

Contribution of geometrical resonances to the Andreev process in double-barrier FISIF and SIFIS structures

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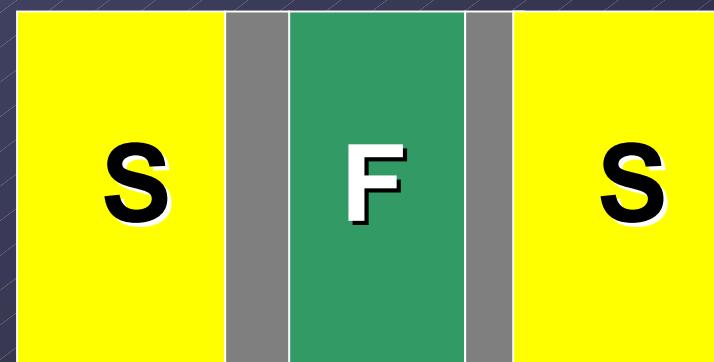
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Outline

- FIS vs. FISIF (SIFIS) junctions – microscopic theory
- Incoherent transport and spin accumulation
- Coherent transport in clean FISIF junctions:
 - Scattering problem
 - Differential conductances (charge and spin)
- Coherent transport in clean SIFIS junctions :
 - Scattering problem
 - dc Josephson current

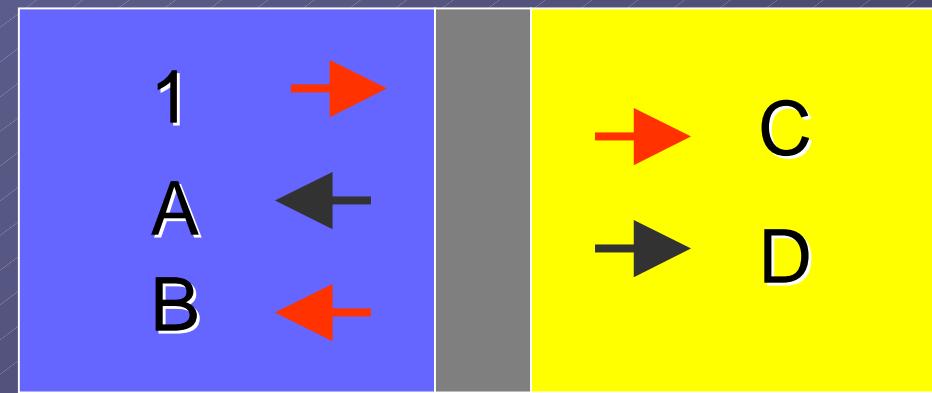
Why FS vs. FSF (SFS) junctions?

- Tunneling spectroscopy of superconductors by spin-polarized currents.
- Interplay of ferromagnetism and superconductivity.
- Ballistic transport – interference effects
- LCs in quantum computers ?



The BTK model

G. E. Blonder, M. Tinkham, and T. M. Klapwijk,
Phys. Rev. B 25, 4515 (1982).



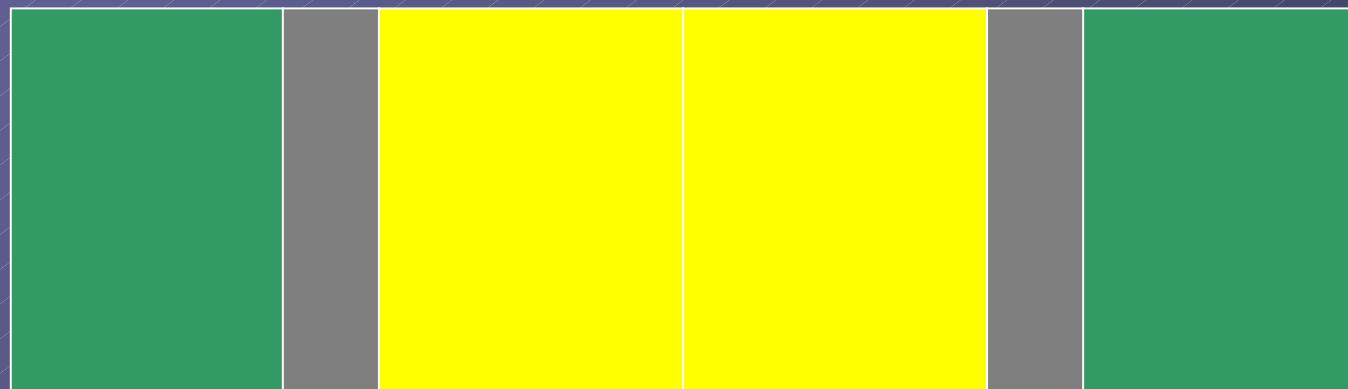
Normal metal

Insulating barrier
of arbitrary strength

conventional
Superconductor

Incoherent transport

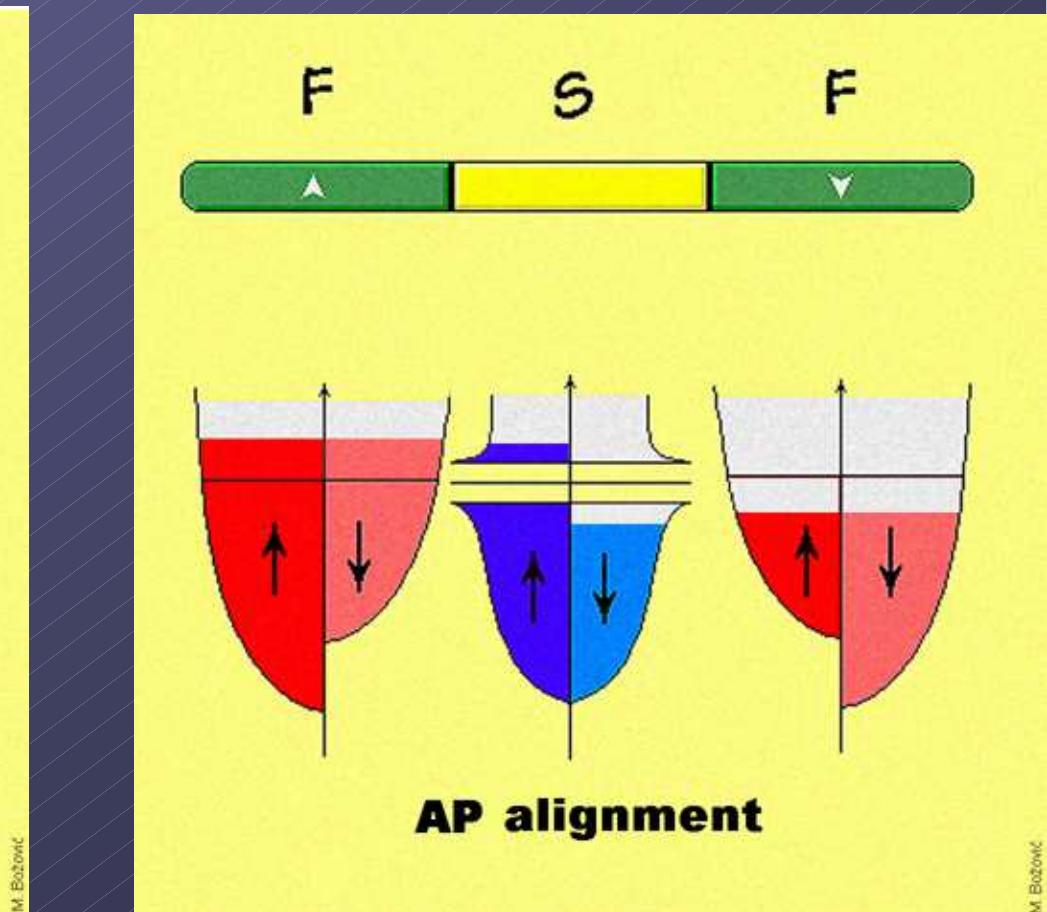
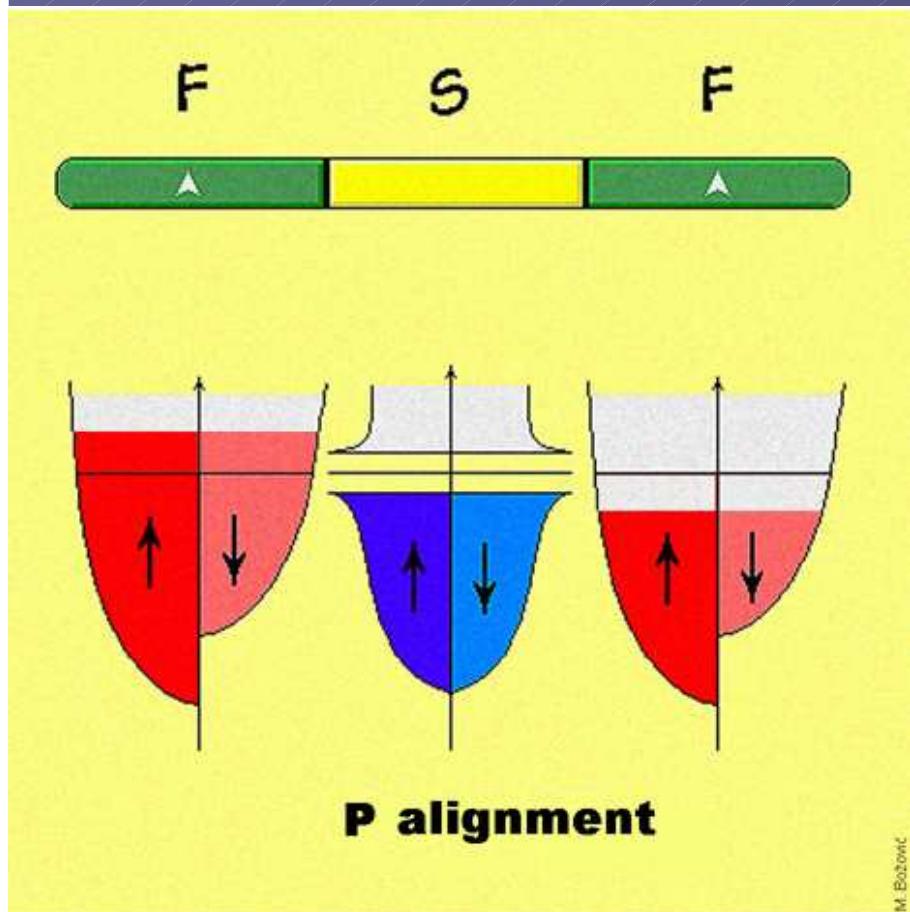
Z. Zheng *et al.*, Phys. Rev. B 62, 14 326 (2000).



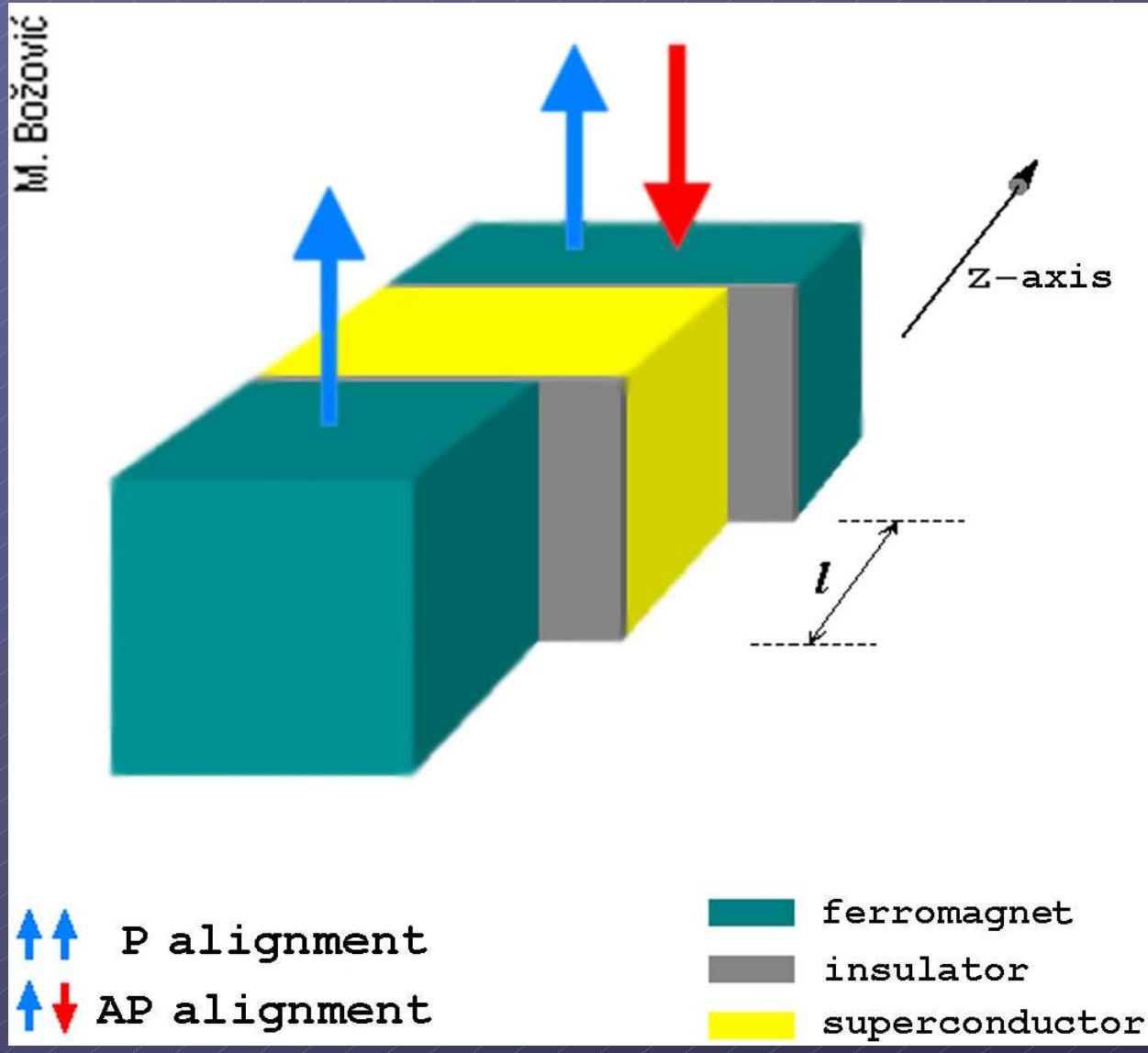
F S + S F

Incoherent transport

S. Takahashi, I. Imamura, and S. Maekawa,
Phys. Rev. Lett. 82, 3911 (1999).



The Model (FISIF)



Scattering Problem

$$\begin{pmatrix} H_0(\mathbf{r}) - \rho_\sigma h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H_0(\mathbf{r}) + \rho_{\bar{\sigma}} h(\mathbf{r}) \end{pmatrix} \Psi_\sigma(\mathbf{r}) = E \Psi_\sigma(\mathbf{r})$$

$$\Psi_\sigma(\mathbf{r}) \equiv \begin{pmatrix} u_\sigma(\mathbf{r}) \\ v_{\bar{\sigma}}(\mathbf{r}) \end{pmatrix} = \exp(i\mathbf{k}_{\parallel,\sigma} \cdot \mathbf{r}) \psi(z)$$

Exchange energy $h(\mathbf{r})/E_F^{(F)} = \textcolor{blue}{X}[\Theta(-z) \pm \Theta(z-l)]$ $\rho_{\uparrow,\downarrow} = \pm 1$

Stepwise pair potential $\Delta(\mathbf{r}) = \Delta \Theta(z) \Theta(l-z)$

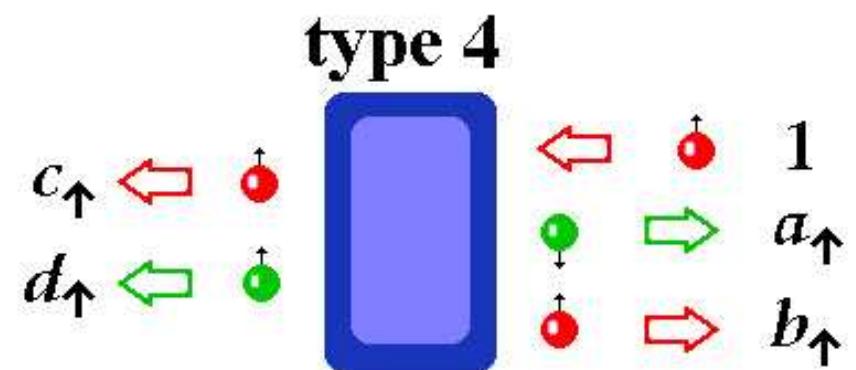
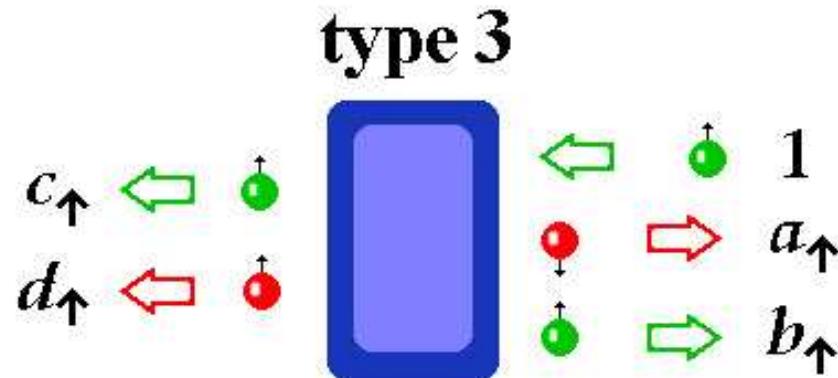
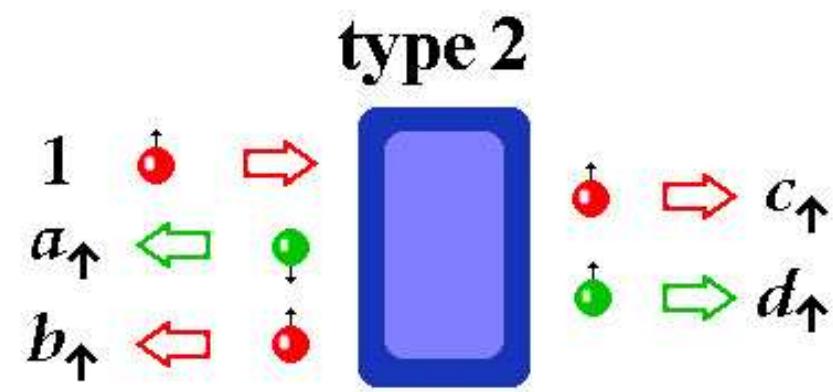
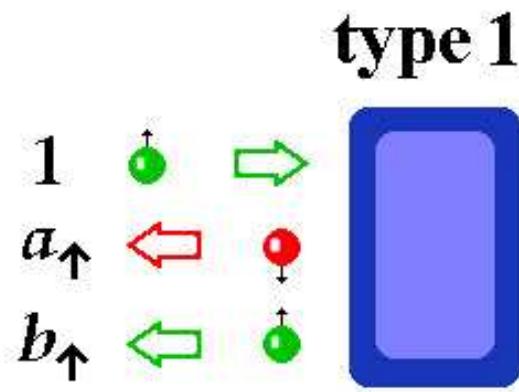
Interface potential $\hat{W}[\delta(z) + \delta(l-z)]$ $\textcolor{blue}{Z} = 2m\hat{W}/\hbar^2 k_F^{(S)}$

FWVM parameter $\kappa = k_F^{(F)}/k_F^{(S)}$

Scattering Problem

A. Furusaki and M. Tsukada,
Solid State Commun. **78**, 299 (1991).

M. Božović



Solutions

$$\psi_1(z) = \begin{cases} [\exp(ik_\sigma^+ z) + b_\sigma(E, \theta) \exp(-ik_\sigma^- z)] \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_\sigma(E, \theta) \exp(ik_\sigma^- z) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & z < 0 \\ [c_1(E, \theta) \exp(iq_\sigma^+ z) + c_2(E, \theta) \exp(-iq_\sigma^+ z)] \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \\ + [c_3(E, \theta) \exp(iq_\sigma^- z) + c_4(E, \theta) \exp(-iq_\sigma^- z)] \begin{pmatrix} \bar{v} \\ \bar{u} \end{pmatrix}, & 0 < z < l \\ c_\sigma(E, \theta) \exp(ik_{\sigma[\bar{\sigma}]}^+ z) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + d_\sigma(E, \theta) \exp(-ik_{\bar{\sigma}[\sigma]}^+ z) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & z > l \end{cases}$$

$$\bar{u} = \sqrt{(1 + \Omega/E)/2} \quad \bar{v} = \sqrt{(1 - \Omega/E)/2}$$

$$\Omega = \sqrt{E^2 - \Delta^2}$$

Wave vector components

Perpendicular component in the **ferromagnets**

$$k_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2) \left(E_F^{(F)} + \rho_{\sigma} h_0 \pm E \right) - \mathbf{k}_{\parallel, \sigma}^2}$$

Perpendicular component in the **superconductor**

$$q_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2) \left(E_F^{(S)} \pm \Omega \right) - \mathbf{k}_{\parallel, \sigma}^2}$$

Conserved parallel component

$$|\mathbf{k}_{\parallel, \sigma}| = \sqrt{(2m/\hbar^2) \left(E_F^{(F)} + \rho_{\sigma} h_0 + E \right) \sin \theta}$$

Wave vector components

Neglecting $E / E_F^{(F)} \ll 1$ and $\Delta / E_F^{(S)} \ll 1$

except in the exponents $\zeta_{\pm} = l(q_{\sigma}^{+} \pm q_{\sigma}^{-})$

the approximated wave-vector components, in units of $k_F^{(S)}$ are

$$\tilde{k}_{\sigma} = \lambda_{\sigma} \cos \theta$$

$$\tilde{q}_{\sigma} = \sqrt{1 - \tilde{\mathbf{k}}_{\parallel, \sigma}^2}$$

$$|\tilde{\mathbf{k}}_{\parallel, \sigma}| = \lambda_{\sigma} \sin \theta$$

$$\lambda_{\sigma} = \kappa \sqrt{1 + \rho_{\sigma} X}$$

Boundary conditions

$$\begin{aligned}\psi_\sigma(z)|_{z=0_-} &= \psi_\sigma(z)|_{z=0_+}, \\ \frac{d\psi_\sigma(z)}{dz}\Big|_{z=0_-} &= \frac{d\psi_\sigma(z)}{dz}\Big|_{z=0_+} - \frac{2m\hat{W}}{\hbar^2}\psi_\sigma(0), \\ \psi_\sigma(z)|_{z=l_-} &= \psi_\sigma(z)|_{z=l_+}, \\ \frac{d\psi_\sigma(z)}{dz}\Big|_{z=l_-} &= \frac{d\psi_\sigma(z)}{dz}\Big|_{z=l_+} - \frac{2m\hat{W}}{\hbar^2}\psi_\sigma(l).\end{aligned}$$

$$a_{\sigma}(E,\theta) \;\; = \;\; \frac{4(\tilde{k}_{\sigma}/\tilde{q}_{\sigma})\Delta\sin(\zeta_-/2)}{\Gamma}\left[\mathcal{A}_+^RE\sin(\zeta_-/2)+i\mathcal{B}_+^R\Omega\cos(\zeta_-/2)\right],$$

$$\begin{array}{rcl} b_{\sigma}(E,\theta) & = & \dfrac{1}{\Gamma}\Big[\mathcal{A}_+^R\mathcal{C}_+\Delta^2-\left(\mathcal{A}_+^R\mathcal{C}_+E^2+\mathcal{B}_+^R\mathcal{D}_+\Omega^2\right)\cos(\zeta_-) \\ \\ & & +\left(\mathcal{A}_-^R\mathcal{C}_-+\mathcal{B}_-^R\mathcal{D}_-\right)\Omega^2\cos(\zeta_+)+i\left(\mathcal{B}_+^R\mathcal{C}_++\mathcal{A}_+^R\mathcal{D}_+\right)E\Omega\sin(\zeta_-) \\ \\ & & -i\left(\mathcal{B}_-^R\mathcal{C}_-+\mathcal{A}_-^R\mathcal{D}_-\right)\Omega^2\sin(\zeta_+)\Big], \end{array}$$

$$c_{\sigma}(E,\theta) \;\; = \;\; \frac{4(\tilde{k}_{\sigma}/\tilde{q}_{\sigma})\Omega e^{-i\tilde{k}_{\tilde{\sigma}}l}}{\Gamma}\times$$

$$\begin{array}{l} \times\Big\{i\left[\mathcal{F}_+\cos(\zeta_+/2)+i\mathcal{E}_+\sin(\zeta_+/2)\right]E\sin(\zeta_-/2) \\ \\ -\left[\mathcal{E}_+\cos(\zeta_+/2)+i\mathcal{F}_+\sin(\zeta_+/2)\right]\Omega\cos(\zeta_-/2)\Big\}, \end{array}$$

$$d_{\sigma}(E,\theta) \;\; = \;\; \frac{4(\tilde{k}_{\sigma}/\tilde{q}_{\sigma})\Delta\Omega e^{i\tilde{k}_{\sigma}l}}{\Gamma}\times$$

$$\times i\left[\mathcal{F}_-\cos(\zeta_+/2)+i\mathcal{E}_-\sin(\zeta_+/2)\right]\sin(\zeta_-/2),$$

$$\begin{array}{l} \Gamma=\mathcal{A}_+^L\mathcal{A}_+^R\Delta^2-\left(\mathcal{A}_+^L\mathcal{A}_+^RE^2+\mathcal{B}_+^L\mathcal{B}_+^R\Omega^2\right)\cos(\zeta_-)+\left(\mathcal{A}_-^L\mathcal{A}_-^R+\mathcal{B}_-^L\mathcal{B}_-^R\right)\Omega^2\cos(\zeta_+) \\ \\ +i\left(\mathcal{A}_+^L\mathcal{B}_+^R+\mathcal{B}_+^L\mathcal{A}_+^R\right)E\Omega\sin(\zeta_-)-i\left(\mathcal{A}_-^L\mathcal{B}_-^R+\mathcal{B}_-^L\mathcal{A}_-^R\right)\Omega^2\sin(\zeta_+). \end{array}$$

$$\begin{array}{lll} \mathcal{A}_\pm^{L(R)}=K_1^{L(R)}\pm K_2^{L(R)}, & \mathcal{B}_\pm^{L(R)}=1\pm K_1^{L(R)}K_2^{L(R)}, & \mathcal{C}_\pm={K_1^L}^*\mp K_2^L, \\ \mathcal{D}_\pm=-(1\mp {K_1^L}^*K_2^L), & \mathcal{E}_\pm=K_2^L\pm K_2^R, & \mathcal{F}_\pm=1\pm K_2^LK_2^R, \end{array}$$

$$\begin{array}{ll} K_1^L=\dfrac{\tilde{k}_{\sigma}+iZ}{\tilde{q}_{\sigma}}, & K_2^L=\dfrac{\tilde{k}_{\bar{\sigma}}-iZ}{\tilde{q}_{\sigma}}, \\ K_1^R=\dfrac{\tilde{k}_{\sigma[\bar{\sigma}]}+iZ}{\tilde{q}_{\sigma}}, & K_2^R=\dfrac{\tilde{k}_{\bar{\sigma}[\sigma]}-iZ}{\tilde{q}_{\sigma}}, \end{array}$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}, \quad \left(\frac{1}{2}\right)^3 = \frac{1}{8}, \quad \left(\frac{1}{2}\right)^4 = \frac{1}{16}, \quad \left(\frac{1}{2}\right)^5 = \frac{1}{32}, \quad \left(\frac{1}{2}\right)^6 = \frac{1}{64}, \quad \left(\frac{1}{2}\right)^7 = \frac{1}{128}, \quad \left(\frac{1}{2}\right)^8 = \frac{1}{256}, \quad \left(\frac{1}{2}\right)^9 = \frac{1}{512}, \quad \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}, \quad \left(\frac{1}{2}\right)^{11} = \frac{1}{2048}, \quad \left(\frac{1}{2}\right)^{12} = \frac{1}{4096}, \quad \left(\frac{1}{2}\right)^{13} = \frac{1}{8192}, \quad \left(\frac{1}{2}\right)^{14} = \frac{1}{16384}, \quad \left(\frac{1}{2}\right)^{15} = \frac{1}{32768}, \quad \left(\frac{1}{2}\right)^{16} = \frac{1}{65536}, \quad \left(\frac{1}{2}\right)^{17} = \frac{1}{131072}, \quad \left(\frac{1}{2}\right)^{18} = \frac{1}{262144}, \quad \left(\frac{1}{2}\right)^{19} = \frac{1}{524288}, \quad \left(\frac{1}{2}\right)^{20} = \frac{1}{1048576}, \quad \left(\frac{1}{2}\right)^{21} = \frac{1}{2097152}, \quad \left(\frac{1}{2}\right)^{22} = \frac{1}{4194304}, \quad \left(\frac{1}{2}\right)^{23} = \frac{1}{8388608}, \quad \left(\frac{1}{2}\right)^{24} = \frac{1}{16777216}, \quad \left(\frac{1}{2}\right)^{25} = \frac{1}{33554432}, \quad \left(\frac{1}{2}\right)^{26} = \frac{1}{67108864}, \quad \left(\frac{1}{2}\right)^{27} = \frac{1}{134217728}, \quad \left(\frac{1}{2}\right)^{28} = \frac{1}{268435456}, \quad \left(\frac{1}{2}\right)^{29} = \frac{1}{536870912}, \quad \left(\frac{1}{2}\right)^{30} = \frac{1}{1073741824}, \quad \left(\frac{1}{2}\right)^{31} = \frac{1}{2147483648}, \quad \left(\frac{1}{2}\right)^{32} = \frac{1}{4294967296}, \quad \left(\frac{1}{2}\right)^{33} = \frac{1}{8589934592}, \quad \left(\frac{1}{2}\right)^{34} = \frac{1}{17179869184}, \quad \left(\frac{1}{2}\right)^{35} = \frac{1}{34359738368}, \quad \left(\frac{1}{2}\right)^{36} = \frac{1}{68719476736}, \quad \left(\frac{1}{2}\right)^{37} = \frac{1}{137438953472}, 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\left(\frac{1}{2}\right)^{118} = \frac{1}{332306998957151746181484355354886144}, \quad \left(\frac{1}{2}\right)^{119} = \frac{1}{664613997914303492362968710709772288}, \quad \left(\frac{1}{2}\right)^{120} = \frac{1}{1329227995828606984725937421419545576}, \quad \left(\frac{1}{2}\right)^{121} = \frac{1}{2658455991657213969451874842839091152}, \quad \left(\frac{1}{2}\right)^{122} = \frac{1}{5316911983314427938903749685678182304}, \quad \left(\frac{1}{2}\right)^{123} = \frac{1}{10633823966628455877807499371356364608}, \quad \left(\frac{1}{2}\right)^{124} = \frac{1}{21267647933256911755614998742712729216}, \quad \left(\frac{1}{2}\right)^{125} = \frac{1}{42535295866513823511229987485425458432}, \quad \left(\frac{1}{2}\right)^{126} = \frac{1}{85070591733027647022459974970850916864}, \quad \left(\frac{1}{2}\right)^{127} = \frac{1}{170141183466055294044919949941701833728}, \quad \left(\frac{1}{2}\right)^{128} = \frac{1}{340282366932110588089839899883403667456}, \quad \left(\frac{1}{2}\right)^{129} = \frac{1}{680564733864221176179679799766807334912}, \quad \left(\frac{1}{2}\right)^{130} = \frac{1}{1361129467728442352359359599533614669824}, \quad \left(\frac{1}{2}\right)^{131} = \frac{1}{2722258935456884704718719199067229339648}, \quad \left(\frac{1}{2}\right)^{132} = \frac{1}{5444517870913769409437438398134458679296}, \quad \left(\frac{1}{2}\right)^{133} = \frac{1}{1088903574182753881887487679626891758592}, \quad \left(\frac{1}{2}\right)^{134} = \frac{1}{2177807148365507763774975359253783517184}, \quad \left(\frac{1}{2}\right)^{135} = \frac{1}{4355614296731015527549950718507567034368}, \quad \left(\frac{1}{2}\right)^{136} = \frac{1}{8711228593462030755099801437015134068736}, \quad \left(\frac{1}{2}\right)^{137} = \frac{1}{17422457186924061510199602874030268137472}, \quad \left(\frac{1}{2}\right)^{138} = \frac{1}{34844914373848123020399205748060536274944}, \quad \left(\frac{1}{2}\right)^{139} = \frac{1}{69689828747696246040798411496121072549888}, \quad \left(\frac{1}{2}\right)^{140} = \frac{1}{139379657495392492081596822992242145097776}, \quad \left(\frac{1}{2}\right)^{141} = \frac{1}{278759314985784984163193645984484290195552}, \quad \left(\frac{1}{2}\right)^{142} = \frac{1}{557518629971569968326387291968968580391104}, \quad \left(\frac{1}{2}\right)^{143} = \frac{1}{1115037259943139936652774583937937160782208}, \quad \left(\frac{1}{2}\right)^{144} = \frac{1}{2230074519886279873305549167875874321564416}, \quad \left(\frac{1}{2}\right)^{145} = \frac{1}{4460149039772559746611098335751748643128832}, \quad \left(\frac{1}{2}\right)^{146} = \frac{1}{8920298079545119493222196671503497286257664}, \quad \left(\frac{1}{2}\right)^{147} = \frac{1}{17840596159090238986444393343006994572515328}, \quad \left(\frac{1}{2}\right)^{148} = \frac{1}{35681192318180477972888786686013989145030656}, \quad \left(\frac{1}{2}\right)^{149} = \frac{1}{71362384636360955945777573372027978290061312}, \quad \left(\frac{1}{2}\right)^{150} = \frac{1}{142724769272721911891555146744055956580122624}, \quad \left(\frac{1}{2}\right)^{151} = \frac{1}{285449538545443823783110293488111913160245248}, \quad \left(\frac{1}{2}\right)^{152} = \frac{1}{570898577090887647566220586976223826320490496}, \quad \left(\frac{1}{2}\right)^{153} = \frac{1}{1141797154181775295132441173952447652640980992}, \quad \left(\frac{1}{2}\right)^{154} = \frac{1}{2283594308363550590264882347904895305281961984}, \quad \left(\frac{1}{2}\right)^{155} = \frac{1}{4567188616727101180529764695809790610563923968}, \quad \left(\frac{1}{2}\right)^{156} = \frac{1}{9134377233454202361059529391619581221127847936}, \quad \left(\frac{1}{2}\right)^{157} = \frac{1}{18268754466908404722119058783239162442255695872}, \quad \left(\frac{1}{2}\right)^{158} = \frac{1}{36537508933816809444238117566478324884511391744}, \quad \left(\frac{1}{2}\right)^{159} = \frac{1}{73075017867633618888476235132956649769022783488}, \quad \left(\frac{1}{2}\right)^{160} = \frac{1}{146150035735267237776952470265913299538045567976}, \quad \left(\frac{1}{2}\right)^{161} = \frac{1}{292300071470534475553904940531826598576091135952}, \quad \left(\frac{1}{2}\right)^{162} = \frac{1}{584600142941068951107808881063653197152182271904}, \quad \left(\frac{1}{2}\right)^{163} = \frac{1}{1169200285882137902215617762127306394304364543808}, \quad \left(\frac{1}{2}\right)^{164} = \frac{1}{2338400571764275804431235524254612788608729087616}, \quad \left(\frac{1}{2}\right)^{165} = \frac{1}{4676801143528551608862471048509225577217458175232}, \quad \left(\frac{1}{2}\right)^{166} = \frac{1}{9353602287057103217724942097018451154434916350464}, \quad \left(\frac{1}{2}\right)^{167} = \frac{1}{18707204574114206435449884194036902308869832700928}, \quad \left(\frac{1}{2}\right)^{168} = \frac{1}{37414409148228412870899768388073804617739665401856}, \quad \left(\frac{1}{2}\right)^{169} = \frac{1}{74828818296456825741799536776147609235479330803712}, \quad \left(\frac{1}{2}\right)^{170} = \frac{1}{149657636592913651483599073552295218509558661607424}, \quad \left(\frac{1}{2}\right)^{171} = \frac{1}{299315273185827302967198147104590437019117323214848}, \quad \left(\frac{1}{2}\right)^{172} = \frac{1}{598630546371654605934396294209180874038234646429696}, \quad \left(\frac{1}{2}\right)^{173} = \frac{1}{119726109274330921186879258841836178066466929285392}, \quad \left(\frac{1}{2}\right)^{174} = \frac{1}{239452218548661842373758517683672356132933858570784}, \quad \left(\frac{1}{2}\right)^{175} = \frac{1}{478904437097323684747517035367344712265867717141568}, \quad \left(\frac{1}{2}\right)^{176} = \frac{1}{957808874194647369495034070734689424531735434283136}, \quad \left(\frac{1}{2}\right)^{177} = \frac{1}{1915617748389294738980068141469378849063470868566272}, \quad \left(\frac{1}{2}\right)^{178} = \frac{1}{3831235496778589477960136282938757698126941737132544}, \quad \left(\frac{1}{2}\right)^{179} = \frac{1}{7662470993557178955920272565877515396253883474265088}, \quad \left(\frac{1}{2}\right)^{180} = \frac{1}{15324941987114357911840545131755031792507766948530176}, \quad \left(\frac{1}{2}\right)^{181} = \frac{1}{30649883974228715823681090263510063585015533897060352}, \quad \left(\frac{1}{2}\right)^{182} = \frac{1}{61299767948457431647362180527020127170031067794120704}, \quad \left(\frac{1}{2}\right)^{183} = \frac{1}{122599535896914863294724361054040254340062135588241408}, \quad \left(\frac{1}{2}\right)^{184} = \frac{1}{245199071793829726589448722108080508680124271176482816}, \quad \left(\frac{1}{2}\right)^{185} = \frac{1}{490398143587659453178897444216161017360248542352965632}, \quad \left(\frac{1}{2}\right)^{186} = \frac{1}{980796287175318906357794888432322034720497084705931264}, \quad \left(\frac{1}{2}\right)^{187} = \frac{1}{1961592574350637812715589776864644069441980169411862528}, \quad \left(\frac{1}{2}\right)^{188} = \frac{1}{3923185148701275625431179553729288138883960338823725056}, \quad \left(\frac{1}{2}\right)^{189} = \frac{1}{7846370297402551250862359107458576277767920677647450112}, \quad \left(\frac{1}{2}\right)^{190} = \frac{1}{15692740594805102501724782214917152555535841353294900224}, \quad \left(\frac{1}{2}\right)^{191} = \frac{1}{31385481189610205003449564429834305111071682706589800448}, \quad \left(\frac{1}{2}\right)^{192} = \frac{1}{62770962379220405006899128859668610222143365413179600896}, \quad \left(\frac{1}{2}\right)^{193} = \frac{1}{125541924758440810013798257719337220444286730826359201792}, \quad \left(\frac{1}{2}\right)^{194} = \frac{1}{251083849516881620027596515438674440888573461652718403584}, \quad \left(\frac{1}{2}\right)^{195} = \frac{1}{502167699033763240055193030877348881777146923305436807168}, \quad \left(\frac{1}{2}\right)^{196} = \frac{1}{1004335398067526480110385061754697763554293846610873614336}, \quad \left(\frac{1}{2}\right)^{197} = \frac{1}{2008670796135052960220770123509395527108587693221747228672}, \quad \left(\frac{1}{2}\right)^{198} = \frac{1}{4017341592270105920441540247018791054217175386443494457344}, \quad \left(\frac{$$

Scattering probabilities

$$A_\sigma(E, \theta) + B_\sigma(E, \theta) + C_\sigma(E, \theta) + D_\sigma(E, \theta) = 1,$$

$$A_\sigma(E, \theta) = \operatorname{Re} \left(\frac{\tilde{k}_{\bar{\sigma}}}{\tilde{k}_\sigma} \right) |a_\sigma(E, \theta)|^2,$$

$$B_\sigma(E, \theta) = |b_\sigma(E, \theta)|^2,$$

$$C_\sigma(E, \theta) = \operatorname{Re} \left(\frac{\tilde{k}_{\sigma[\bar{\sigma}]}}{\tilde{k}_\sigma} \right) |c_\sigma(E, \theta)|^2,$$

$$D_\sigma(E, \theta) = \operatorname{Re} \left(\frac{\tilde{k}_{\bar{\sigma}[\sigma]}}{\tilde{k}_\sigma} \right) |d_\sigma(E, \theta)|^2,$$

Two limits

- Metallic limit ($Z = 0$)
Andreev reflection vanishes at geometrical resonances:

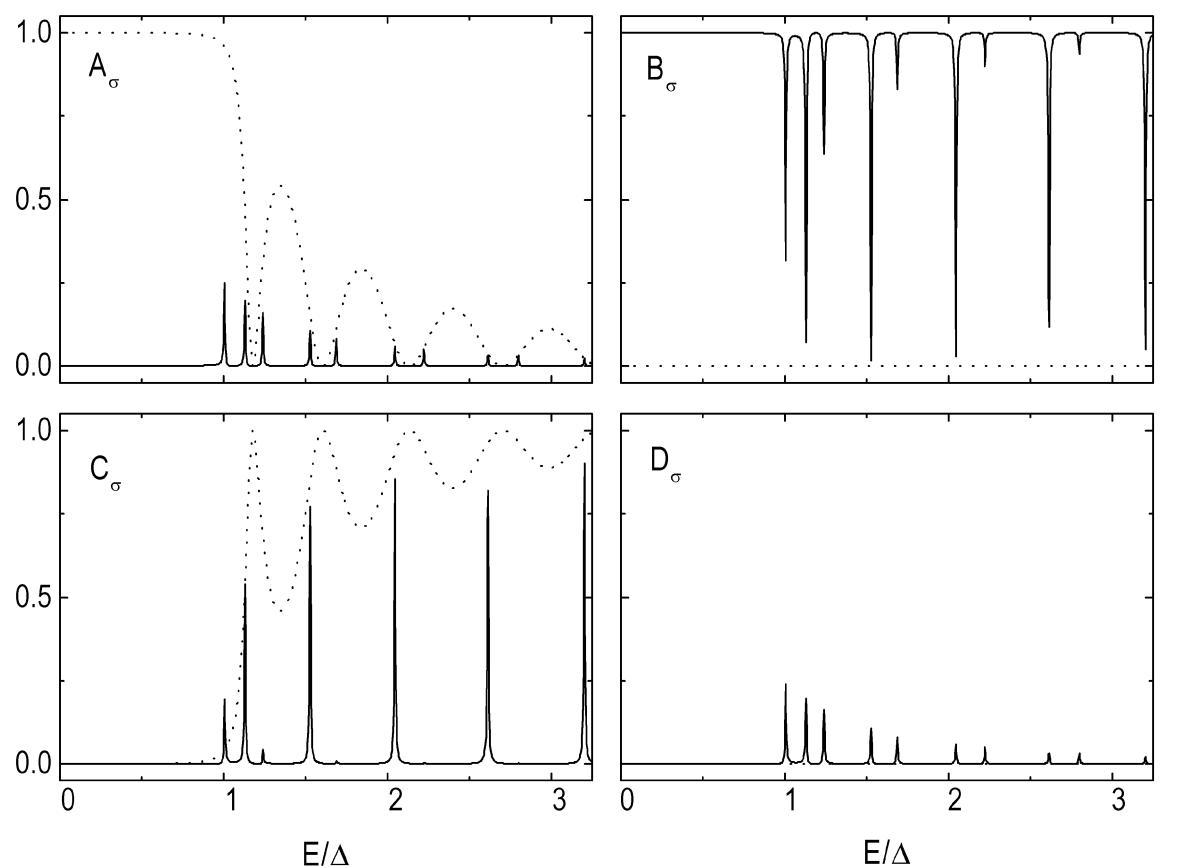
$$A_\sigma = D_\sigma = 0 \quad \text{when} \quad l(q_\sigma^+ - q_\sigma^-) = 2n\pi$$

$$q_\sigma^\pm = \sqrt{(2m/\hbar^2)[E_F^{(S)} \pm \Omega] - \mathbf{k}_{\parallel,\sigma}^2} \quad \Omega = \sqrt{E^2 - \Delta^2}$$

- Tunnel limit ($Z \rightarrow \infty$)
Transport through the bound states:

$$lq_\sigma^+ = n_1\pi \quad lq_\sigma^- = n_2\pi \quad n_1 - n_2 = 2n$$

Metallic vs. Tunnel **NSN** junction



$$X = 0$$

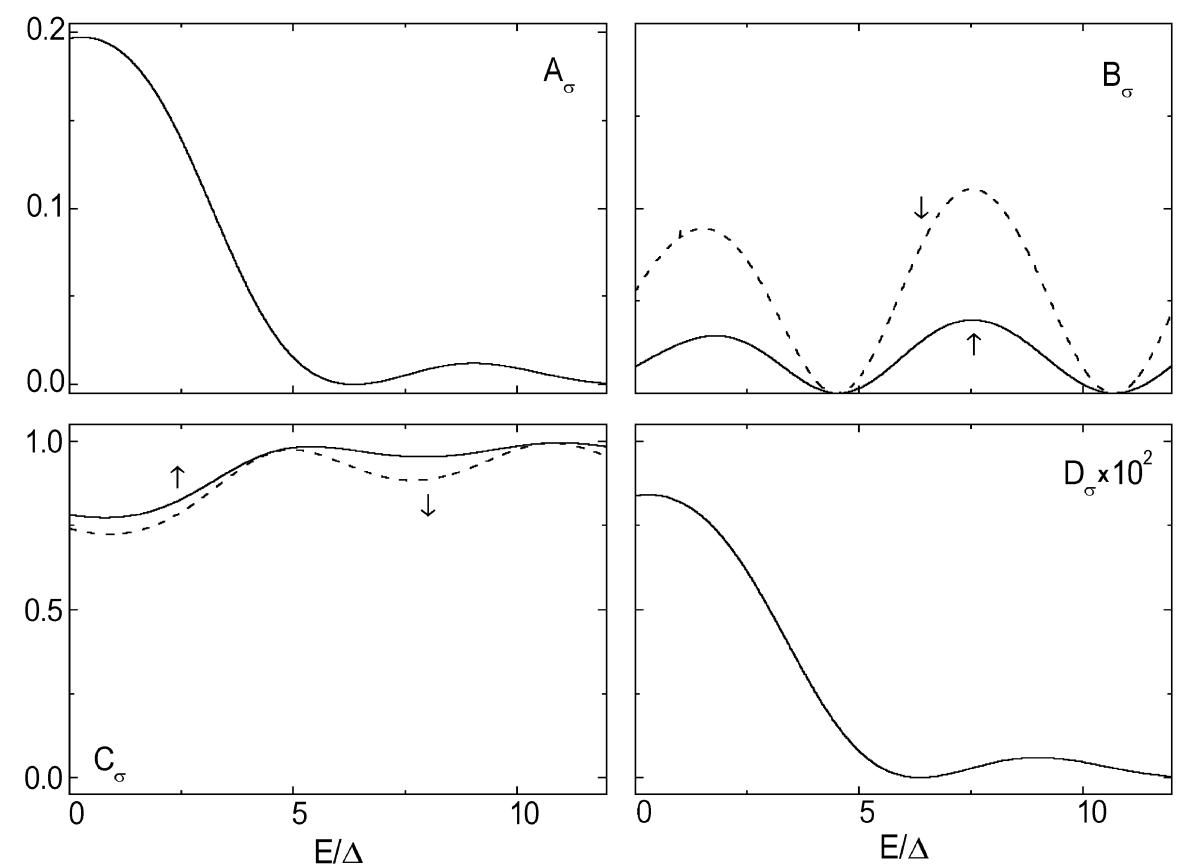
$$lk_F^{(S)} = 10^4 \quad [l/\xi_0 \approx 10]$$

$$\theta = 0$$

$$\kappa = 1, \Delta/E_F^{(S)} = 10^{-3}$$

M. Božović and Z. Radović in
*Supercond. and Rel. Ox.: Phys. and
nanoeng. V, Proc. of SPIE*, vol. 4811
(Seattle, 2002), p. 216.

FSF double junction (P alignment)



$Z=0, X=0.5$

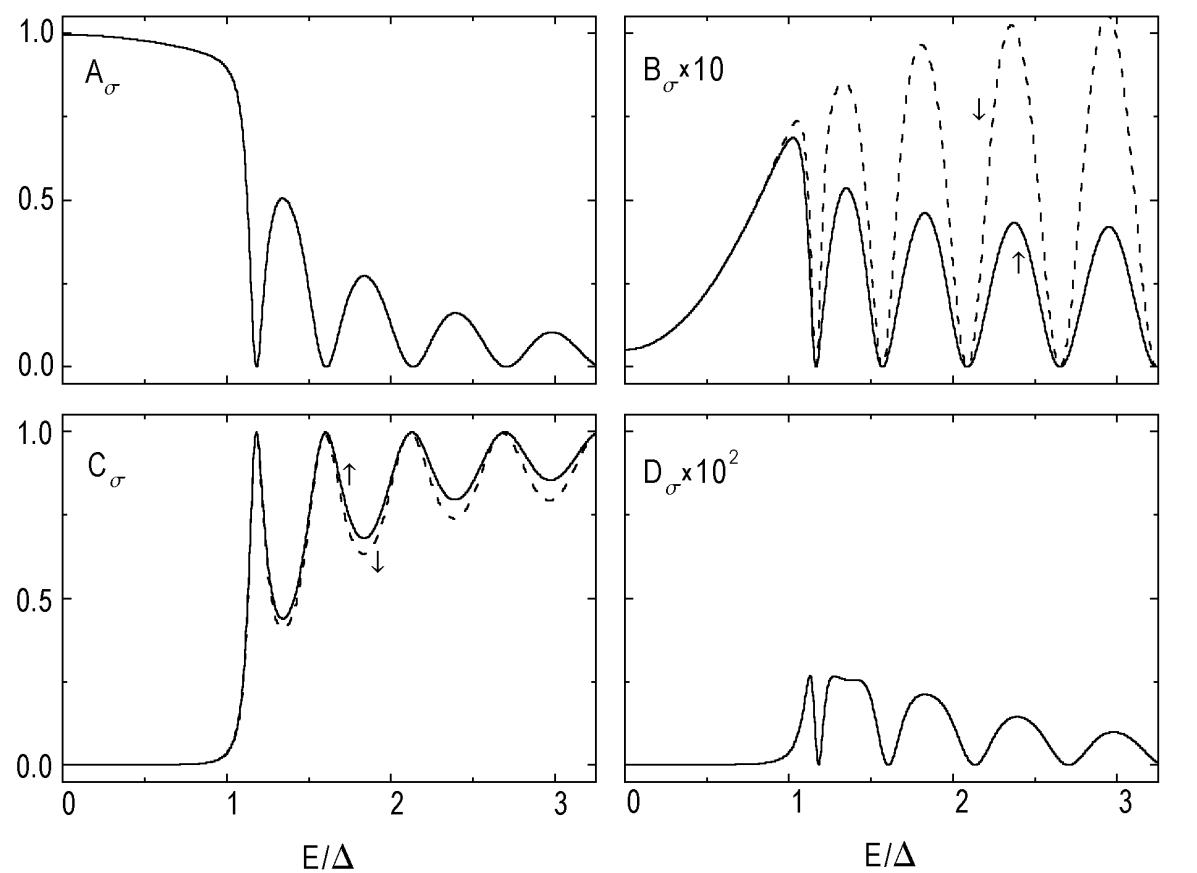
$lk_F^{(S)}=10^3$ [$l/\xi_0 \approx 1$]

$\theta=0$

$\kappa=1, \Delta/E_F^{(S)}=10^{-3}$

M. Božović and Z. Radović,
Phys. Rev. B **66**, 134524 (2002)

FSF double junction (P alignment)



$Z=0, X=0.5$

$lk_F^{(S)}=10^4$ [$l/\xi_0 \approx 10$]

$\theta=0$

$\kappa=1, \Delta/E_F^{(S)}=10^{-3}$

M. Božović and Z. Radović,
Phys. Rev. B **66**, 134524 (2002)

Current

$$j_q(V) = \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3\mathbf{k}}{(2\pi)^3} e \mathbf{v}_\sigma \cdot \hat{\mathbf{z}} \delta f(\mathbf{k}, V)$$

charge

$$I_q(V) = \frac{1}{e} \int_{-\infty}^{\infty} dE [f_0(E - eV/2) - f_0(E + eV/2)] G_q(E)$$

spin

$$I_s(V) = \frac{1}{e} \int_{-\infty}^{\infty} dE [f_0(E - eV/2) - f_0(E + eV/2)] G_s(E)$$

Differential conductances (at $T=0$)

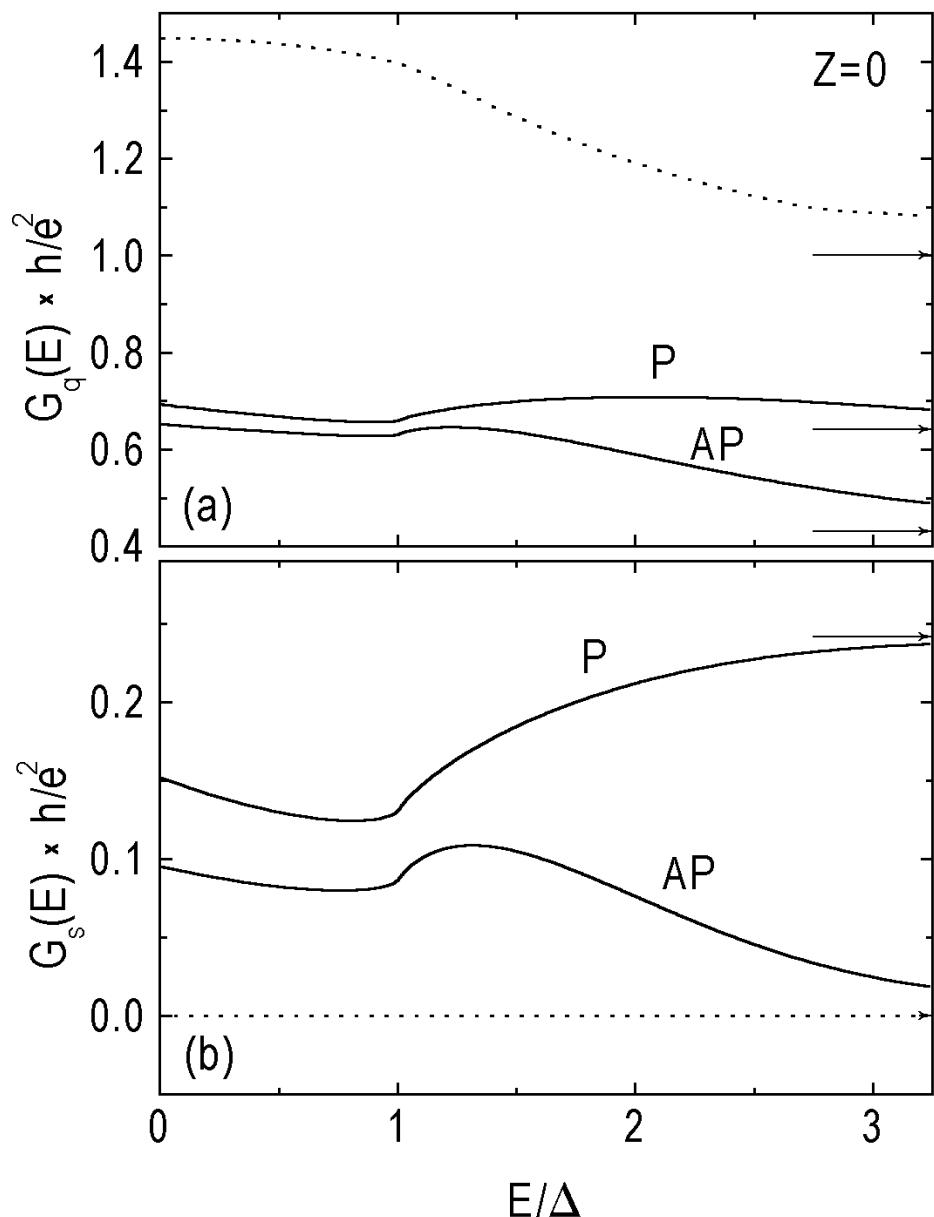
charge

$$G_q(E) = \frac{e^2}{2h} \sum_{\sigma} \lambda_{\sigma}^2 \int_0^{\theta_{c1,\sigma}} d\theta \sin \theta \cos \theta [1 + A_{\sigma}(E, \theta) - B_{\sigma}(E, \theta)]$$

spin

$$G_s(E) = \frac{e^2}{2h} \sum_{\sigma} \rho_{\sigma} \lambda_{\sigma}^2 \int_0^{\theta_{c1,\sigma}} d\theta \sin \theta \cos \theta [1 - A_{\sigma}(E, \theta) - B_{\sigma}(E, \theta)]$$

$$\lambda_{\sigma} = \kappa \sqrt{1 + \rho_{\sigma} X} \quad \theta_{c1,\uparrow} = \arcsin(1/\lambda_{\uparrow}) \quad \theta_{c1,\downarrow} = \pi / 2$$



FSF

double junction

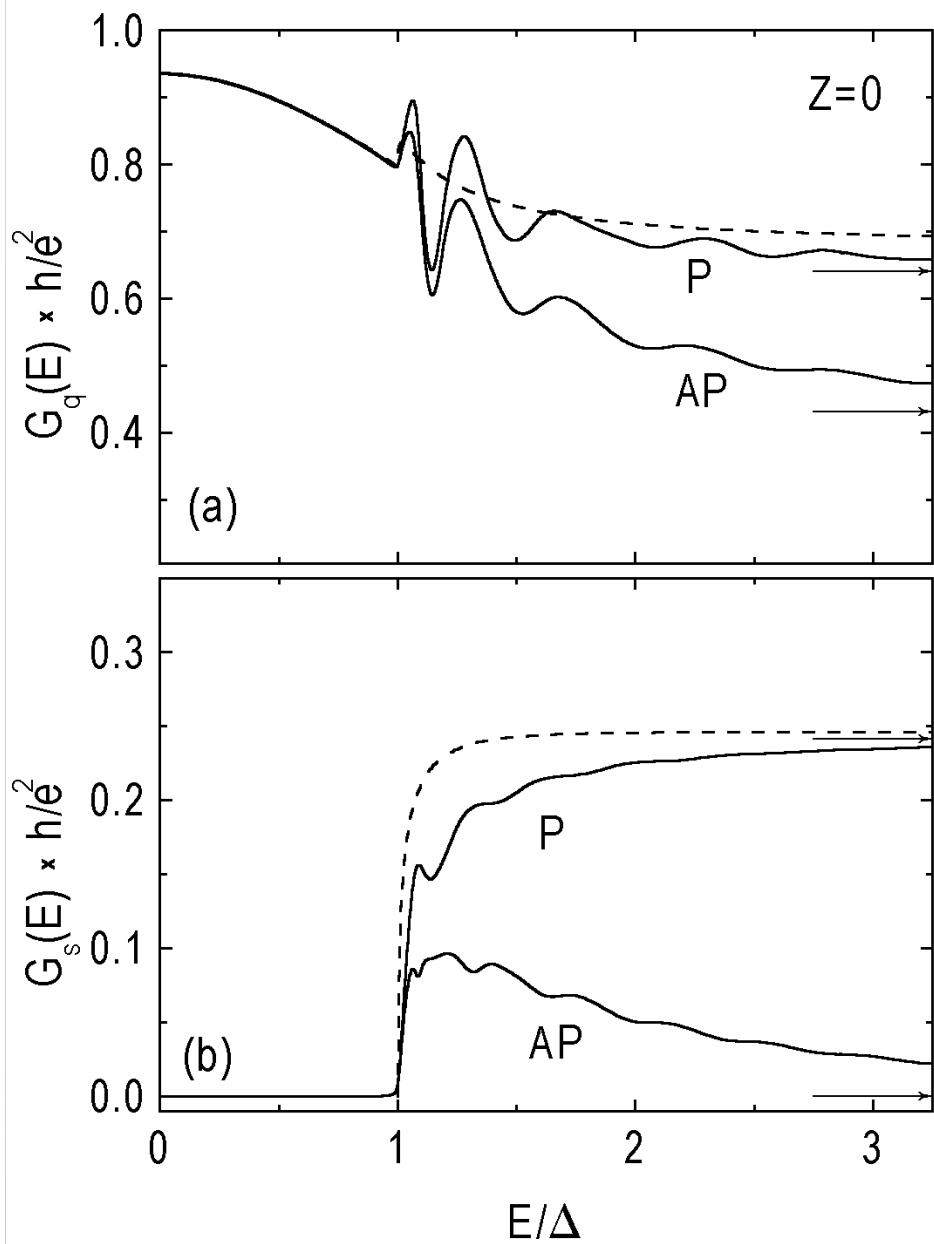
(thin S film)

$Z=0, X=0.5$

$$lk_F^{(S)} = 10^4 \quad [l/\xi_0 \approx 1]$$

$$\kappa=1, \Delta/E_F^{(S)}=10^{-3}$$

M. Božović and Z. Radović,
Phys. Rev. B **66**, 134524 (2002)



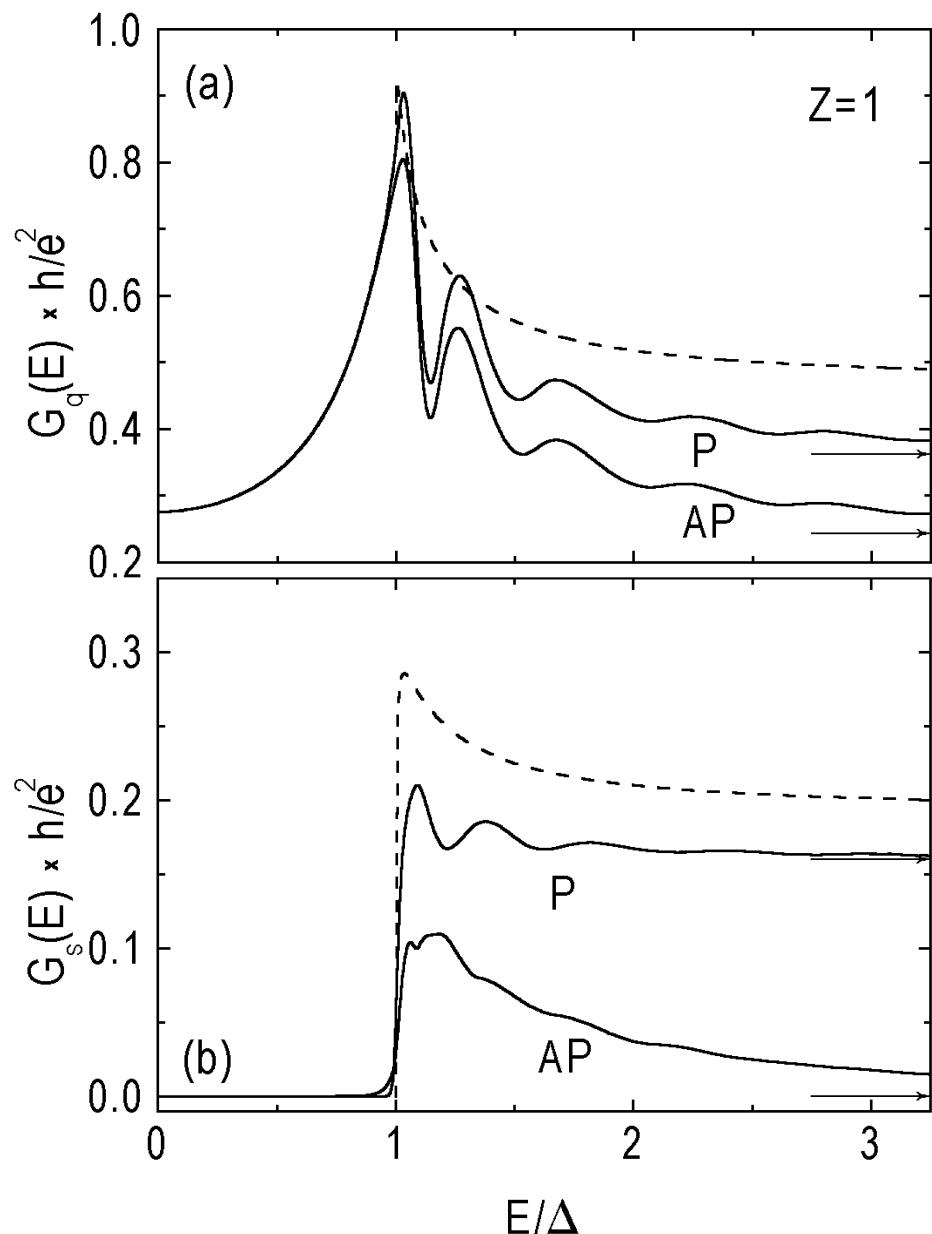
FSF
double junction
(thick S film)

$Z=0, X=0.5$

$lk_F^{(S)} = 10^4$ [$l/\xi_0 \approx 10$]

$\kappa=1, \Delta/E_F^{(S)} = 10^{-3}$

M. Božović and Z. Radović,
Phys. Rev. B 66, 134524 (2002)



FSF
double junction
(thick S film)

$Z=1, X=0.5$

$lk_F^{(S)} = 10^4$ [$l/\xi_0 \approx 10$]

$\kappa=1, \Delta/E_F^{(S)} = 10^{-3}$

M. Božović and Z. Radović,
Phys. Rev. B **66**, 134524 (2002)

Transparent NSN double junction

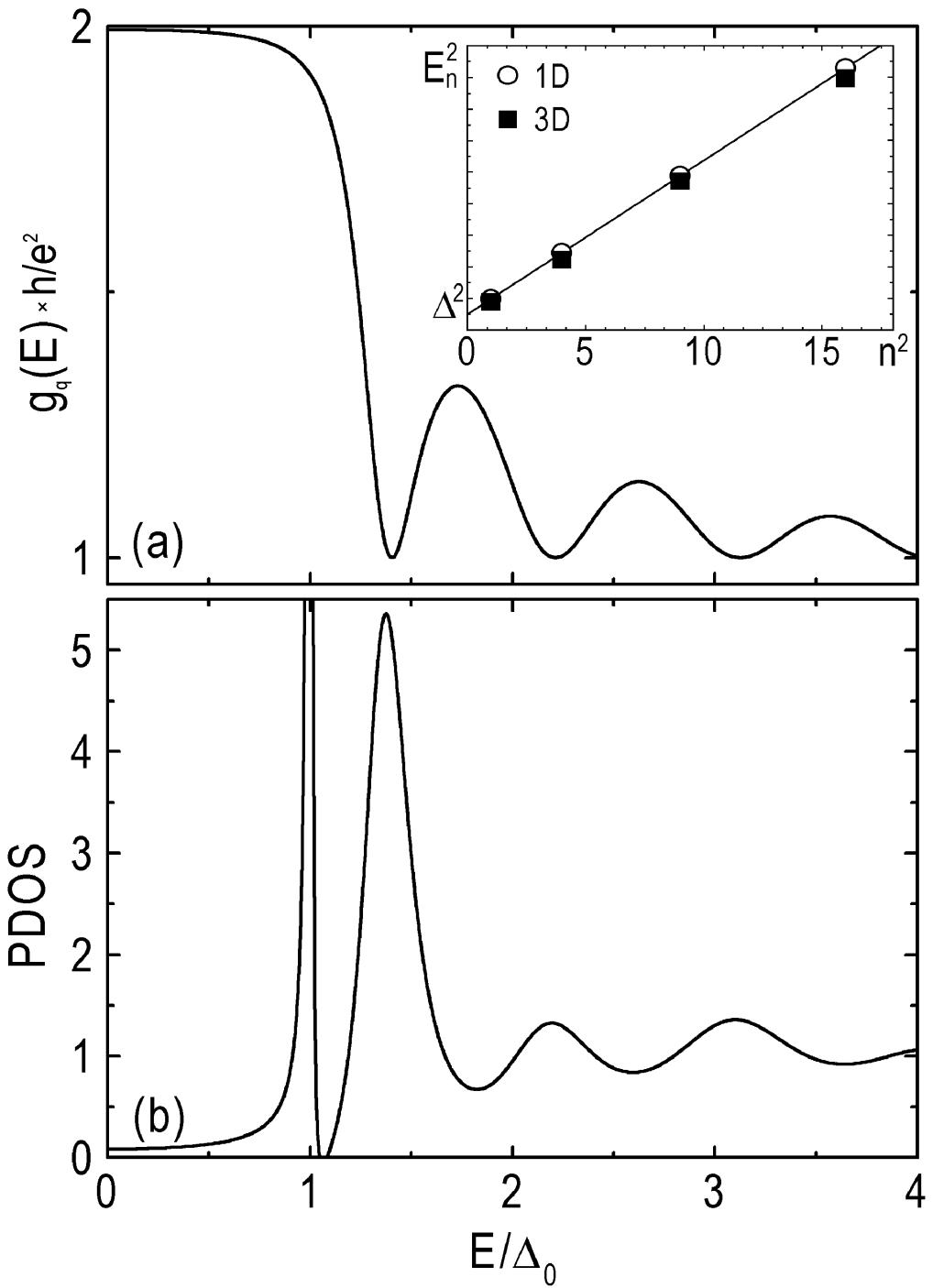
$$A_\sigma(E, \theta) = \left| \frac{\Delta \sin(\zeta_-/2)}{E \sin(\zeta_-/2) + i\Omega \cos(\zeta_-/2)} \right|^2$$

$$\begin{aligned} \tilde{N}(z, \theta, E) = & \frac{1}{\Gamma(E) \cos \theta} \times \\ & \operatorname{Re} \left\{ 2E^2(E^2 + \Omega^2) + 2E^2\Delta^2 \cos \zeta \right. \\ & + [\cos(\zeta(z/d - 1)) + \cos(\zeta z/d)] \\ & \times [\Delta^4 - \Delta^2(E^2 + \Omega^2) \cos \zeta] \\ & \left. + 2E^2\Delta^2[\sin(\zeta(z/d - 1)) - \sin(\zeta z/d)] \sin \zeta \right\} \end{aligned}$$

$$\Gamma(E) = [(E^2 + \Omega^2) \cos \zeta - \Delta^2]^2 + 4E^2\Omega^2 \sin^2 \zeta$$

$$\frac{N(z, E)}{N(0)} = \int_0^{\pi/2} d\theta \sin \theta \cos \theta \tilde{N}(z, \theta, E)$$

M. Božović, Z. Pajović, and Z. Radović, Physica C, in press (2003).



NSN
double junction
(1D, thick S film)

$$\theta = 0$$

$$Z = 0$$

$$X = 0$$

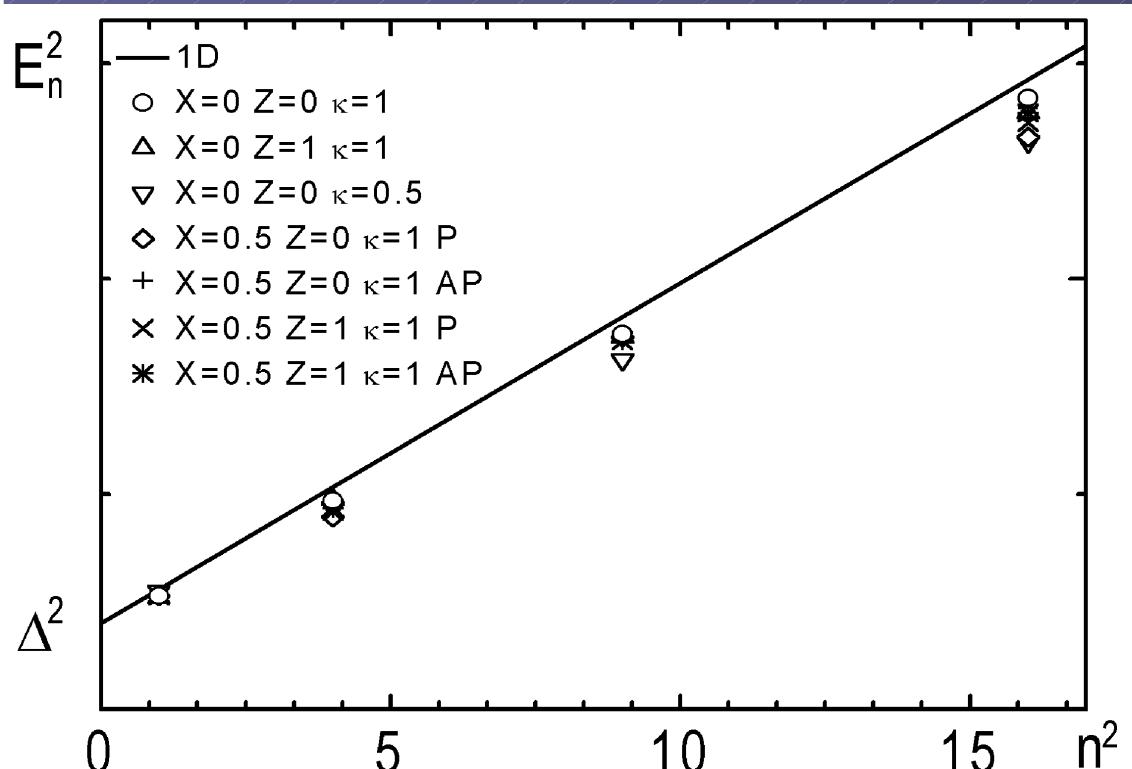
$$l/\xi_0 \approx 10$$

$$\kappa = 1, \Delta_0 / E_F^{(S)} = 10^{-3}$$

M. Božović, Z. Pajović, and
 Z. Radović, Physica C,
 in press (2003).

How to infer Δ and v_F in the superconductor?

Conductance minima satisfy: $E_n^2 = \Delta^2 + \left(\frac{\pi \hbar v_F^{(S)}}{l} \right)^2 n^2$

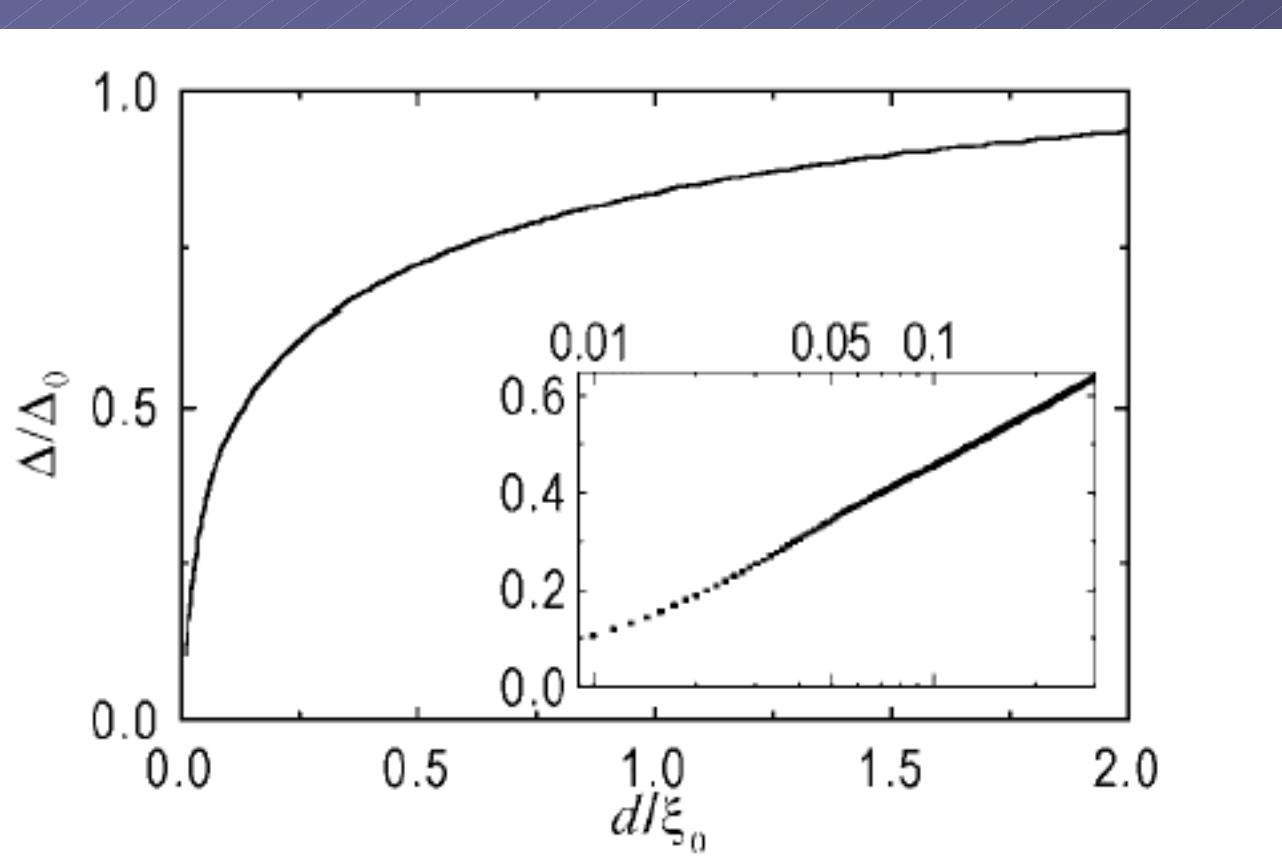


Ballistic
spectroscopy

O. Nesher and G. Koren,
Phys. Rev. B **60**, 9287 (1999).

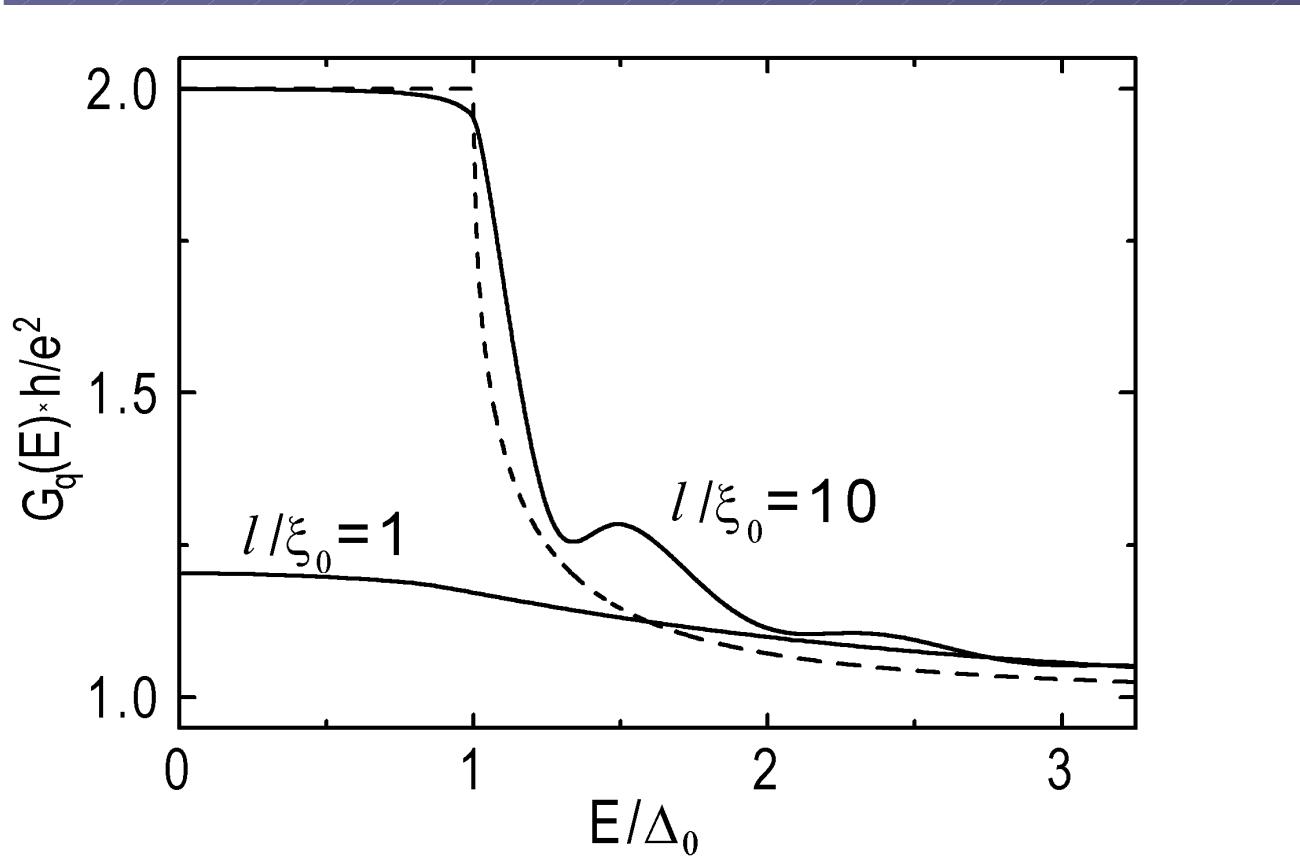
M. Božović and Z. Radović in
Supercond. and Rel. Ox.: Phys. and nanoeng. V, Proc. of SPIE,
vol. 4811 (Seattle, 2002), p. 216.

Transparent NSN double junction: self-consistent pair potential in thin films



M. Božović, Z. Pajović, and Z. Radović,
Physica C, in press (2003).

Transparent NSN double junction: the conductance spectra



3D

$Z=0$

$X=0$

$\kappa=1$

$\Delta/E_F^{(S)} = 10^{-3}$

M. Božović, Z. Pajović, and Z. Radović,
Physica C, in press (2003).

NSN

double junction (thick S film)

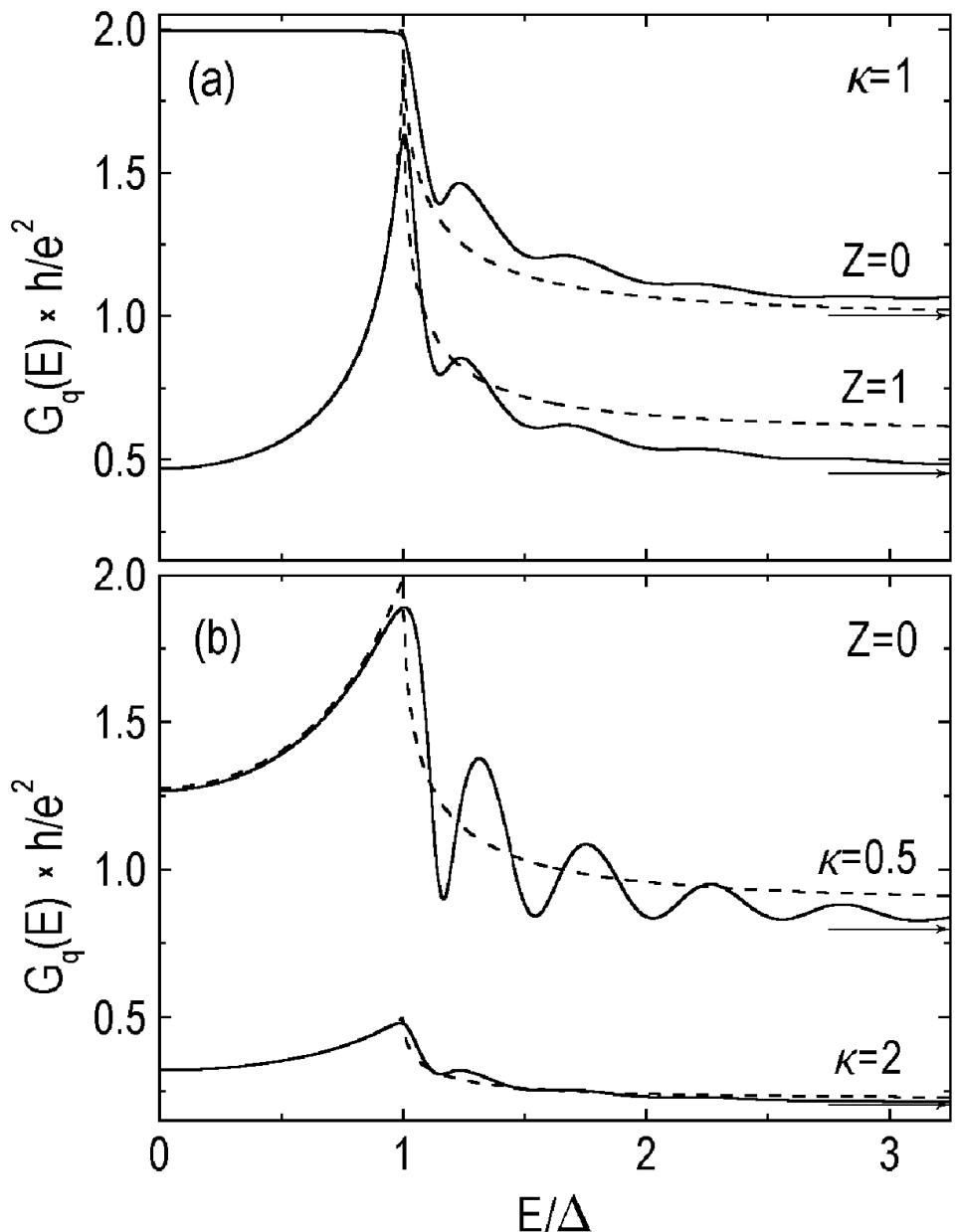
Influence of κ and Z

$X = 0$

$$lk_F^{(S)} = 10^4 \quad [l/\xi_0 \approx 10]$$

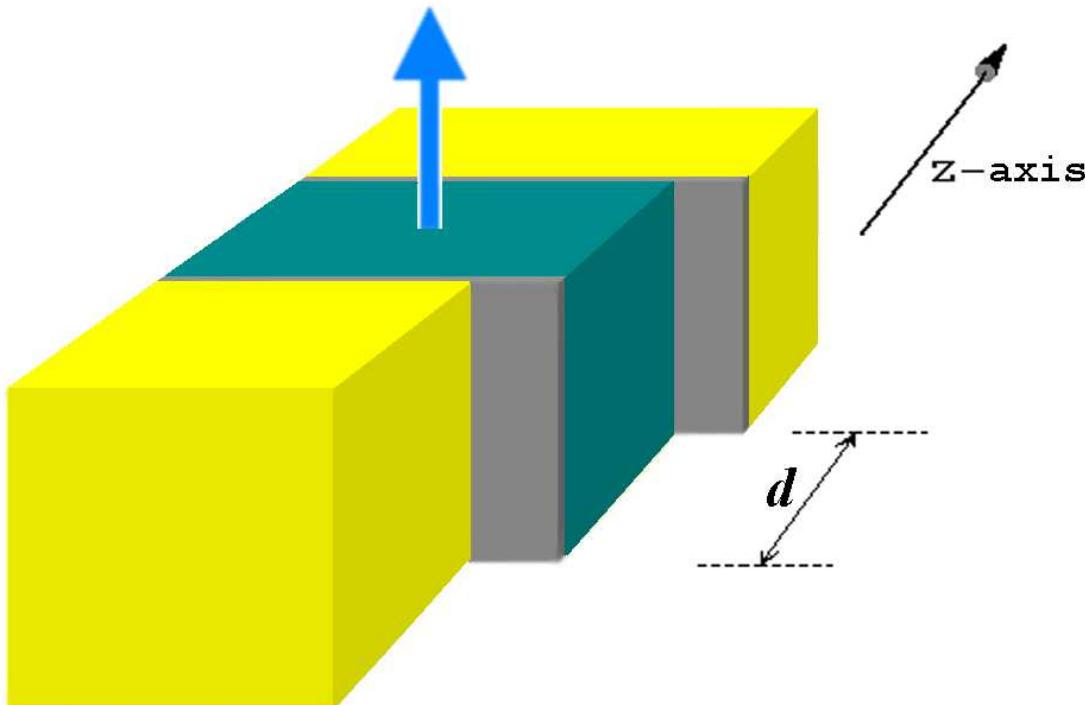
$$\Delta/E_F^{(S)} = 10^{-3}$$

M. Božović and Z. Radović in
*Supercond. and Rel. Ox.: Phys. and
nanoeng. V, Proc. of SPIE*, vol. 4811
(Seattle, 2002), p. 216.



The Model (**SIFIS**)

M. Božović



- ferromagnet
- insulator
- superconductor

Scattering Problem

$$\begin{pmatrix} H_0(\mathbf{r}) - \rho_\sigma h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -H_0(\mathbf{r}) + \rho_{\bar{\sigma}} h(\mathbf{r}) \end{pmatrix} \Psi_\sigma(\mathbf{r}) = E \Psi_\sigma(\mathbf{r})$$

$$\Psi_\sigma(\mathbf{r}) \equiv \begin{pmatrix} u_\sigma(\mathbf{r}) \\ v_{\bar{\sigma}}(\mathbf{r}) \end{pmatrix} = \exp(i\mathbf{k}_\parallel \cdot \mathbf{r}) \psi_\sigma(z)$$

Exchange energy $h(\mathbf{r})/E_F^{(F)} = \textcolor{blue}{X}\Theta(z)\Theta(l-z)$ $\rho_{\uparrow,\downarrow} = \pm 1$

Stepwise pair potential $\Delta(\mathbf{r}) = \Delta[\Theta(-z) \pm \Theta(z-l)]$

Interface potential $\hat{W}[\delta(z) + \delta(l-z)]$ $\textcolor{blue}{Z} = 2m\hat{W}/\hbar^2 k_F^{(S)}$

FWVM parameter $\kappa = k_F^{(F)}/k_F^{(S)}$ $\textcolor{blue}{Z}_\theta = Z/\cos\theta$

Solutions

$$\psi_1(z) = \begin{cases} [\exp(iq^+z) + b_1(E, \theta)\exp(-iq^+z)] \begin{pmatrix} ue^{i\phi_L/2} \\ \bar{v}e^{-i\phi_L/2} \end{pmatrix} + a_1(E, \theta)\exp(iq^-z) \begin{pmatrix} \bar{v}e^{i\phi_L/2} \\ \bar{u}e^{-i\phi_L/2} \end{pmatrix}, & z < 0 \\ [C_1(E, \theta)\exp(ik_\sigma^+z) + C_2(E, \theta)\exp(-ik_\sigma^+z)] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ + [C_3(E, \theta)\exp(ik_{\bar{\sigma}}^-z) + C_4(E, \theta)\exp(-ik_{\bar{\sigma}}^-z)] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & 0 < z < d \\ c_1(E, \theta)\exp(iq^+z) \begin{pmatrix} ue^{i\phi_R/2} \\ \bar{v}e^{-i\phi_R/2} \end{pmatrix} + d_1(E, \theta)\exp(-iq^-z) \begin{pmatrix} \bar{v}e^{i\phi_R/2} \\ \bar{u}e^{-i\phi_R/2} \end{pmatrix}, & z > d \end{cases}$$

$$\bar{u} = \sqrt{(1 + \Omega/E)/2} \quad \bar{v} = \sqrt{(1 - \Omega/E)/2}$$

$$\Omega = \sqrt{E^2 - \Delta^2}$$

Wave vector components

Perpendicular component in the ferromagnet

$$k_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2) \left(E_F^{(F)} + \rho_{\sigma} h_0 \pm E \right) - \mathbf{k}_{\parallel}^2}$$

Perpendicular component in the superconductors

$$q_{\sigma}^{\pm} = \sqrt{(2m/\hbar^2) \left(E_F^{(S)} \pm \Omega \right) - \mathbf{k}_{\parallel}^2}$$

Conserved parallel component

$$|\mathbf{k}_{\parallel}| = \sqrt{(2m/\hbar^2) \left(E_F^{(S)} + \Omega \right) \sin \theta}$$

Wave vector components

Neglecting $\Omega / E_F^{(S)} \ll 1$ and $\Delta / E_F^{(S)} \ll 1$

except in the exponents $\zeta_\sigma^\pm = d(k_\sigma^+ \pm k_{\bar{\sigma}}^-)$

the reduced wave-vector components,
in units of $k_F^{(S)} \cos \theta$ are

$$\beta_\sigma = \frac{\sqrt{\lambda_\sigma^2 - \sin^2 \theta}}{\cos \theta}$$

where

$$\lambda_\sigma = \kappa \sqrt{1 + \rho_\sigma X}$$

The Josephson current

$$I = \frac{4\pi k_B T \Delta^2}{eR} \int_0^{\pi/2} d\theta \sin \theta \cos \theta \sum_{\omega_n, \sigma} \frac{\beta_\sigma \beta_{\bar{\sigma}} \sin \phi}{G_n}$$

$$G_n = 8\Delta^2 \beta_\sigma \beta_{\bar{\sigma}} \cos \phi - \mathcal{G}_1^- \cos(\zeta_\sigma^-) + \mathcal{G}_1^+ \cos(\zeta_\sigma^+) + i\mathcal{G}_2^- \sin(\zeta_\sigma^-) - i\mathcal{G}_2^+ \sin(\zeta_\sigma^+)$$

$$\begin{aligned} \mathcal{G}_1^\pm &= - \left\{ \omega_n (\beta_\sigma \mp \beta_{\bar{\sigma}}) + \Omega_n [1 + Z_\theta^2 - iZ_\theta (\beta_\sigma \pm \beta_{\bar{\sigma}}) \mp \beta_\sigma \beta_{\bar{\sigma}}] \right\}^2 \\ &\quad - \left\{ \omega_n (\beta_\sigma \mp \beta_{\bar{\sigma}}) - \Omega_n [1 + Z_\theta^2 + iZ_\theta (\beta_\sigma \pm \beta_{\bar{\sigma}}) \mp \beta_\sigma \beta_{\bar{\sigma}}] \right\}^2 \\ \mathcal{G}_2^\pm &= 4i\Omega_n (1 + Z_\theta^2 \mp \beta_\sigma \beta_{\bar{\sigma}}) [i\omega_n (\beta_\sigma \mp \beta_{\bar{\sigma}}) + \Omega_n Z_\theta (\beta_\sigma \pm \beta_{\bar{\sigma}})] \end{aligned}$$

$$\Omega_n = \sqrt{\omega_n^2 + \Delta^2} \qquad \omega_n = \pi k_B T (2n+1)$$

The normal resistance

$$\frac{R}{R_N} = \int_0^{\pi/2} d\theta \sin \theta \cos \theta \sum_{\sigma} (1 - |b_N|^2)$$

$$b_N = \frac{2Z_{\theta}\beta_{\sigma} \cos(dk_{\sigma}^{+}) + (1 + Z_{\theta}^2 - \beta_{\sigma}^2) \sin(dk_{\sigma}^{+})}{2i(1 + iZ_{\theta})\beta_{\sigma} \cos(dk_{\sigma}^{+}) + (1 + 2iZ_{\theta} - Z_{\theta}^2 + \beta_{\sigma}^2) \sin(dk_{\sigma}^{+})}$$

$$R = 2\pi^2 \hbar / S e^2 k_F^{(F)^2}$$

Generalization of the Furusaki-Tsukada formula ...

A. Furusaki and M. Tsukada,
Phys. Rev. B 43, 10164 (1991).

... to the ballistic double-barrier SNS junction

Z. Radović, N. Lazarides, and N. Flytzanis,
Phys. Rev. B, in press (2003)

SNS junction ($X=0$)

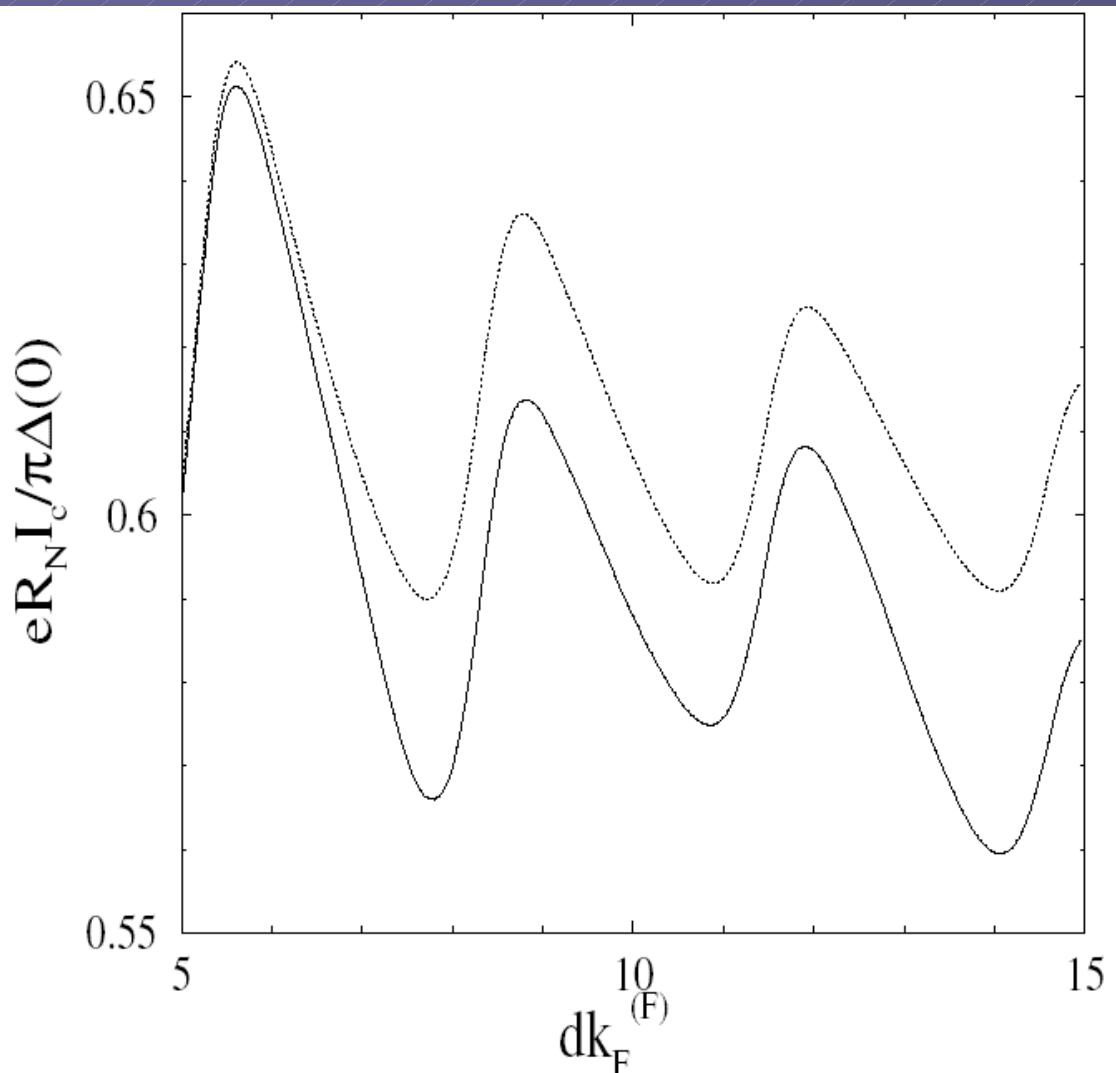
$$I = \frac{\pi k_B T \Delta^2}{eR} \int_0^{\pi/2} d\theta \sin \theta \cos \theta \sum_{\omega_n} \frac{\sin \phi}{\Gamma_n}$$

$$\begin{aligned}\Gamma_n = & \Delta^2 \cos \phi + (K^2 \Omega_n^2 + \omega_n^2) \cosh \left(\frac{2\omega_n d}{\hbar v_N} \right) + 2K\omega_n \Omega_n \sinh \left(\frac{2\omega_n d}{\hbar v_N} \right) \\ & - (K^2 - 1 - 2Z_\theta^2) \Omega_n^2 \cos(2k_N d) + 2Z_\theta (K^2 - 1 - Z_\theta^2)^{1/2} \Omega_n^2 \sin(2k_N d)\end{aligned}$$

$$K = \frac{1}{2} \left(\beta + \frac{1 + Z_\theta^2}{\beta} \right) \quad \beta = \frac{k_N}{k_S} \quad v_N = \frac{\hbar k_N}{m}$$

$$\begin{aligned}k_N &= \sqrt{k_F^{(N)^2} - \mathbf{k}_\parallel^2} & k_S &= \sqrt{k_F^{(S)^2} - \mathbf{k}_\parallel^2} \\ |\mathbf{k}_\parallel| &= k_F^{(S)} \sin \theta\end{aligned}$$

SFS double junction: the maximum current I_c



$T/T_c = 0.1$

$Z = 1$

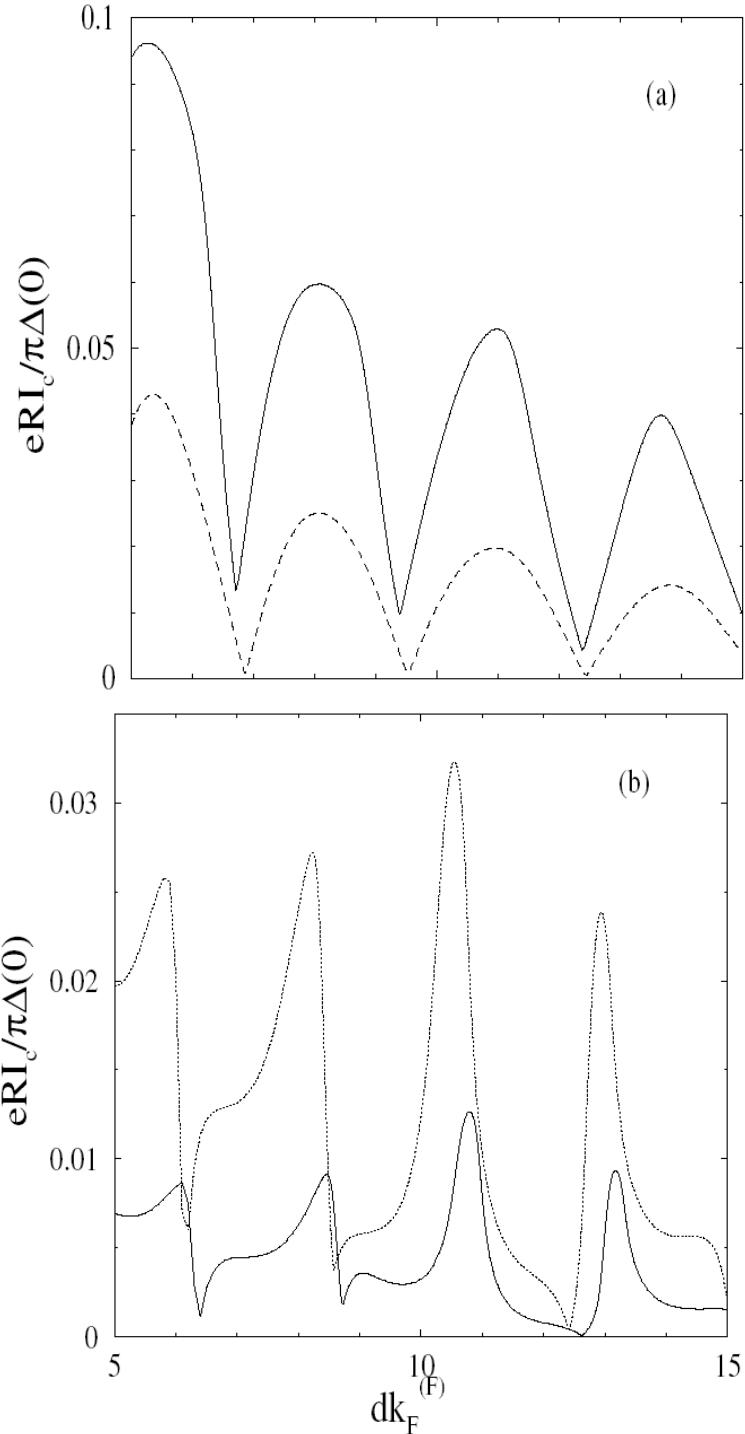
$X = 0.01$ (solid)

$X = 0$ (dotted)

$\kappa = 1$

$\Delta/E_F^{(S)} = 10^{-3}$

Z. Radović, N. Lazarides,
and N. Flytzanis,
Phys. Rev. B, in press (2003)



Strong ferromagnet

$X = 0.9$

$\Delta/E_F^{(S)} = 10^{-3}$

Top panel:

$Z = 0$

$Z = 1$

$\kappa = 1$

$T/T_c = 0.1$ (solid)

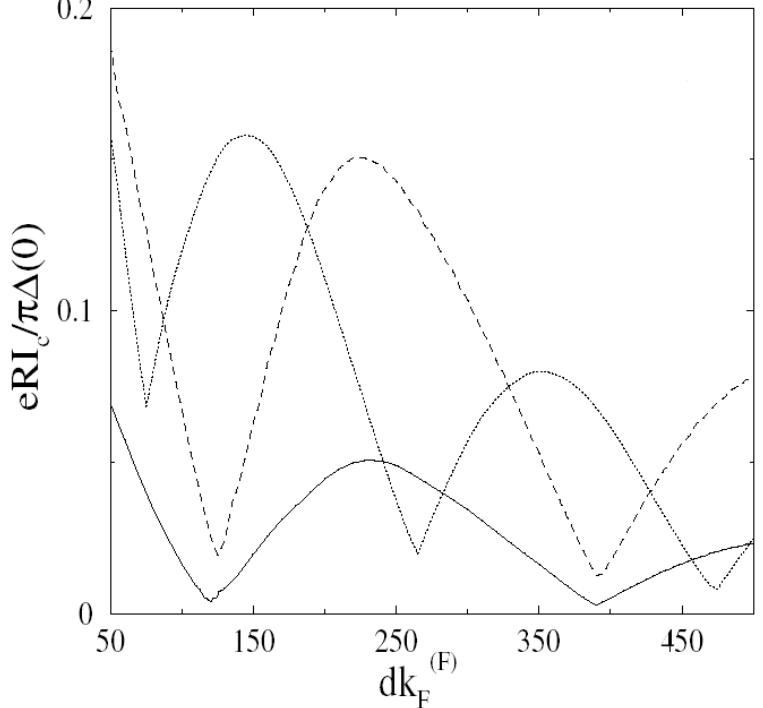
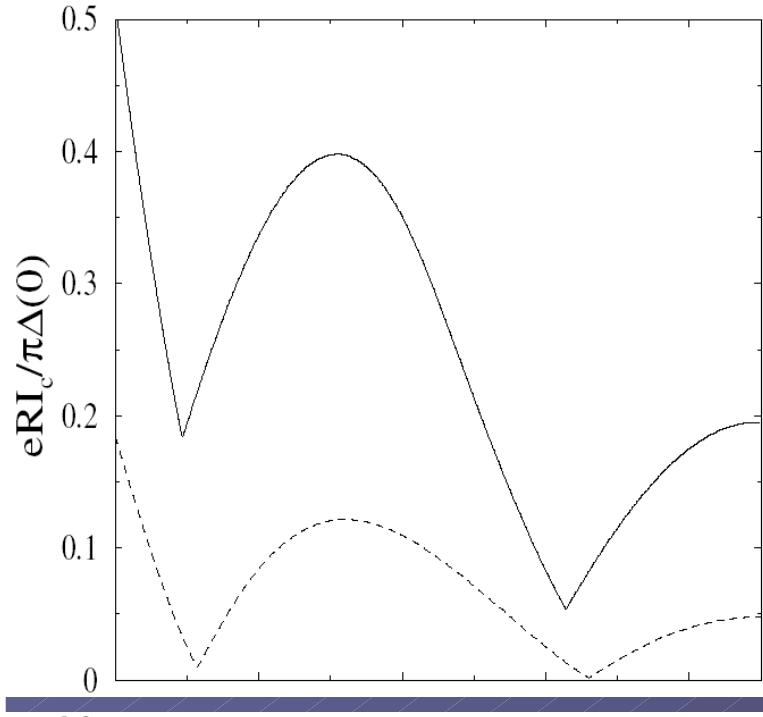
$T/T_c = 0.1$

$T/T_c = 0.7$ (dotted)

$\kappa = 0.7$ (solid)

$\kappa = 1$ (dotted)

Z. Radović, N. Lazarides, and
N. Flytzanis,
Phys. Rev. B, in press (2003)



Weak ferromagnet

$X = 0.01$

$\Delta / E_F^{(S)} = 10^{-3}$

Top panel:

$Z = 0$

$\kappa = 1$

$T / T_c = 0.1$ (solid)

$T / T_c = 0.7$ (dashed) $Z = 0, \kappa = 0.7$ (dotted)

Bottom panel:

$T / T_c = 0.1$

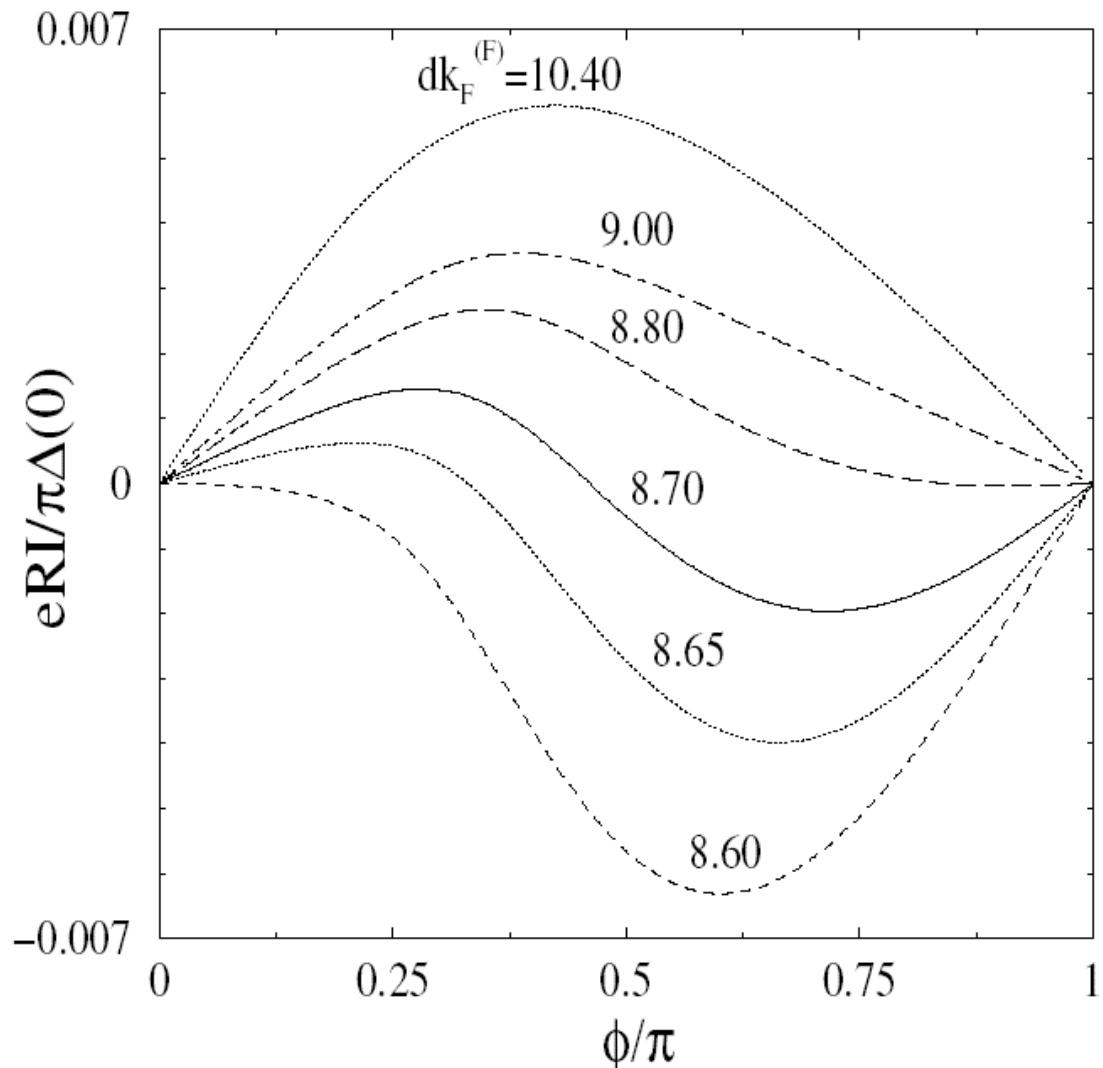
$Z = 1, \kappa = 0.7$ (solid)

$Z = 1, \kappa = 1$ (dashed)

$Z = 0, \kappa = 0.7$ (dotted)

Z. Radović, N. Lazarides, and N. Flytzanis,
Phys. Rev. B, in press (2003)

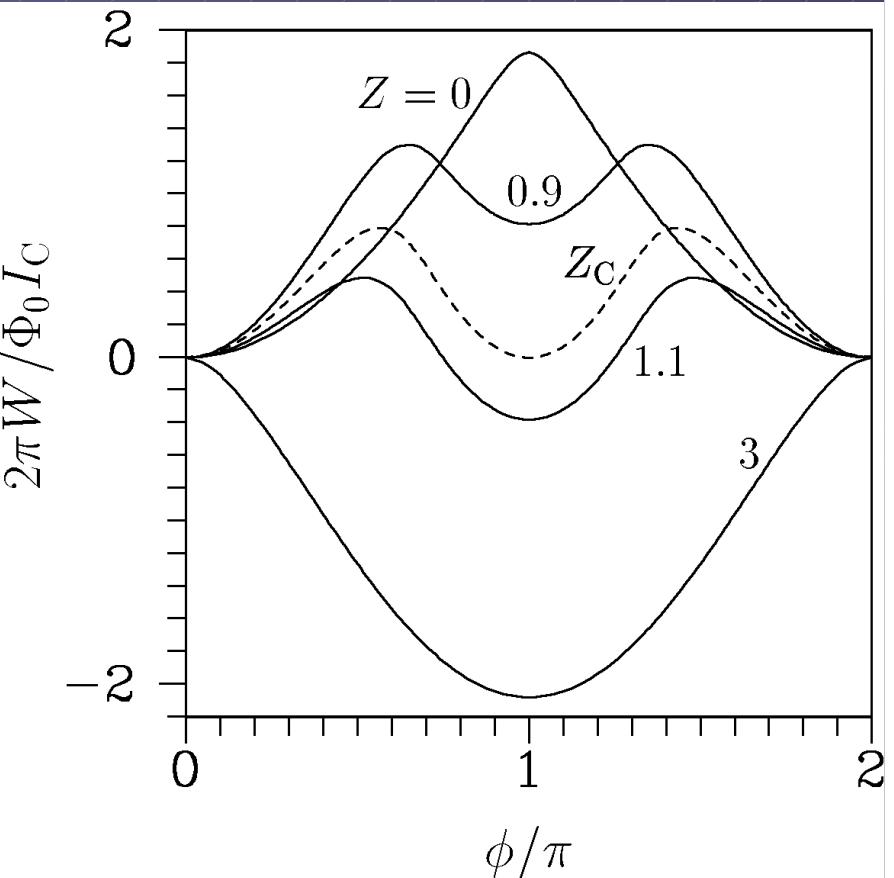
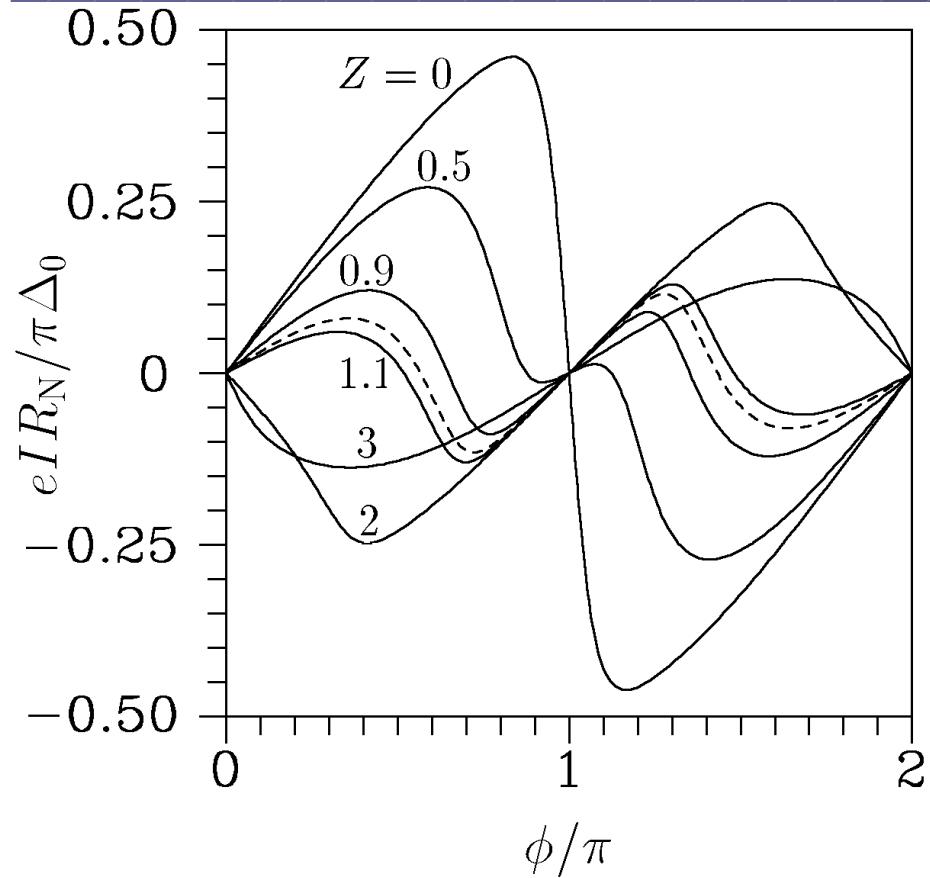
SFS double junction: the current-phase relation close to the transition



$Z = 1$
 $X = 0.9$
 $\kappa = 0.7$
 $\Delta/E_F^{(S)} = 10^{-3}$

Z. Radović, N. Lazarides,
and N. Flytzanis,
Phys. Rev. B, in press (2003)

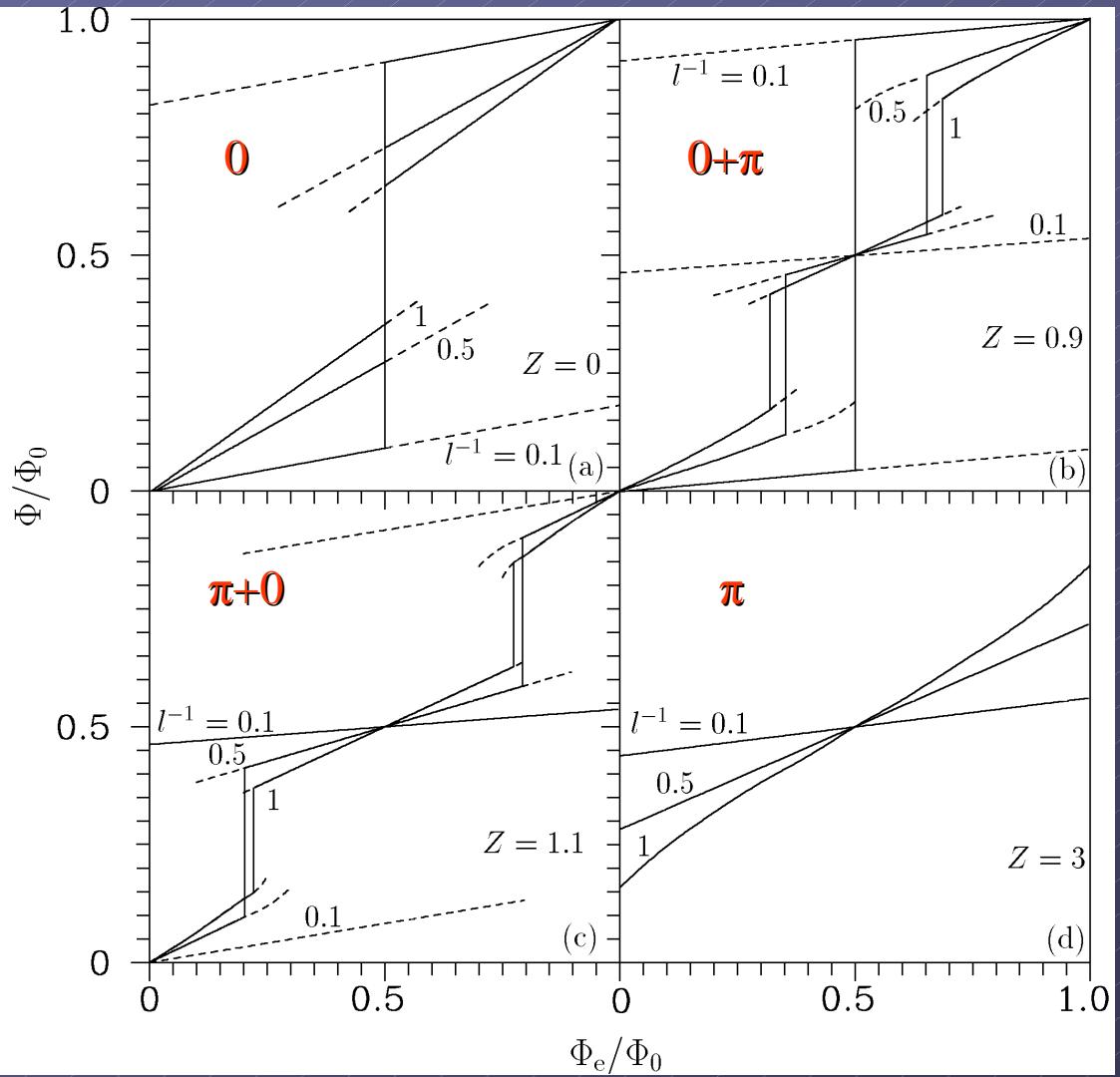
Coexistence of stable and metastable 0 and π states



$$Z = X d k_F$$

Z. Radović *et al.*,
Phys. Rev. B **63**, 214512 (2001).

Magnetic flux vs. external flux in SQUIDs



Effectively two times smaller flux quantum

$$l = \frac{2\pi}{\Phi_0} L I_c$$

Z. Radović *et al.*,
Phys. Rev. B **63**, 214512 (2001).

Temperature-induced 0- π transition

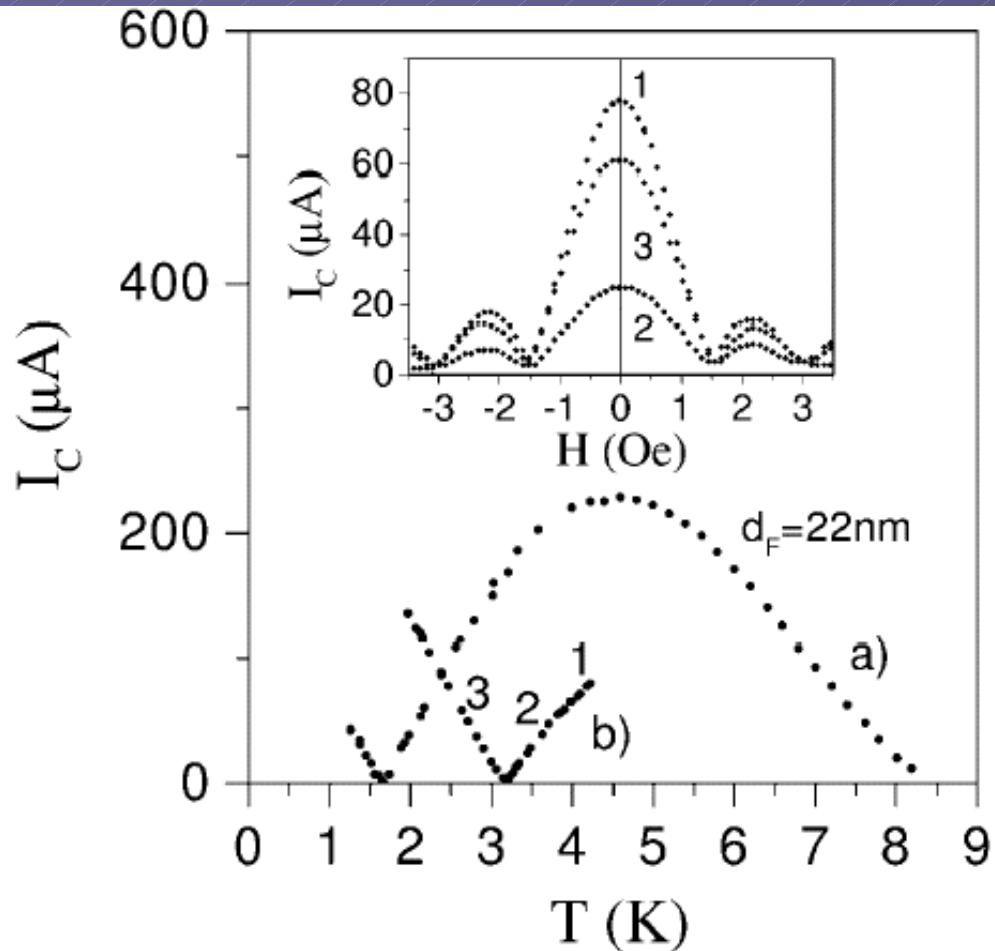
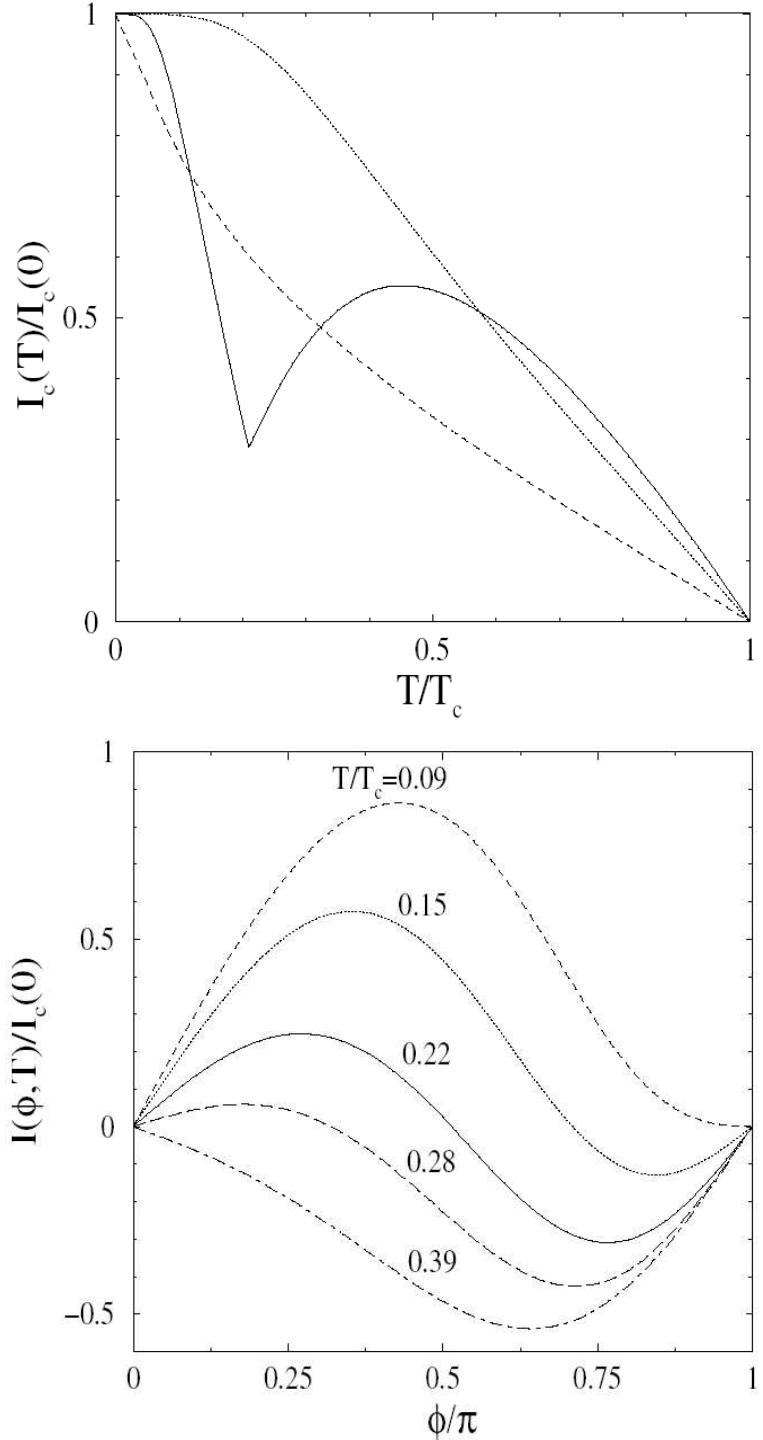


FIG. 3. Critical current I_c as a function of temperature T for two junctions with $\text{Cu}_{0.48}\text{Ni}_{0.52}$ and $d_F = 22 \text{ nm}$ [17]. Inset: I_c versus magnetic field H for the temperatures around the cross-over to the π state as indicated on curve b : (1) $T = 4.19 \text{ K}$, (2) $T = 3.45 \text{ K}$, (3) $T = 2.61 \text{ K}$.

V. V. Ryazanov *et al.*,
Phys. Rev. Lett. **86**, 2427
(2001).



Temperature-induced 0- π transition

finite
transparency

$Z = 1.2$ $X = 0.92$ $\kappa = 1$

$\Delta / E_F^{(S)} = 10^{-3}$

Top panel:

$dk_F^{(F)} = 17$ (dotted)

$dk_F^{(F)} = 17.23$ (solid) five values of T

$dk_F^{(F)} = 17.4$ (dashed)

strong
ferromagnet

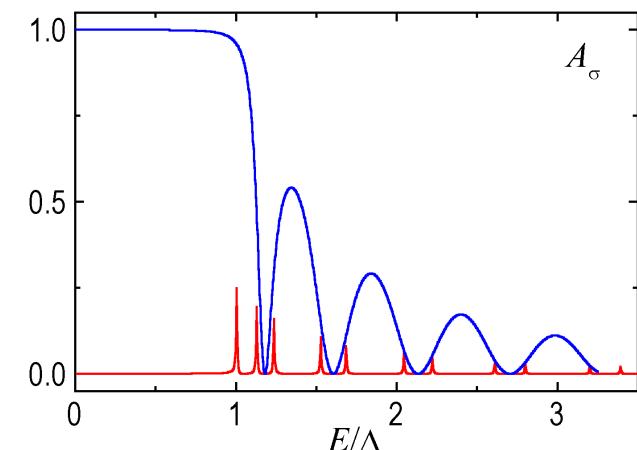
Bottom panel:

$dk_F^{(F)} = 17.23$

Z. Radović, N. Lazarides, and N. Flytzanis,
Phys. Rev. B, in press (2003)

Conclusion

- Features of finite size and coherency in clean FISIF:
 - (1) Subgap transport of electrons
(reduction of the excess current in thin S film)
 - (2) Oscillations of differential conductances
(vanishing of the Andreev reflection at geometrical resonances)
 - (3) Resonances in **metallic** vs. bound states in **tunnel junctions**

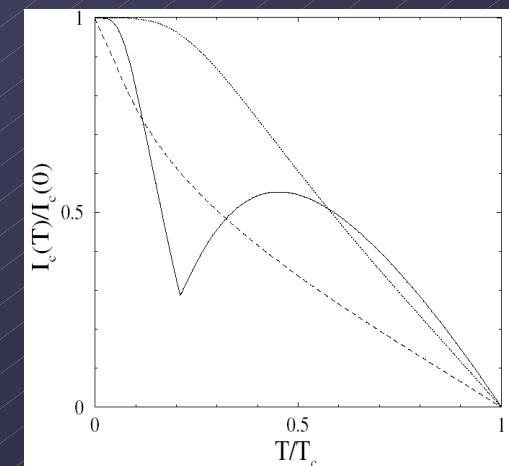
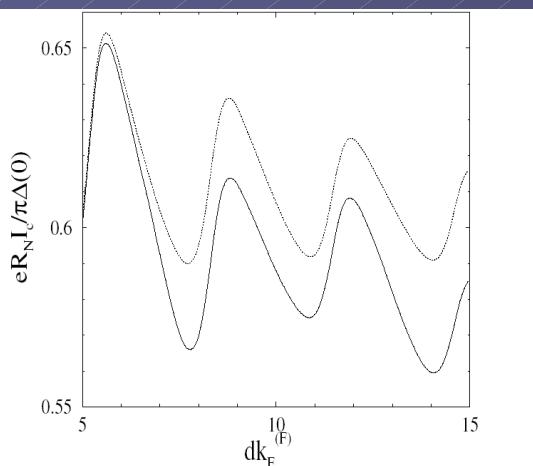


Conclusion

- Non-trivial spin polarization of the current without excess spin accumulation in S, i.e. without destruction of superconductivity, even in the AP alignment.
- Reliable ballistic spectroscopy of quasiparticle excitations in superconductors – measurements of Δ and v_F .

Conclusion

- Features of finite size and coherency in clean SIFIS:
 - (1) Geometrical oscillations of the maximum Josephson current
 - (2) Oscillations related to the crossovers between 0 and π states
 - (3) Temperature-induced 0- π transition when junction is close to the crossover at $T=0$, with finite transparency and strong ferromagnet



Conclusion

- Region of coexisting 0 and π states is considerably large.
Possible application:
 π SQUID with improved accuracy which operates with effectively 2x smaller flux quantum