ENERGY PUMPING IN TIME-DEPENDENT RANDOM MATRIX ENSEMBLES

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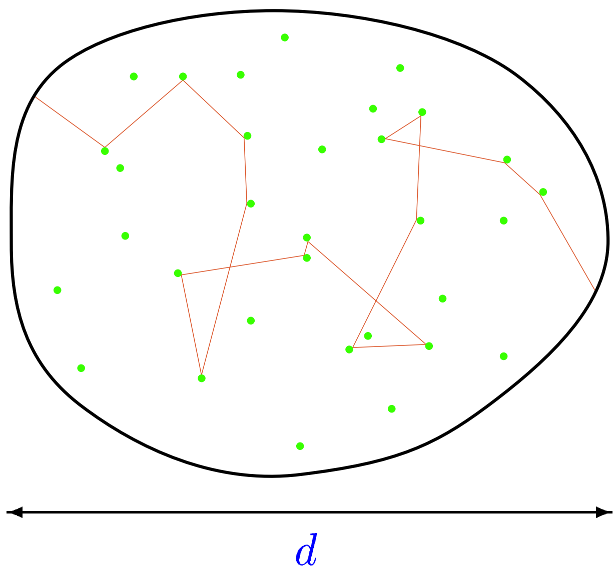
OUTLINE

- *Time-dependent random matrices*
- *Two basic phenomena in dynamics*
  - *Adiabatic and Kubo regimes of dissipation*
  - *Localization in the energy space*
- *Keldysh σ-model for parametrically-driven systems*
  - *Saddle point $\longrightarrow$ kinetic equation*
  - *Fluctuations $\longrightarrow$ quantum corrections*
- *Results*
  - *Linearly growing perturbation*
  - *(Multi-) periodic perturbation*

RANDOM MATRIX THEORY IN CONDENSED MATTER PHYSICS



closed disordered metal grain or quantum dot

Thouless energy: $E_c = \dfrac{\hbar D}{d^2}$

Universality:

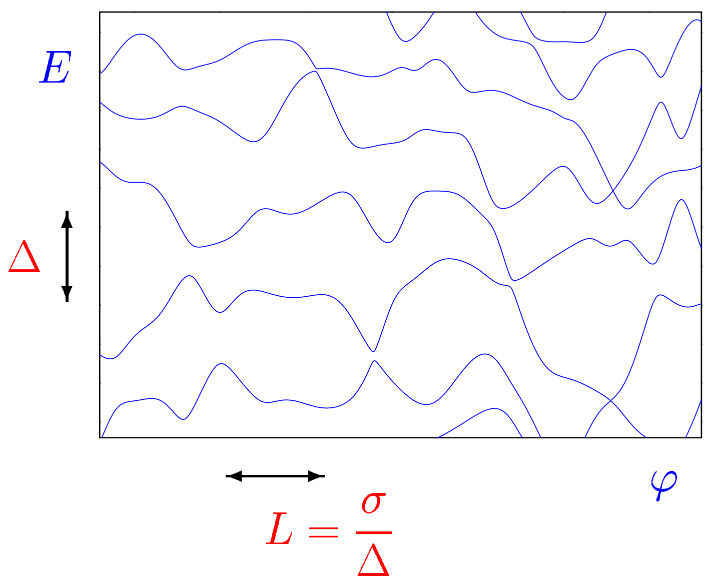
In the frequency range $|E - E'| \ll E_c$ the spectral statistics is proven to be that of the RMT of the corresponding symmetry: GOE, GUE, GSE (Efetov, 1982)

Follows from consideration of the 0D SUSY σ-model.

ROUTE TO TIME-DEPENDENT RANDOM MATRICES

- **Eigenvalue statistics**: $H\Psi = E\Psi$ — everything is known

- **Parametric eigenvalue statistics**: $H[\varphi]\,\Psi = E[\varphi]\,\Psi$



Parameters of the spectrum:

- Δ — mean level spacing
- σ — mean level velocity:

$$\sigma^2 = \left\langle (\partial E_i / \partial \varphi)^2 \right\rangle$$

- **Time-dependent problem**. Let $\varphi(t)$ be a function of time.

$$i \frac{\partial \Psi(t)}{\partial t} = H[\varphi(t)]\,\Psi(t)$$

Energy is not a conserving quantity anymore

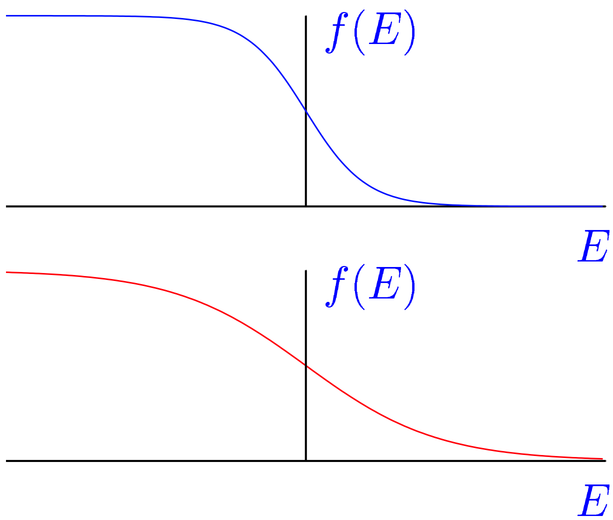
$$\langle E(t) \rangle = ???$$

WHAT IS TO LOOK FOR?

Spreading of the wave function due to interlevel transitions

[math] Evolution of the initial state $\Psi_n(0) = \delta_{n,0}$

[phys] Evolution of the distribution of noninteracting fermions



Pauli principle + interlevel transitions $\longrightarrow$ Growth of $\langle E(t) \rangle$

Energy absorption + inelastic relaxation $\longrightarrow$ Energy dissipation

$\Downarrow$

Heating

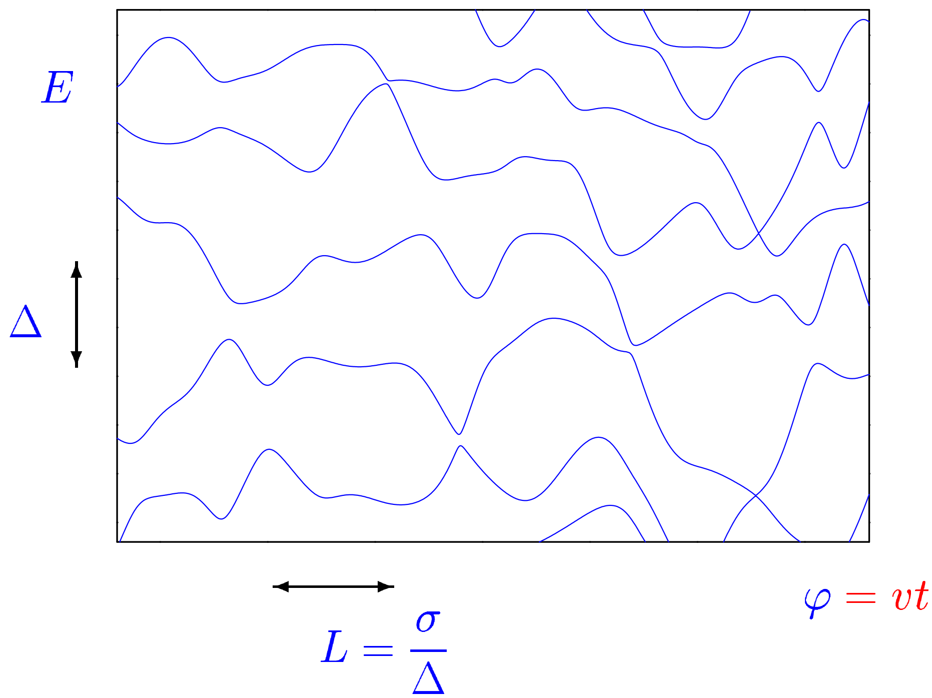
TWO BASIC PHENOMENA

- **Adiabatic** & **Kubo** regimes of dissipation
  - $*$ distinguished by $v = d\varphi/dt$
  - $*$ local property

- **Dynamical localization** in the energy space due to interference
  - $*$ for re-entrant $\varphi(t)$
  - $*$ global property

TWO REGIMES OF DISSIPATION IN CLOSED SYSTEMS

What is the meaning of adiabatic spectrum for a time-dependent perturbation?



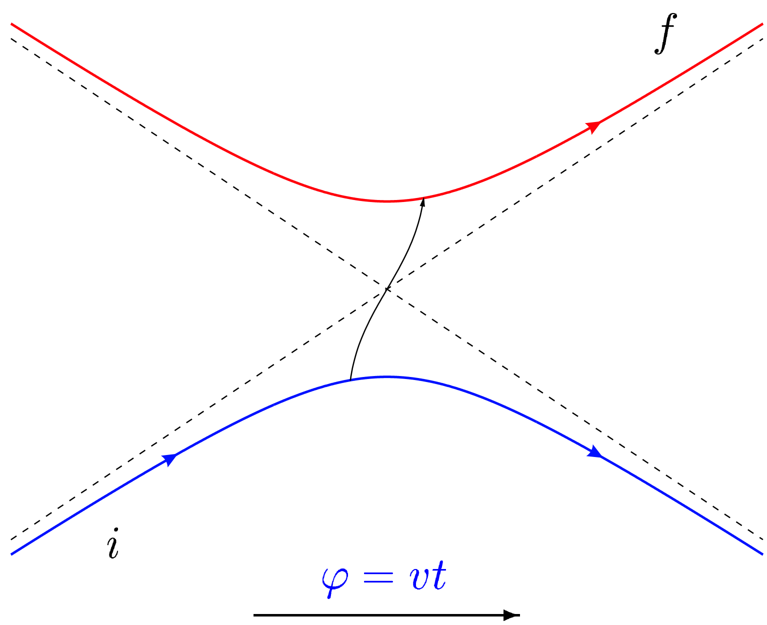
Levels acquire a width $\Gamma_v \sim \Delta\sqrt{\frac{v}{v_K}}$, with the critical velocity:

$$v_K \sim \frac{\Delta^2}{\sigma}$$

- $v \ll v_K \quad \longrightarrow$ discrete spectrum
- $v \gg v_K \quad \longrightarrow$ continuous spectrum

Avoided crossing

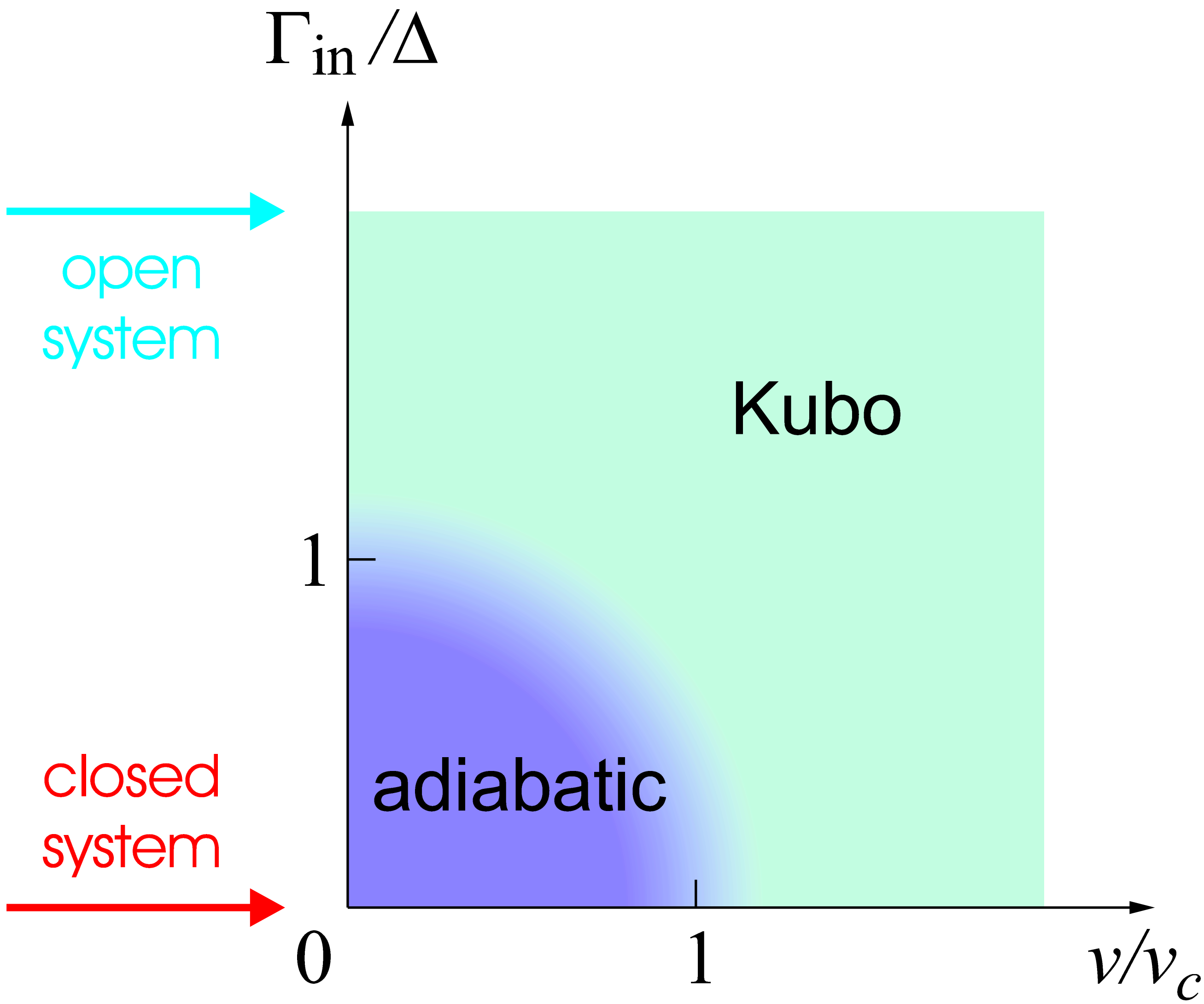
$$H[\varphi] = \begin{pmatrix} \varepsilon & \sigma\varphi \\ \sigma\varphi & -\varepsilon \end{pmatrix}, \qquad E_\pm[\varphi] = \pm\sqrt{\varepsilon^2 + \sigma^2\varphi^2}$$



$$\varphi = vt$$

Probability to jump to the other branch:

$$w_{i\to f} = \exp\left(-\frac{\pi\varepsilon^2}{\sigma v}\right)$$



Adiabatic regime:

- discrete spectrum
- Landau-Zener transitions
- dissipation rate depends on the spectral statistics

Kubo regime:

- continuous spectrum
- Kubo formula is valid
- Ohmic dissipation $W_K = \eta_K v^2$

RESULTS FOR THE RANDOM-MATRIX ENSEMBLES

(Wilkinson, 1988)

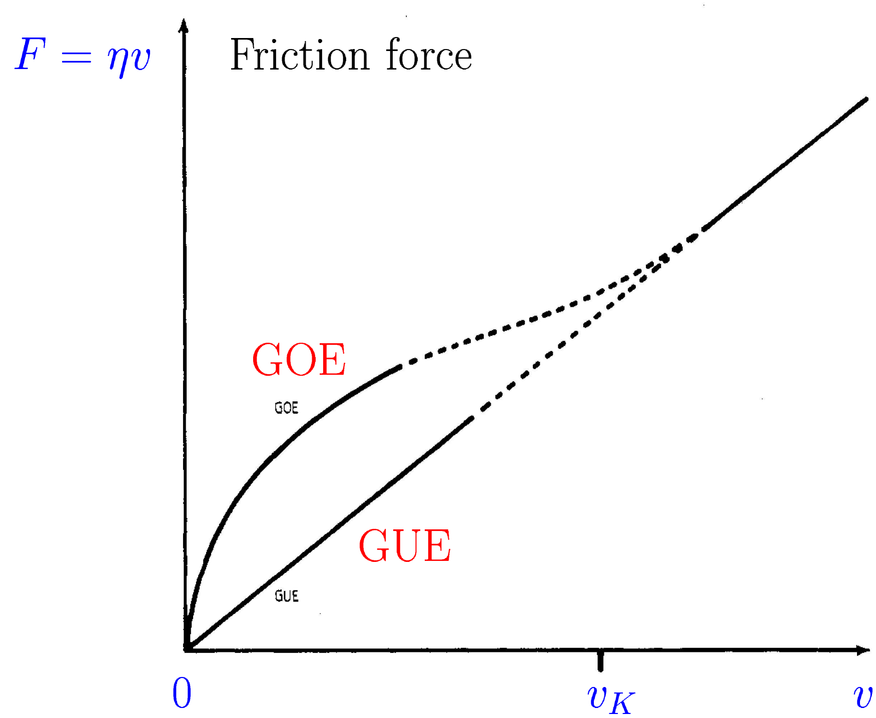
$$W = dE/dt = \eta v^2$$

• **In the Kubo regime**, $\eta_K = \dfrac{\pi \sigma^2}{\Delta^2}$ for $v \gg v_K$.

• **In the adiabatic regime**, to find W one has to average the probability of Landau-Zener tunneling over the distribution of avoided crossings. At $\varepsilon \ll \Delta$, it is given by the pair correlation function $R_2(\varepsilon) \propto \varepsilon^\beta$ (where $\beta = 1, 2, 4$ for GOE, GUE, GSE):

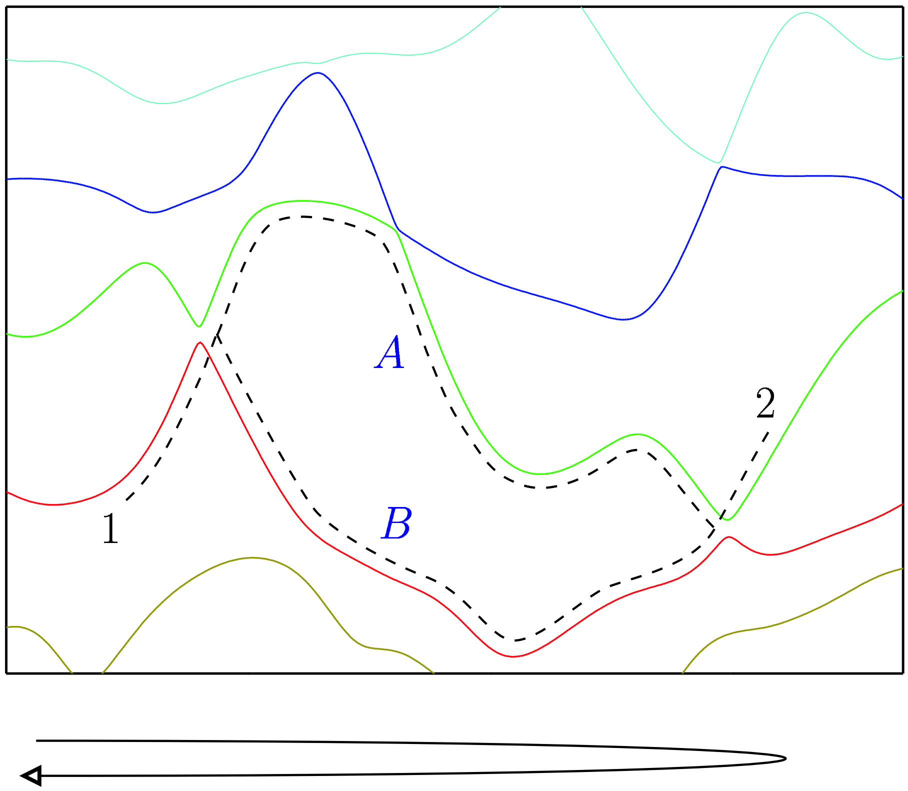
$$\int \exp\left(-\frac{\varepsilon^2}{\sigma v}\right) R_2(\varepsilon)\, d\varepsilon \qquad \longrightarrow \qquad \varepsilon \propto \sqrt{v}$$

In this way Wilkinson found $\eta \sim \eta_K \left(\dfrac{v}{v_K}\right)^{\frac{\beta}{2}-1}$ for $v \ll v_K$.



LOCALIZATION IN THE ENERGY SPACE

(Handwaving arguments in the adiabatic regime)



- Monotonous $\varphi(t)$ $\longrightarrow$ interference is ineffective

$$|A+B|^2 = |A|^2 + |B|^2 + 2\,\mathrm{Re}(AB^*) \longrightarrow |A|^2 + |B|^2$$

- Re-entrant $\varphi(t)$ $\longrightarrow$ interference *may be* important

$$|AA + AB + BA + BB|^2 = |AA|^2 + |BB|^2 + 4|AB|^2$$

$$\text{if} \quad \int_{\longrightarrow} \frac{E(\varphi)\,d\varphi}{d\varphi/dt} = \int_{\longleftarrow} \frac{E(\varphi)\,d\varphi}{d\varphi/dt}$$

Enhanced return probability $\Rightarrow$ Localization

THE MODEL

We consider a time-dependent matrix Hamiltonian

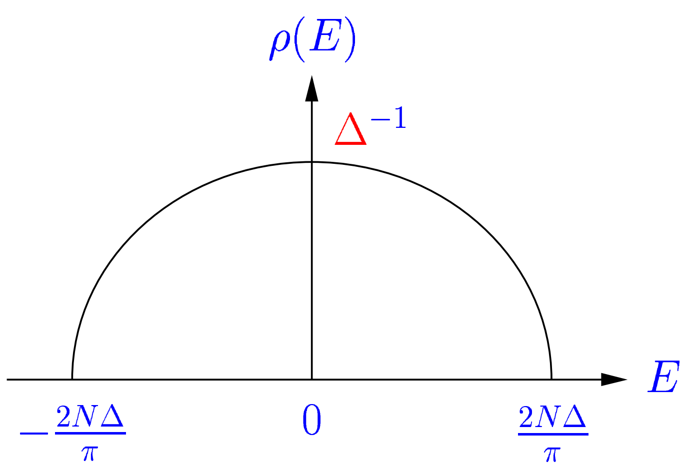
$$H(t) = H_0 + V\varphi(t),$$

where H_0 and V are random $N \times N$ matrices from the Gaussian Orthogonal Ensembles ($H^T = H$) with the variances

$$\langle (H_0)_{mn}(H_0)_{m'n'} \rangle = \frac{N\Delta^2}{\pi^2}\,[\delta_{mn'}\delta_{nm'} + \delta_{mm'}\delta_{nn'}],$$

$$\langle V_{mn}V_{m'n'} \rangle = \frac{\Gamma\Delta}{\pi}\,[\delta_{mn'}\delta_{nm'} + \delta_{mm'}\delta_{nn'}]$$

The DOS for an instant Hamiltonian (with $\dot{\varphi} = 0$) is given by the Wigner semicircle:



The parameter Γ determines the sensitivity of the spectrum to variation of φ:

$$\left\langle \left(\frac{\partial E_n}{\partial \varphi} \right)^2 \right\rangle = \frac{2\Gamma\Delta}{\pi}$$

σ-MODEL IN THE KELDYSH FORMALISM
Outline of the derivation

- Keldysh partition function via the functional integral over Grassmannian fields $\Psi(t)$:

$$Z = \int D\Psi D\Psi^* \exp\left\{ i \int_{\rightleftarrows} dt\ \Psi^\dagger(t) \left[i\tau_3 \frac{\partial}{\partial t} - H(t) \right] \Psi(t) \right\}$$

- Averaging over H_0 and V generates the quartic term

$$\left\{ \Psi_i^\dagger(t) \Psi_j(t) \right\} \left\{ \Psi_j^\dagger(t') \Psi_i(t') \right\}$$

- Decoupling by the Hubbard-Stratonovich matrix field $Q_{tt'}$

- Evaluation of the resulting Gaussian integral over Ψ:

$$S[Q] = -\frac{N}{2} \mathrm{Tr} \ln \left[\frac{\pi}{N\Delta} \delta(t-t') \frac{\partial}{\partial t'} + \gamma(t,t') Q_{tt'} \right]$$

$$+ \frac{N}{4} \int dt\, dt'\, \gamma(t,t')\, \mathrm{tr}\, Q_{tt'} Q_{t't}$$

where

$$\gamma(t,t') = 1 - \frac{\pi\Gamma}{N\Delta} [\varphi(t) - \varphi(t')]^2$$

- Expansion of the action $S[Q]$ over $1/N$

KELDYSH σ-MODEL

The low-energy effective theory is formulated in terms of the matrix Q-field ($Q^2 = 1$).

$Q_{tt'}^{\alpha\beta}$ acts in:

- **time** space – t is a continuous index
- 2×2 **Keldysh** space (σ_i)
- 2×2 **Particle-Hole** space (τ_i)

The σ-model action (e^{-S})

$$S[Q] = \frac{\pi i}{2\Delta} \operatorname{Tr} \hat{E} \tau_3 Q + \frac{\pi \Gamma}{8\Delta} \int dt\, dt' \, [\varphi(t) - \varphi(t')]^2 \operatorname{tr} Q_{tt'} Q_{t't}$$

E-term

responsible for the RMT energy **level statistics**

(encoded in the rich structure of $Q_{EE'}$, Altland & Kamenev, 2000)

kinetic term

accounts for **interlevel transitions** of the time-dependent Hamiltonian $H[\varphi(t)]$

• In the stationary case ($\varphi = \text{const}$), the Keldysh Green function Q is diagonal in the energy representation:

$$\Lambda = \begin{pmatrix} 1 & 2F^{(0)} \\ 0 & -1 \end{pmatrix} \otimes \tau_3,$$

$F(E) = 1 - 2f(E)$, and $f(E)$ is the fermion distribution function.

QUANTUM KINETIC EQUATION

Variation of the action with the constraint $Q^2 = \mathbf{1}$ yields the saddle point equation $[Q, \delta S/\delta Q] = 0$:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial t'}\right) Q_{tt'} = \frac{\Gamma}{2} \int d\tau \, [(\varphi(t)-\varphi(\tau))^2 - (\varphi(\tau)-\varphi(t'))^2] \, Q_{t\tau} Q_{\tau t'}$$

• In the non-stationary case, one can seek the solution using the stationary ansatz, but with the distribution function $F_{tt'}$ depending on both of its time indices. Then the saddle point equation becomes the **quantum kinetic equation**:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial t'}\right) F^{(0)}_{tt'} = -\Gamma(\varphi(t) - \varphi(t'))^2 \, F^{(0)}_{tt'}.$$

The Wigner-transformed function

$$F(E,t) = \int d\tau e^{iE\tau} F(t + \tau/2, t - \tau/2)$$

after averaging over fast oscillations in t obeys the **diffusion equation in the energy space**:

$$\frac{\partial F^{(0)}(E,t)}{\partial t} = D \frac{\partial^2 F^{(0)}(E,t)}{\partial E^2}$$

where $D = \Gamma\overline{(d\varphi/dt)^2}$ is the diffusion coefficient.

ENERGY DISSIPATION RATE

The energy absorption rate is determined by $F_{tt'}$:

$$W(t) \equiv \frac{\partial \langle E \rangle}{\partial t} = -\frac{1}{2}\int E \frac{\partial F(E,t)}{\partial t}\frac{dE}{\Delta} = -\frac{i\pi}{\Delta}\lim_{\eta\to 0}\frac{\partial^2 F_{t+\eta/2,\, t-\eta/2}}{\partial t\,\partial \eta}$$

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In the saddle-point approximation employing the diffusion equation we get

$$W = -\frac{D}{2\Delta}\int E\,\frac{\partial^2 F^{(0)}}{\partial E^2}\,dE = \frac{D}{2\Delta}\int \frac{\partial F^{(0)}}{\partial E}\,dE = \frac{D}{\Delta}$$

Thus we obtain Ohmic dissipation

$$W_K = \frac{\Gamma}{\Delta}\overline{\left(\frac{d\varphi}{dt}\right)^2}$$

coinciding with the result in the Kubo regime (Wilkinson, 1998). Valid provided $v \gg v_K$ and NO interference.

Where are Landau-Zener and interference?

They are in the fluctuation corrections to the saddle point

STRUCTURE OF THE Q-MANIFOLD

The symmetries of the Q matrix:

- the Q-manifold is compact $\longleftrightarrow$ fermionic system
- $Q^T = \sigma_1 \tau_2 Q \tau_2 \sigma_1$ $\longleftrightarrow$ PH symmetry

can be naturally implemented by

$$Q = U_F^{-1} P U_F, \qquad U_F = \begin{pmatrix} 1 & F^{(0)} \\ 0 & -1 \end{pmatrix}$$

The matrix P obeys: $P^\dagger = P, \quad P^T = \sigma_1 \tau_2 P \tau_2 \sigma_1.$

- The *saddle point* corresponds to $P_0 = \sigma_3 \tau_3$.

- The whole manifold can be parametrized as

$$P = \sigma_3 \tau_3 \frac{1 + W/2}{1 - W/2}$$

where

$$W = \left(\begin{array}{cc|cc} 0 & a & b & 0 \\ -a^\dagger & 0 & 0 & -b^T \\ \hline -b^\dagger & 0 & 0 & a^T \\ 0 & b^* & -a^* & 0 \end{array} \right)_K$$

SOFT MODES: DIFFUSONS AND COOPERONS

Cooperons: $\langle a_{t+\eta/2,t-\eta/2}\, a^*_{t'+\eta'/2,t'-\eta'/2}\rangle = \dfrac{\Delta}{\pi}\,\delta(t-t')\,C_t(\eta,\eta')$

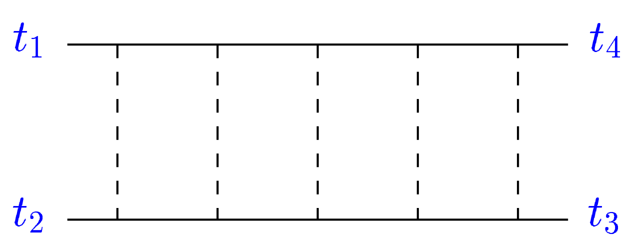
Diffusons: $\langle b_{t+\eta/2,t-\eta/2}\, b^*_{t'+\eta'/2,t'-\eta'/2}\rangle = \dfrac{2\Delta}{\pi}\,\delta(\eta-\eta')\,D_\eta(t,t')$

$$C_t(\eta,\eta') = \theta(\eta-\eta')\ \exp\left\{-\frac{\Gamma}{2}\int_{\eta'}^{\eta}[\varphi(t+\tau/2)-\varphi(t-\tau/2)]^2\,d\tau\right\}$$

$$D_\eta(t,t') = \theta(t-t')\ \exp\left\{-\Gamma\int_{t'}^{t}[\varphi(\tau+\eta/2)-\varphi(\tau-\eta/2)]^2\,d\tau\right\}$$

$\uparrow$

Dephasing
by the time-dependent perturbation (Vavilov, Aleiner, 1999 & Yudson, Kanzieper, Kravtsov, 2001)



Each impurity line carries a nonzero frequency $\Longrightarrow$ both diffusons and cooperons decay with time.

ONE-LOOP QUANTUM CORRECTION

Fluctuations induce corrections to the distribution function $F_{tt'}$:

vanishes due to causality [$\theta(t=0)=0$]

One-loop interference correction to the Kubo absorption rate W_0 for arbitrary $\varphi(t)$:

$$\delta W(t) = \frac{\Gamma}{\pi}\int_0^\infty \partial_t\varphi(t)\,\partial_t\varphi(t-\xi)\,C_{t-\xi/2}(\xi,-\xi)\,d\xi$$

THE CASE OF THE LINEAR BIAS $\varphi = vt$

The cooperon and diffuson have the form

$$C_t(\eta, \eta') = \theta(\eta - \eta') \exp\left\{-\frac{\Omega^3}{6}(\eta^3 - \eta'^3)\right\}$$

$$D_\eta(t, t') = \theta(t - t') \exp\left\{-\Omega^3\eta^2(t - t')\right\}$$

and decay at the time scale Ω^{-1}, where $\Omega^3 = \Gamma v^2$.

For a monotonous perturbation, there is no interference and quantum corrections are responsible for the crossover from the **Kubo** to **adiabatic** regimes of dissipation.

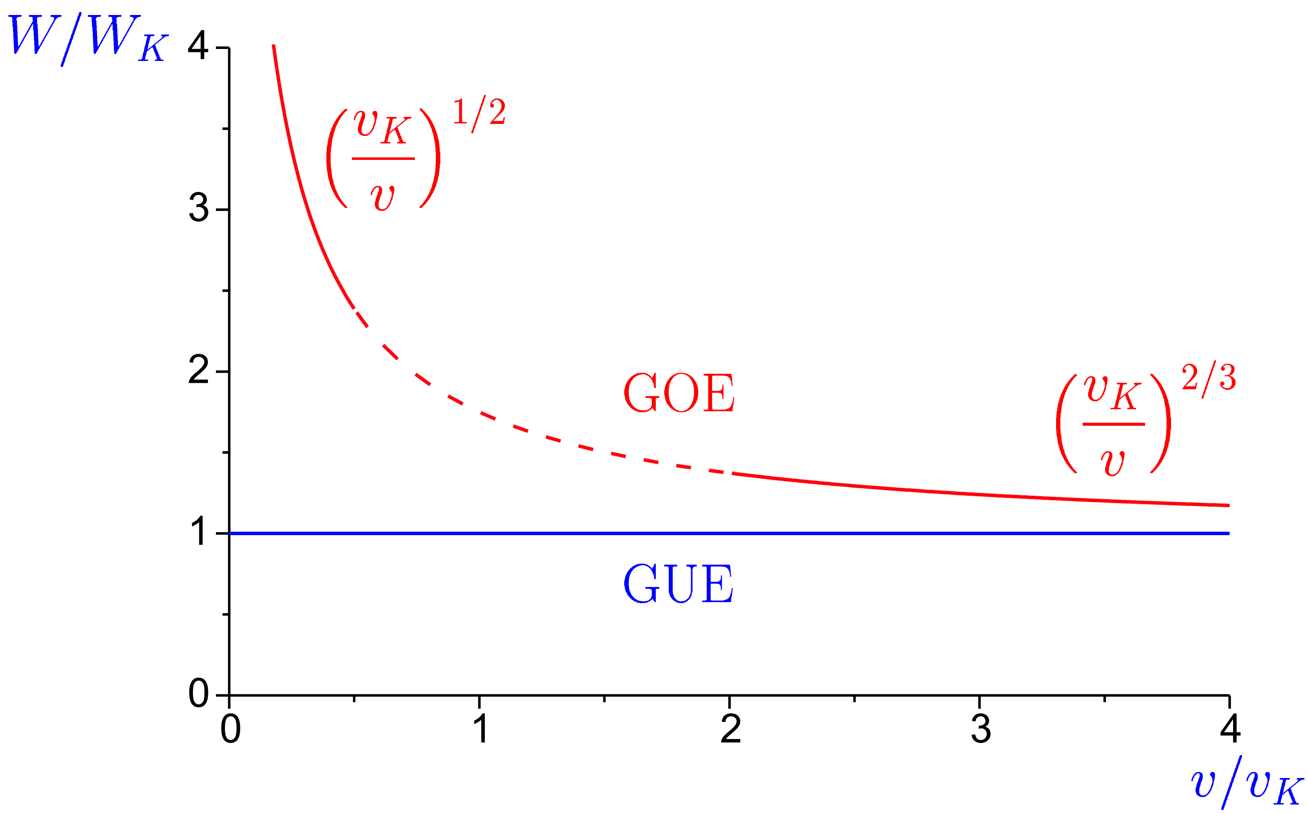
Loop expansion parameter: $\dfrac{\Delta}{\Omega} = \pi \left(\dfrac{v_K}{v}\right)^{2/3}$

$\uparrow$

small in the Kubo regime $v \gg v_K$

RESULTS FOR THE LINEAR BIAS $\varphi = vt$

Dissipation rate vs. velocity



Analytic expressions at $v \gg v_K$

for GOE: $$\frac{W}{W_K} = 1 + \frac{\Gamma(\frac{1}{3})}{3^{2/3}} \left(\frac{v_K}{v}\right)^{2/3} + \cdots$$

for GUE: $$\frac{W}{W_K} = 1$$

WEAK DYNAMIC LOCALIZATION

1. Monochromatic perturbation

$$\varphi(t) = \theta(t) \sin \omega t$$

To study the long-time, period-averaged dynamics at $t, \xi \gg 1/\omega$ we can approximate

$$C_{t-\xi/2}(\xi, -\xi) \approx \exp\left\{-2\Gamma\xi \cos^2[\omega(t - \xi/2)]\right\}$$

The cooperon is equal to unity at *no-dephasing points*

$$\xi_k = 2t - (2k+1)\pi/\omega$$

Performing Gaussian integration near ξ_k and summation over ξ_k we obtain a growing in time quantum interference correction to the ohmic absorption rate in the limit $t \gg 1/\omega, 1/\Gamma$:

$$\frac{W(t)}{W_K} = 1 - \sqrt{\frac{t}{t_*}}, \qquad t_* = \frac{\pi^3\Gamma}{2\Delta^2}$$

- Role of the phase relaxation time t_φ.

- Remarkable correspondence to the weak localization correction to the conductivity in a quasi-1D disordered wire.

$$\delta W(t) \longleftrightarrow \delta\sigma_1(t_\varphi)$$

2. General periodic perturbation

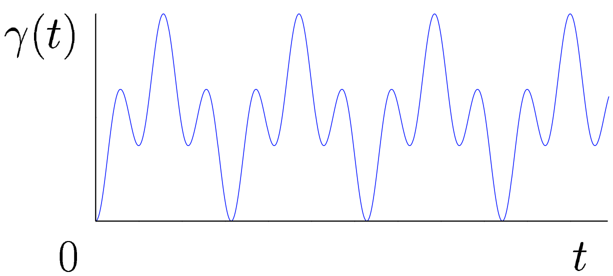
$$\varphi(t) = \theta(t) \sum_n A_n \sin(n\omega t - \alpha_n)$$

At long times the cooperon becomes

$$C_{t-\xi/2}(\xi, -\xi) \approx e^{-\xi\,\gamma(t-\xi/2)}, \quad \gamma(t) = 2\Gamma \sum_n A_n^2 \cos^2[n\omega t - \alpha_n]$$

Existence of the no-dephasing points is equivalent to the generalized time-reversal symmetry of the perturbation:

$$\varphi(-t+\tau) = \varphi(t+\tau)$$



a regular array of zeros

⇓

one-loop correction as for the monochromatic case

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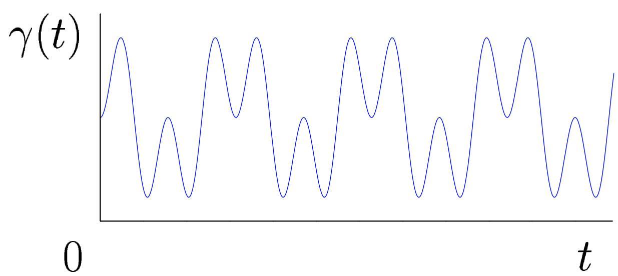
$$\frac{\delta W(t)}{W_K} \sim -\sqrt{\frac{t}{t_*}}$$

$$\varphi(-t+\tau) \neq \varphi(t+\tau)$$



a gap

⇓

one-loop correction is small; two-loop correction as for GUE

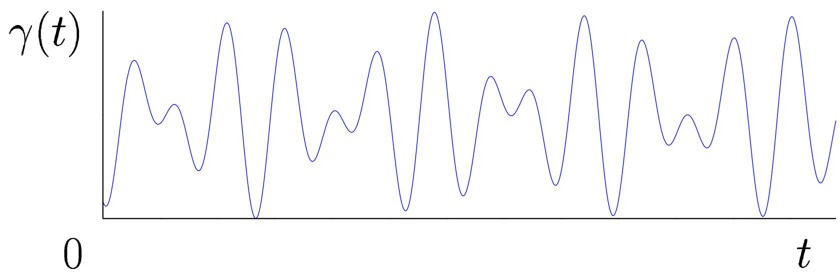
⇓

$$\frac{\delta W(t)}{W_K} \sim -\frac{t}{t_*}$$

WEAK DYNAMIC LOCALIZATION

3. d incommensurate frequencies

$$\varphi(t) = \sum_{n=1}^{d} A_n \sin(\omega_n t - \alpha_n)$$



a pseudo-gap

Result for the case when all $A_n = 1$:

$$\frac{W(t)}{W_K} = 1 - \frac{\Delta}{\pi\Gamma}\int_0^{\Gamma t} dz\, e^{-zd}\, [I_0(z)]^{d-1}\, \frac{dI_0(z)}{dz},$$

where $I_0(z)$ is the modified Bessel function.

$d = 2$: $$\frac{W(t)}{W_K} = -\frac{\Delta}{2\pi^2\Gamma} \ln \Gamma t$$

$d > 2$: $$\frac{W(t)}{W_K} \propto -t^{1-d/2} \longrightarrow \text{const}$$

- Complete analogy with the behavior of the WL correction in d dimensions.

APPLICATIONS

- *Quantum dot whose shape is being changed by a low-frequency gate voltage*
- *Quantum dot in a microwave electric field*
- *Vortex motion in impure superconductors*
- ...

CONCLUSION

- *Keldysh σ-model approach to study energy pumping in the parametrically-driven random-matrix ensembles.*

- *We calculated the leading quantum correction to the Ohmic absorption rate.*

∗ *Linearly growing perturbation: Quantum correction to the Kubo formula, which reveals the discreteness of the spectrum of the stationary Hamiltonian.*

∗ *Weak dynamic localization: For d incommensurate frequencies it behaves similar to the WL correction to conductivity of d-dimensional samples.*

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