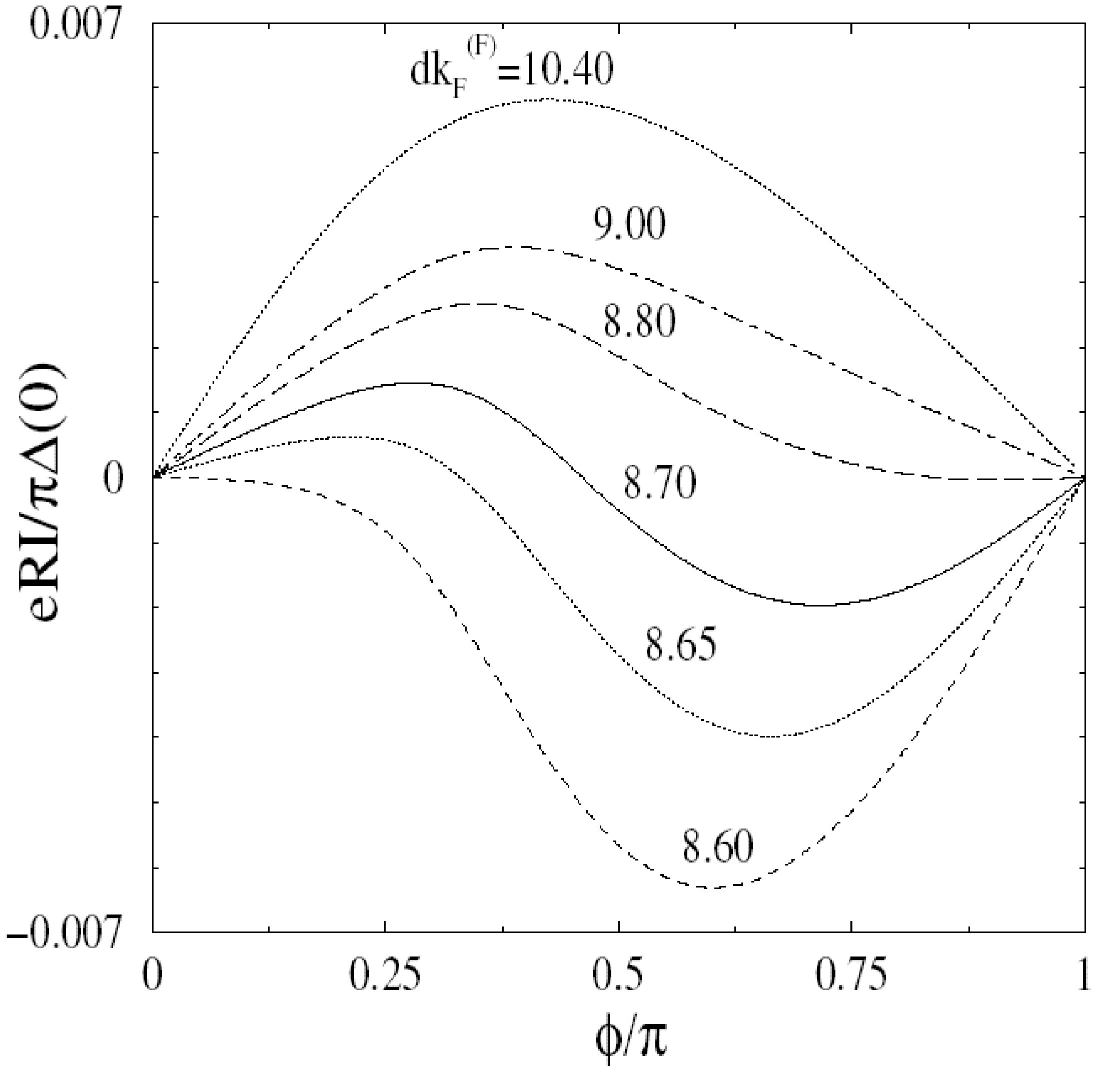
$Z = 1,\ \kappa = 0.7$ (solid)

$Z = 1,\ \kappa = 1$ (dashed)

$Z = 0,\ \kappa = 0.7$ (dotted)

Z. Radović, N. Lazarides, and N. Flytzanis, Phys. Rev. B, in press (2003)

SFS double junction: the current-phase relation close to the transition



$Z = 1$

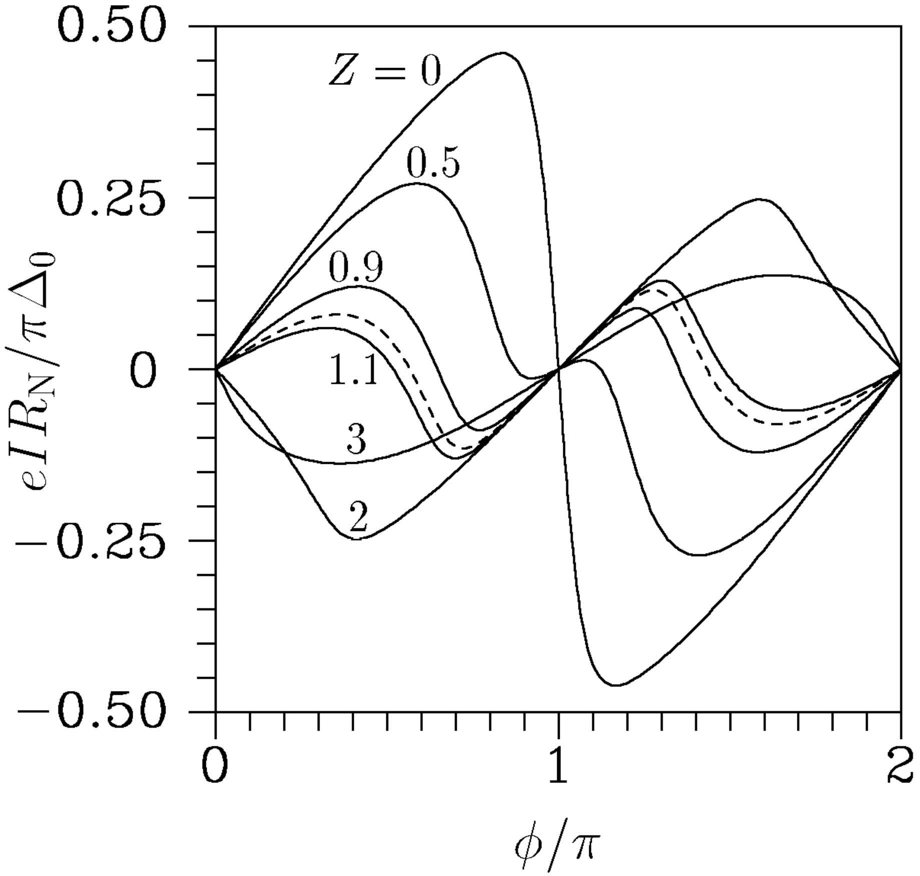
$X = 0.9$

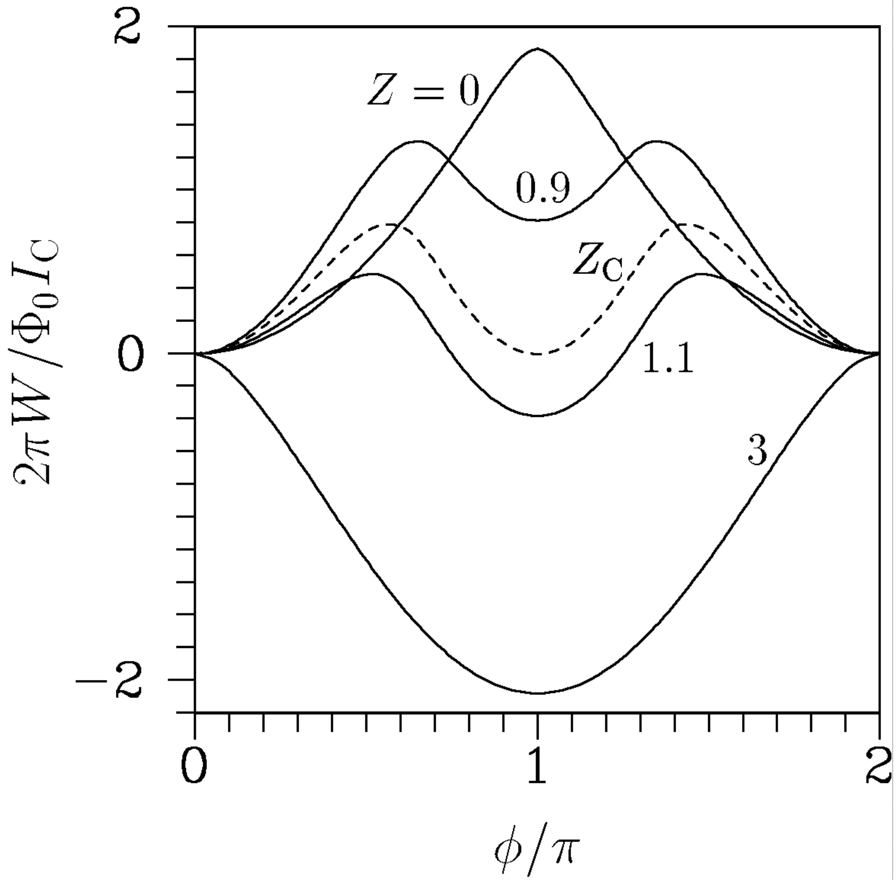
$\kappa = 0.7$

$\Delta / E_F^{(S)} = 10^{-3}$

Z. Radović, N. Lazarides, and N. Flytzanis, Phys. Rev. B, in press (2003)

Coexistence of stable and metastable 0 and π states

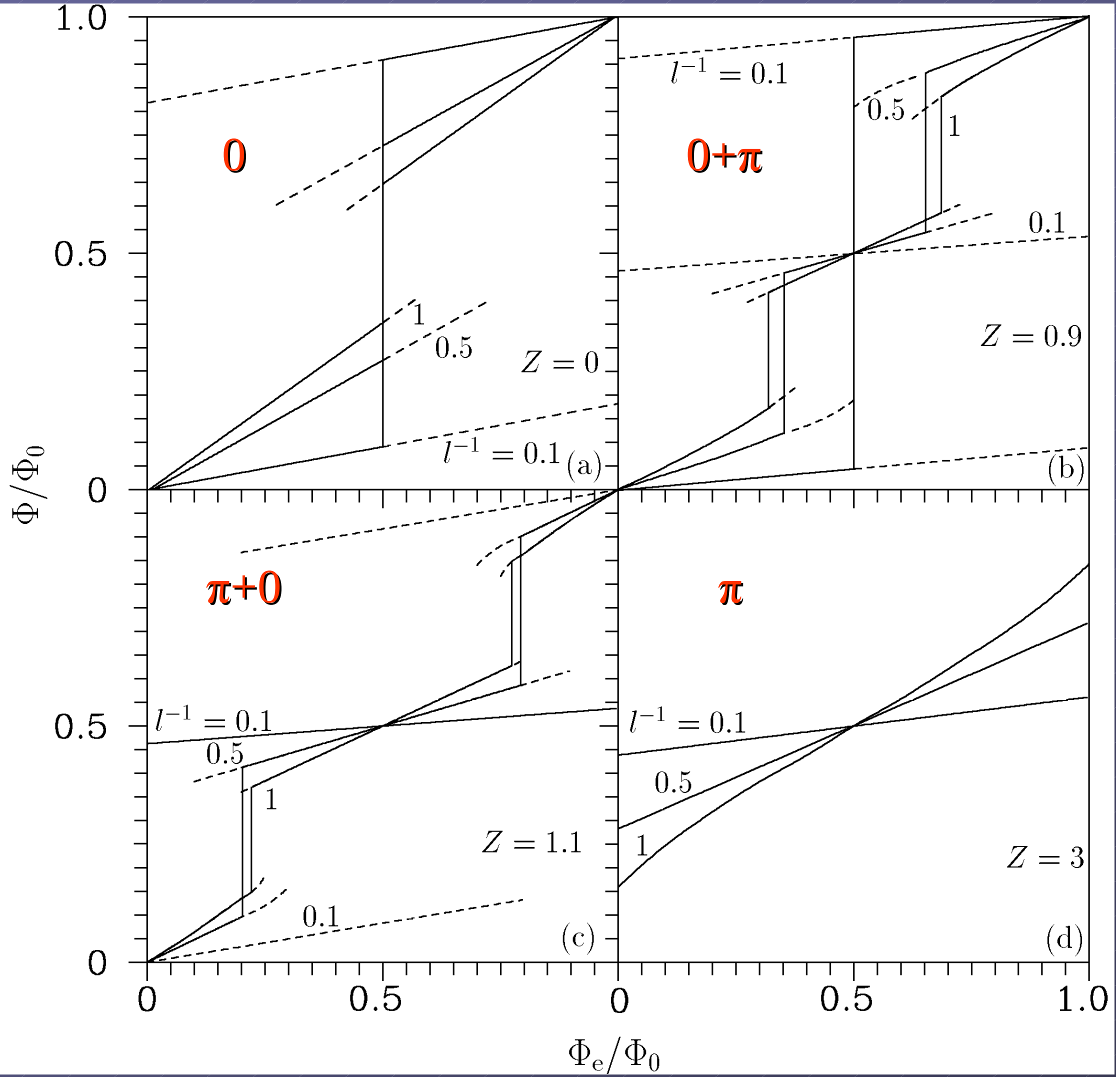




$Z = X\, d\, k_F$

Z. Radović *et al.*,
Phys. Rev. B **63**, 214512 (2001).

Magnetic flux vs. external flux in SQUIDs



Effectively two times smaller flux quantum

$$l = \frac{2\pi}{\Phi_0} L I_c$$

Z. Radović *et al.*,
Phys. Rev. B **63**, 214512 (2001).

Temperature-induced 0-π transition

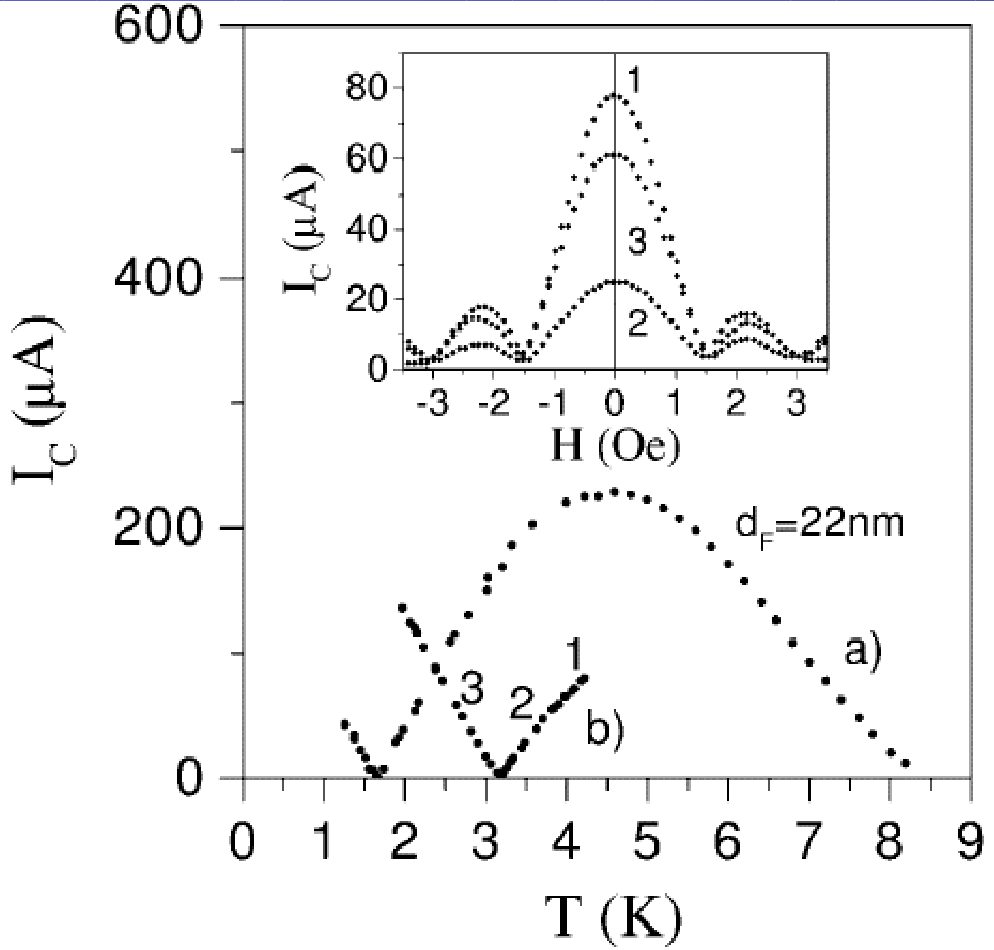
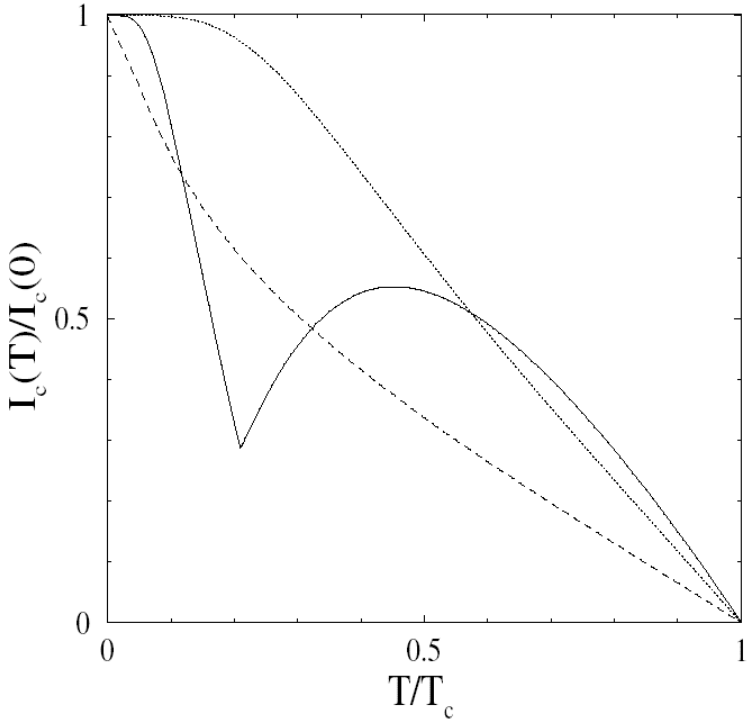
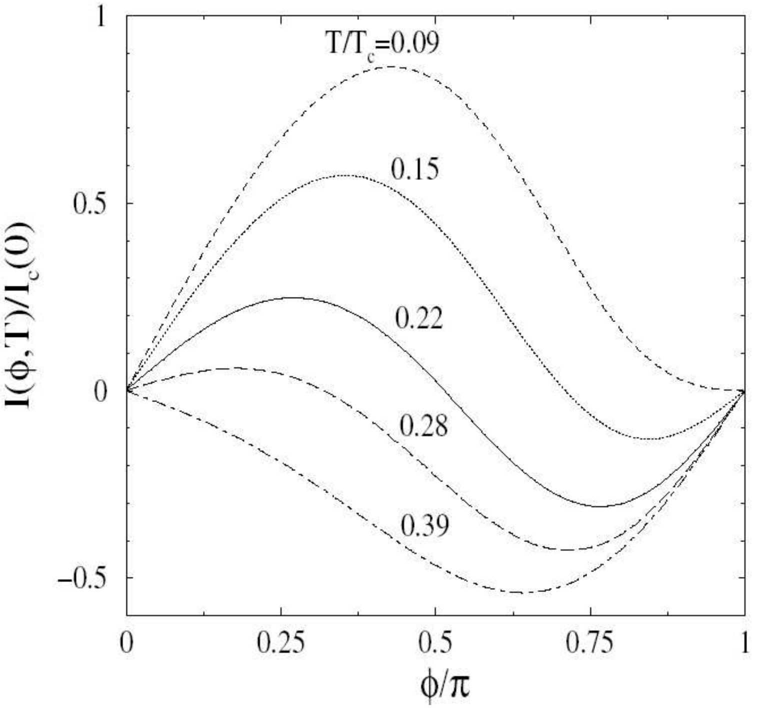


FIG. 3. Critical current I_c as a function of temperature T for two junctions with $Cu_{0.48}Ni_{0.52}$ and $d_F = 22$ nm [17]. Inset: I_c versus magnetic field H for the temperatures around the cross-over to the π state as indicated on curve b: (1) $T = 4.19$ K, (2) $T = 3.45$ K, (3) $T = 2.61$ K.

V. V. Ryazanov *et al.*, Phys. Rev. Lett. **86**, 2427 (2001).





Temperature-induced 0-π transition

finite transparency → $Z = 1.2$

strong ferromagnet → $X = 0.92$

$$Z = 1.2 \quad X = 0.92 \quad \kappa = 1$$

$$\Delta / E_F^{(S)} = 10^{-3}$$

Top panel:

$dk_F^{(F)} = 17$ (dotted)

$dk_F^{(F)} = 17.23$ (solid)

$dk_F^{(F)} = 17.4$ (dashed)

Bottom panel:

$dk_F^{(F)} = 17.23$

five values of T

Z. Radović, N. Lazarides, and N. Flytzanis, Phys. Rev. B, in press (2003)

Conclusion

- Features of finite size and coherency in clean FISIF:

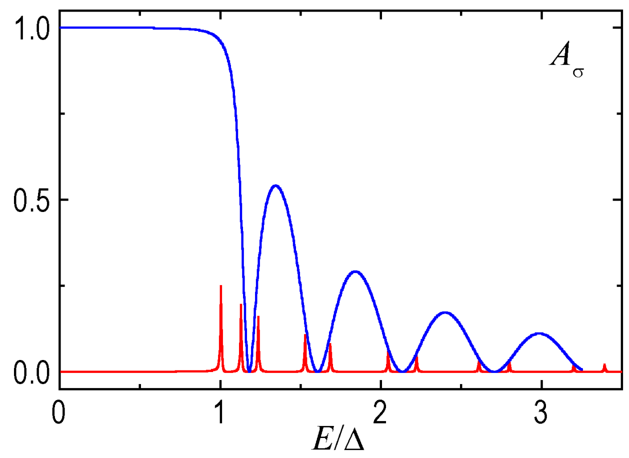
(1) Subgap transport of electrons

(reduction of the excess current in thin S film)

(2) Oscillations of differential conductances

(vanishing of the Andreev reflection at geometrical resonances)

(3) Resonances in metallic vs. bound states in tunnel junctions



Conclusion

- Non-trivial spin polarization of the current without excess spin accumulation in S, i.e. without destruction of superconductivity, even in the AP alignment.

- Reliable ballistic spectroscopy of quasiparticle excitations in superconductors – measurements of Δ and v_F.

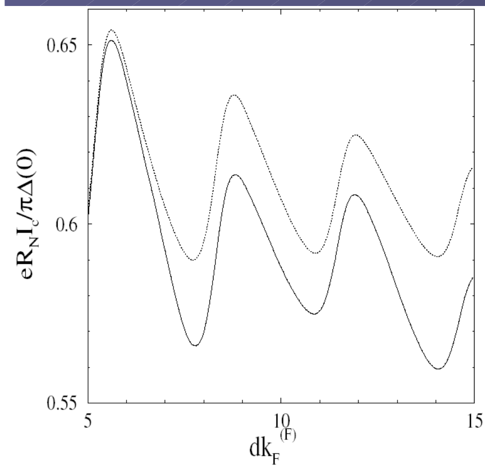
Conclusion

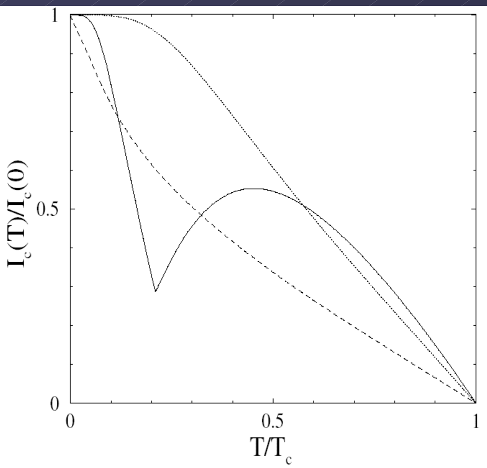
- Features of finite size and coherency in clean SIFIS:

(1) Geometrical oscillations of the maximum Josephson current

(2) Oscillations related to the crossovers between 0 and π states

(3) Temperature-induced 0-π transition when junction is close to the crossover at *T*=0, with finite transparency and strong ferromagnet





Conclusion

- Region of coexisting 0 and π states is considerably large.

  Possible application:

  π SQUID with improved accuracy which operates with effectively 2x smaller flux quantum