NATO Advanced Research Workshop --- Chernogolovka, Russia --- June 11, 2003

Effects of 1/f Noise on Decoherence in Superconducting Flux Qubits

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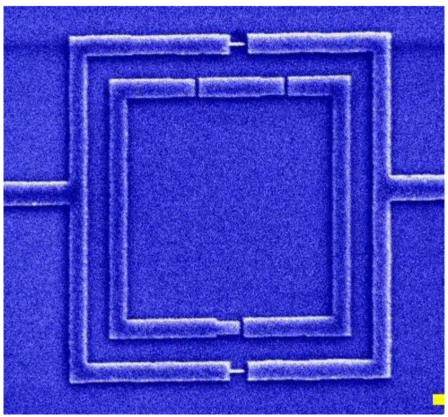
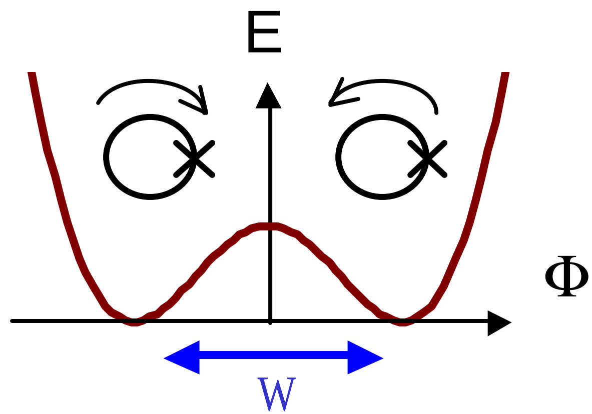
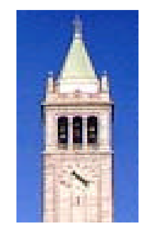


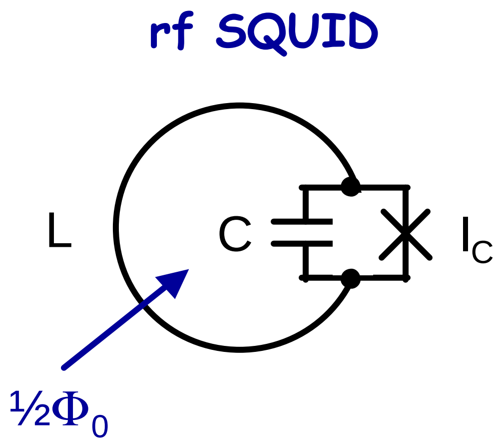


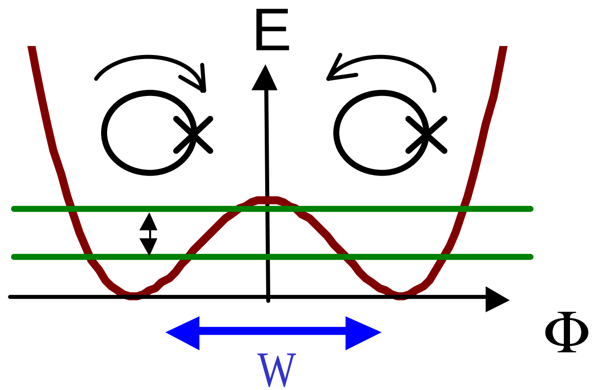
Trevis Crane, Sergey Frolov, and Madalina Colci
University of Illinois at Urbana-Champaign

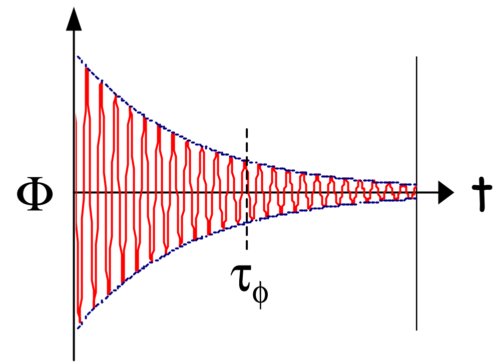
Britton Plourde, Paul Reichardt, Tim Robertson, and John Clarke
University of California at Berkeley



Superconducting flux qubit







Superconducting flux qubits --- the experimental picture

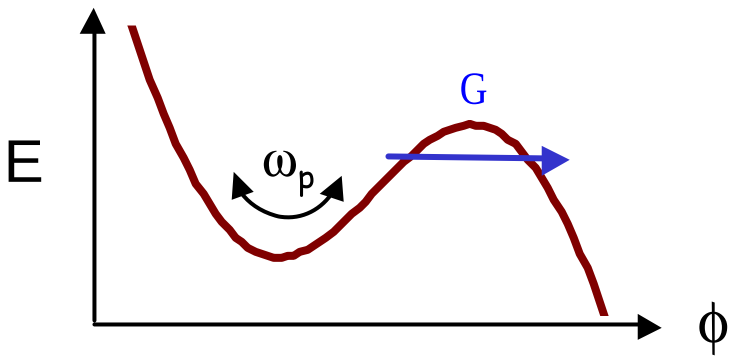
Design	Group	Quantum oscillations	Level splitting	Rabi oscillations	Ramsey fringes	Spin echoes	τ_ϕ
rf SQUID (excited)	Friedman et al. (Lukens) --Stony Brook		✓				---
3-junction qubit	van der Wal et al. (Mooij) -- Delft		✓	✓	✓		200 ns
3-junction qubit	Chiorescu,Nakamura et al. (Mooij) -- Delft		✓	✓	✓	✓	500 ns
single JJ qubit	Martinis et al. NIST Boulder			✓	✓		50 ns
single JJ qubit	Yu et al. (Han) – Kansas			✓			---
quantronium (hybrid charge/phase qubit)	Vion et al. (Esteve, Devoret) -- Saclay			✓	✓	✓	500 ns

Decoherence in superconducting flux qubits

A key challenge of Qubit Engineering is to minimize internal and external noise sources that reduce decoherence times …

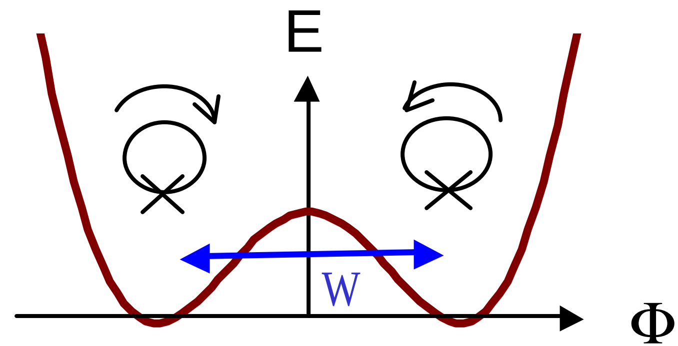
consider <u>low frequency noise</u> in flux qubits ® $f \ll \omega_p$, $1/\Gamma$, Ω, .….

Macroscopic Quantum Tunneling (MQT)



- <u>High frequency noise</u> suppresses the quantum tunneling rate Γ
- <u>Low frequency noise</u> has little effect on the tunneling rate (but can perturb the junction potential)

Macroscopic Quantum Coherence (MQC)



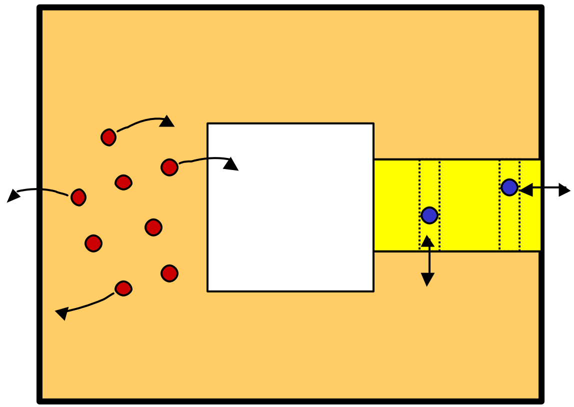
- <u>High frequency noise</u> suppresses the coherent oscillation frequency Ω and shortens decoherence times
- <u>Low frequency noise</u> causes dephasing by fluctuating the qubit potential via internal critical current fluctuations and external magnetic flux noise

Sources of low frequency noise in superconducting circuits

Flux noise … vortex motion

Vortex hopping in SC film induces flux changes in SQUID

dominates for large loops and large junctions



Critical current noise … charge motion

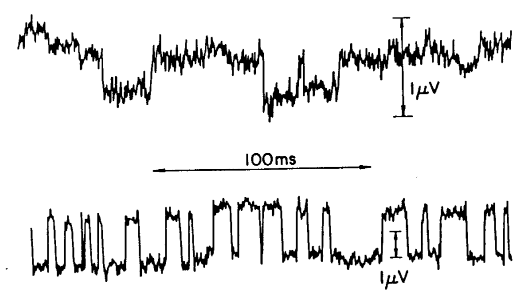
Charge trapping in Josephson junction barrier blocks tunneling in localized region → change in critical current

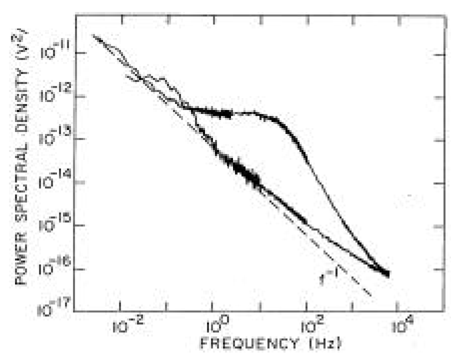
dominates for small loops and small junctions

Single vortex/charge trap creates switching noise → Lorentzian in power spectrum

Multiple vortices/charge traps add to give ~1/f noise spectrum (Dutta-Horn model)

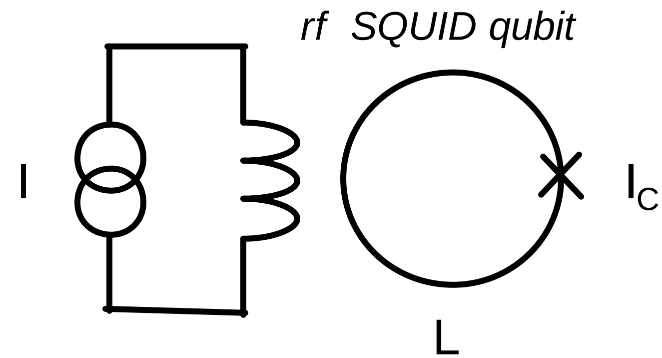
Example: charge traps in tunnel junctions (Wakai, Van Harlingen)



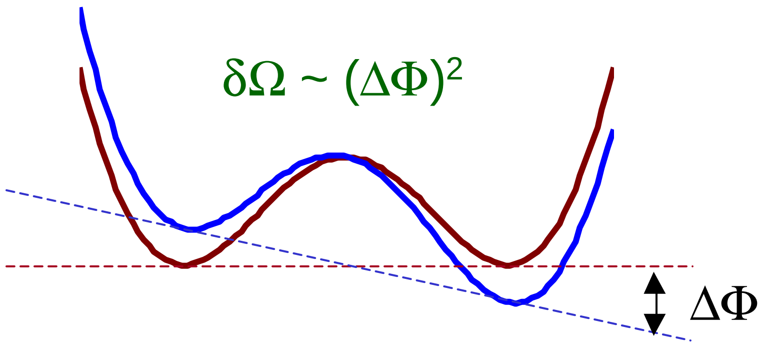


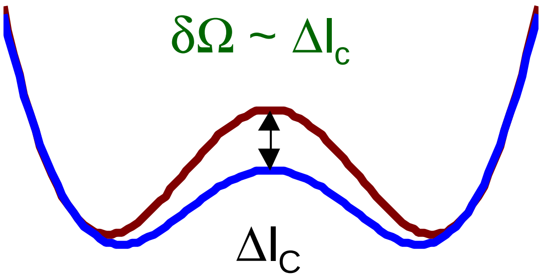
Effect of low frequency noise on the measurement of coherent quantum oscillations

1/f flux noise in qubit <u>and</u> coil current noise modulates flux bias from ½ Φ_0

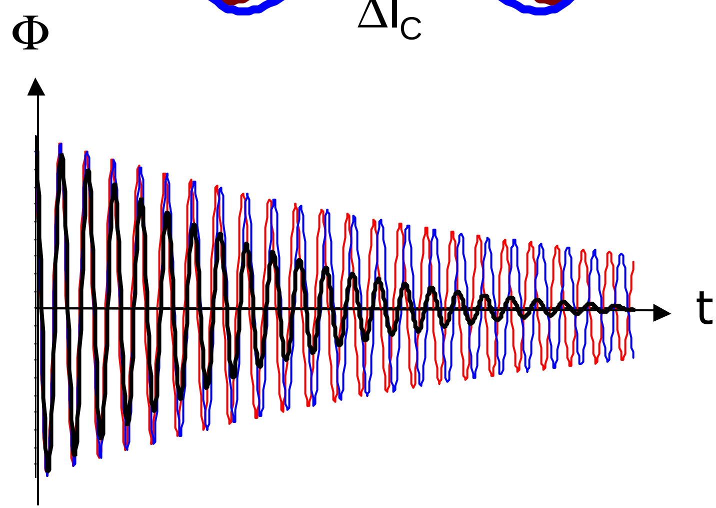


1/f critical current noise modulates tunneling barrier height





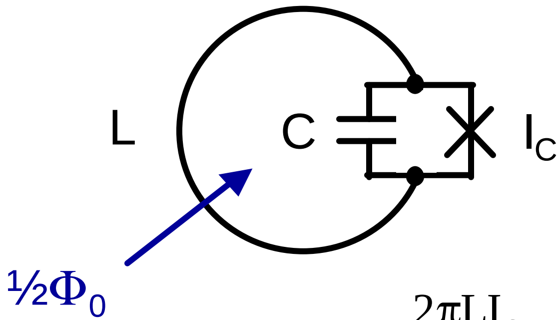
Fluctuation of the tunneling frequency causes phase noise → results in an <u>effective</u> decoherence since Ω is different for each successive point of a distribution measurement

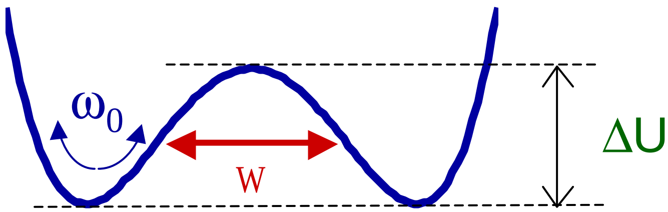


dephasing factor: $\Lambda = \left(\frac{d\Omega}{dI_c}\right) \Big/ \left(\frac{\Omega}{I_c}\right)$

rf SQUID qubit

A.J. Leggett; J. R. Friedman, Nature





$$\beta_L = \frac{2\pi L I_c}{\Phi_0} > 1$$

Barrier height: $$\Delta U = \frac{3E_J}{4\pi^2}(\beta_L - 1)^2$$

Plasma frequency: $$\omega_0 = 2\sqrt{\frac{\beta_L - 1}{LC}}$$

"Degree of classicality" : $$\lambda = \sqrt{\frac{8 I_c C \Phi_0^{\,3}}{\pi^3 \hbar^2}}$$

Tunneling frequency: $$\Omega = \omega_0 \exp\left[-\frac{(\beta_L - 1)^{3/2}\lambda}{\sqrt{2}}\right]$$

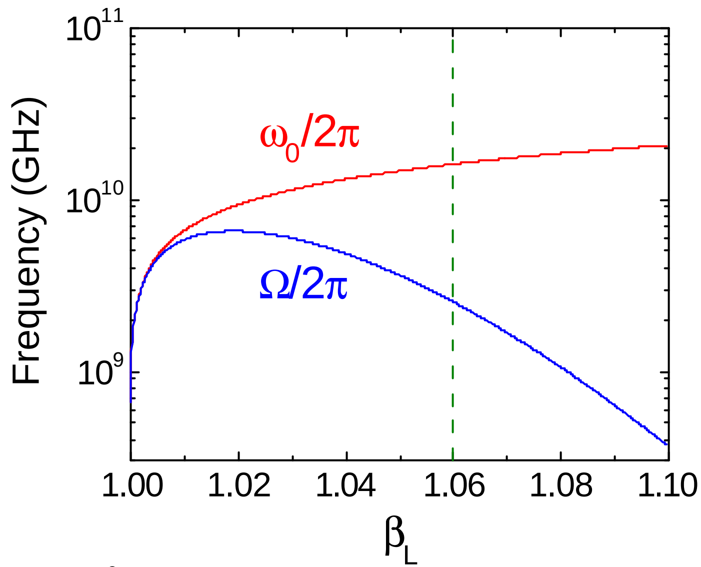
rf SQUID qubit --- ground state ---Stony Brook parameters*

J. R. Friedman *et al*., Nature 406, 43 (2000)

I_c = 1.46 μA
L = 240 pH
} β_L = 1.06

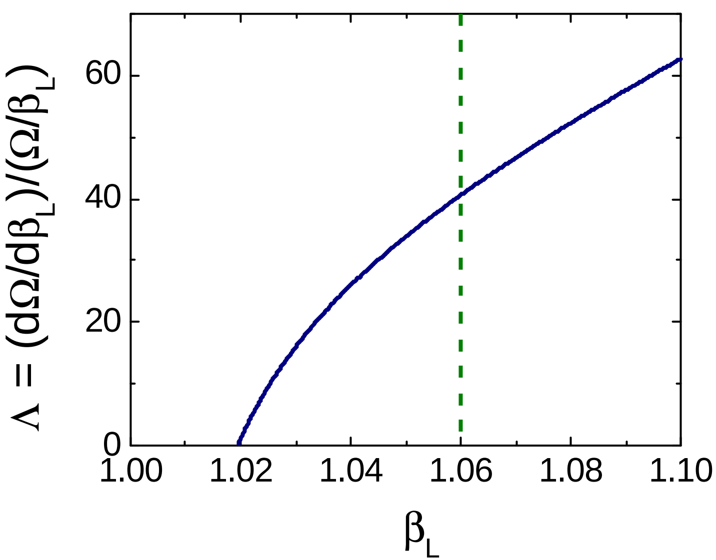
A = 2 μm^2 C = 103 fF

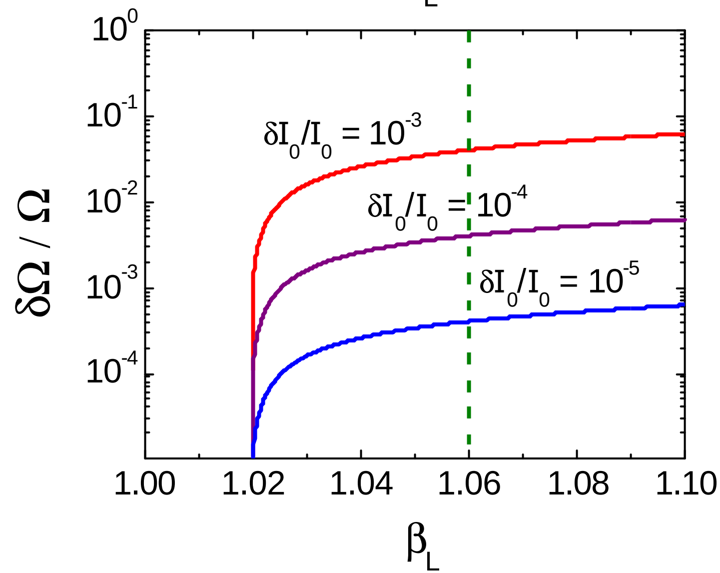
$\omega_0/2\pi$ = 98.5 GHz $\Omega/2\pi$ = 2.52 GHz



Ground state:
Λ = 40.6

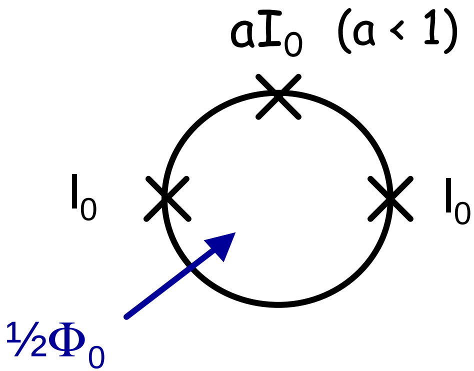
* Excited state:
Λ = 35

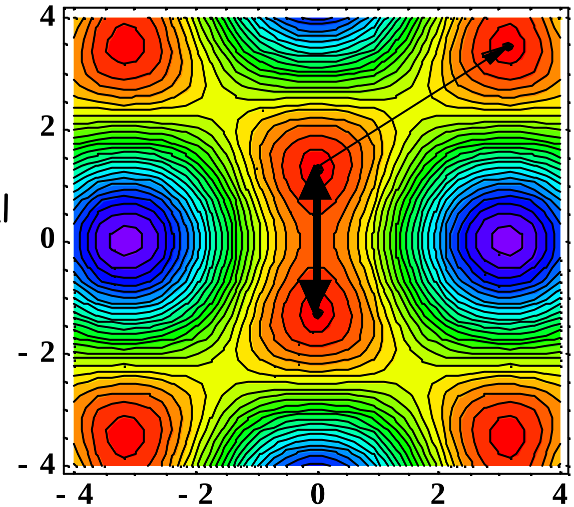




Three-junction qubit

T. P. Orlando *et al.*, PRL 60, 15398 (1999)





Josephson energy: $E_J = I_0 F_0 / 2\pi$

Charging energy: $E_C = e^2 / 2C$

Plasma frequency: $\omega_m = \frac{1}{\hbar}\left[\frac{4(4a^2-1)}{a(1+2a)} E_J E_C\right]^{1/2}$

Phase minimum : $\phi_m^* = \cos^{-1}\left(\frac{1}{2a}\right)$

Tunneling frequency: (intracell) $W(E_J) = \omega_m \exp\left[-\frac{4a(1+2a)E_J}{E_C}\right]^{1/2}\left(\sin\phi_m^* - \frac{\phi_m^*}{2a}\right)$

Three-junction qubit --- Delft parameters

C. H. van der Waal et al., Science 290, 773 (2000)

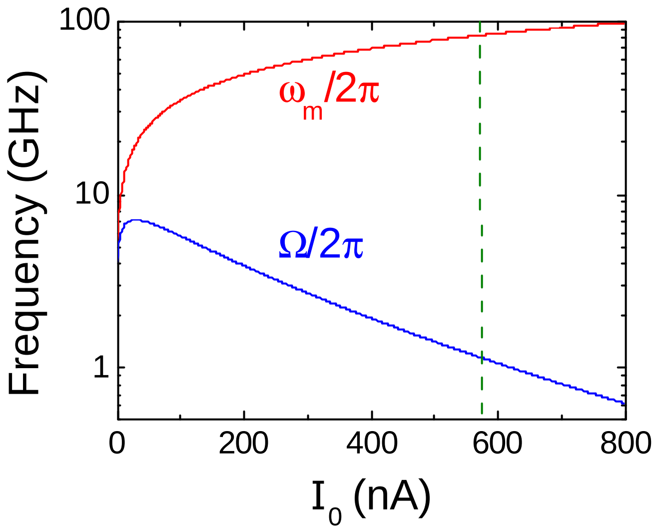
I_c = 570 nA a = 0.82 L = 11 pH

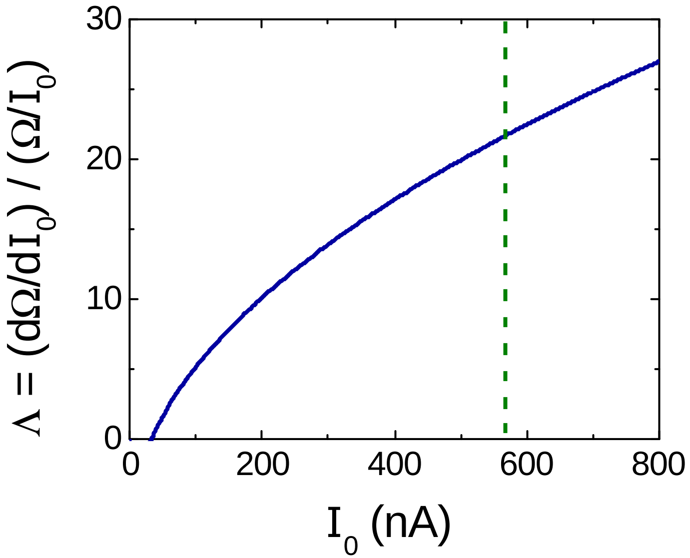
A = 2 μm^2 C = 2.6 fF

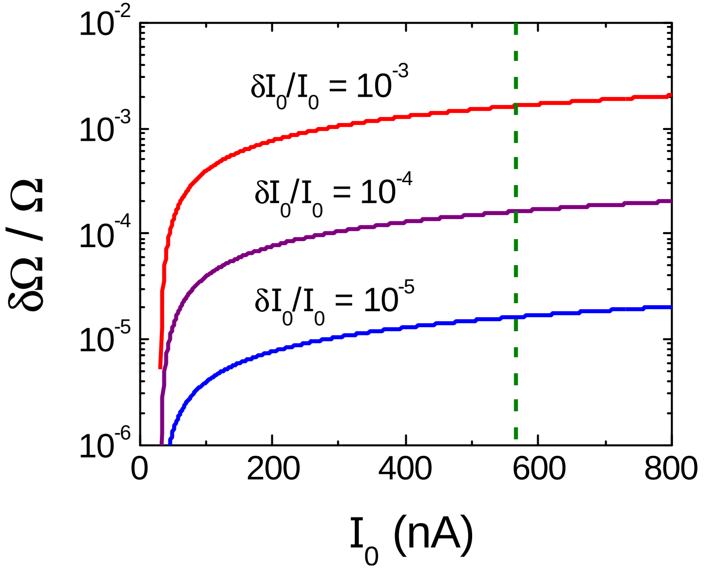
$\omega_0/2\pi$ = 81.4 GHz $\Omega/2\pi$ = 1.14 GHz

Modulation of only smallest junction: Λ = 20.4

For all junctions: Λ = 14.0

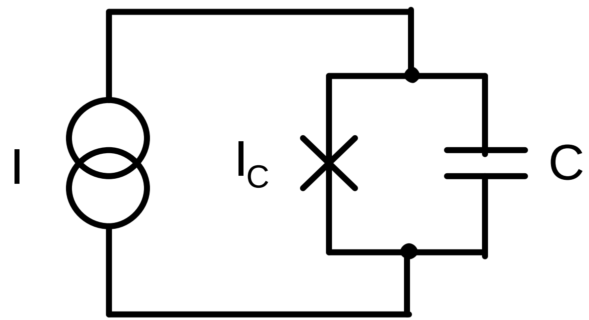




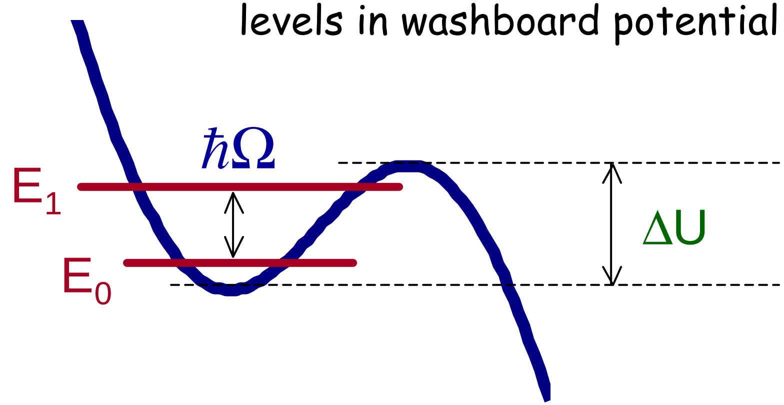


Single Josephson junction qubit

Current-biased single junction



Oscillations between energy levels in washboard potential well



Barrier height:

$$DU = \frac{2\sqrt{2}\, I_c\, F_0}{3\pi}\left(1 - \frac{I}{I_c}\right)^{3/2}$$

Plasma frequency:

$$\omega_p = \left(\frac{2\sqrt{2}\,\pi I_c}{C\ F_0}\right)^{1/2}\left(1 - \frac{I}{I_c}\right)^{1/4}$$

Tunneling frequency:

$$W = \frac{E_1 - E_0}{\hbar} \approx \omega_p$$

Single junction qubit --- NIST parameters

J. M. Martinis *et al., preprint*

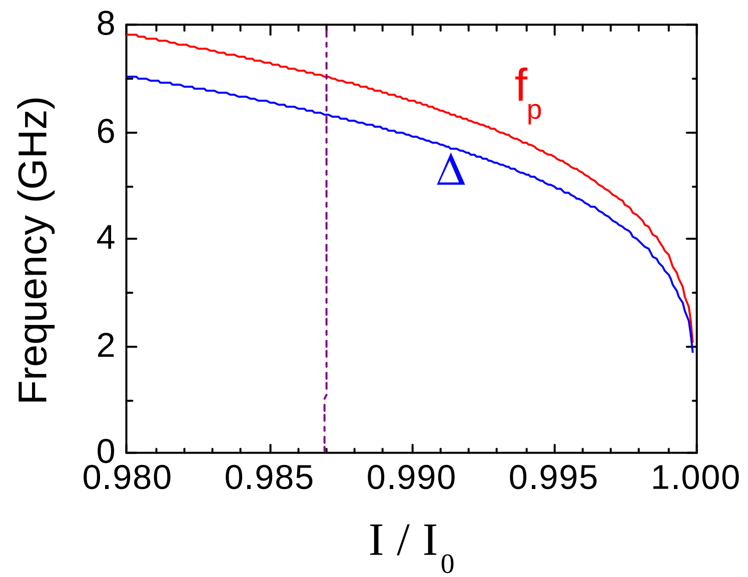
$I_c = 20\ \mu A$

$I_b = 19.33\ \mu A$

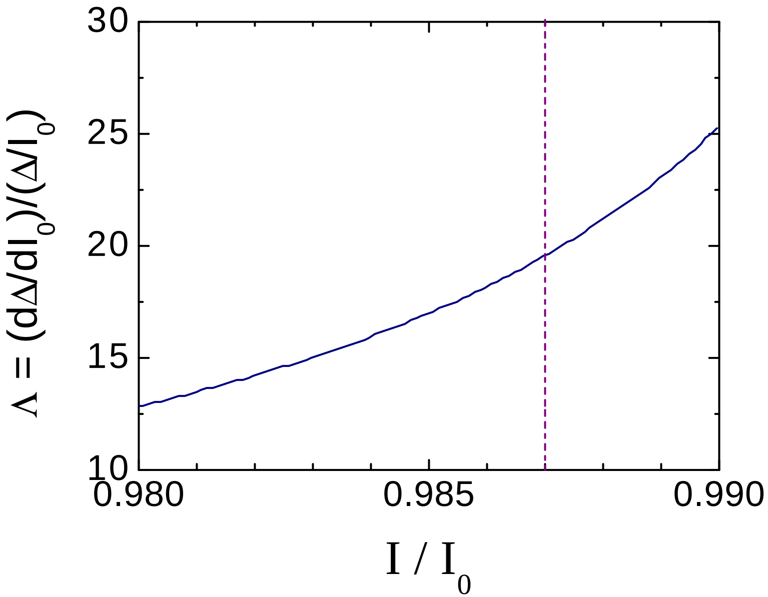
} $I/I_c = 0.987$

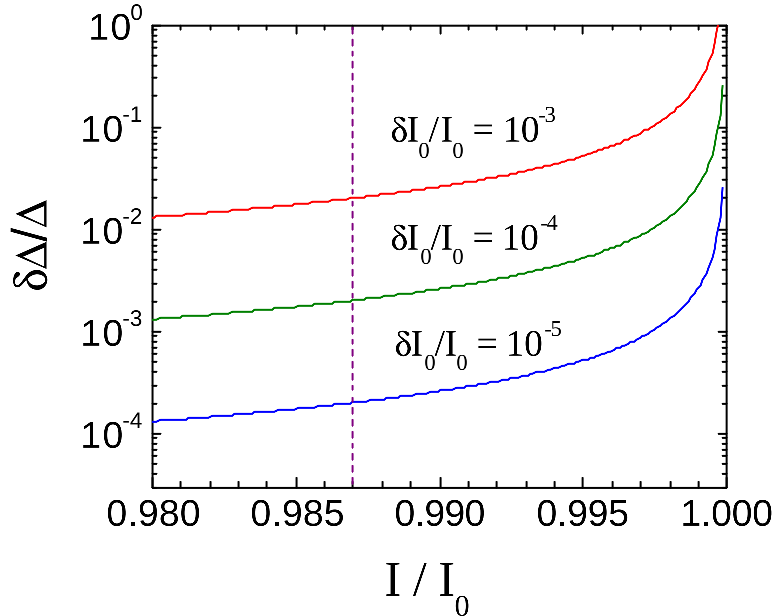
$A = 100\ \mu m^2 \rightarrow C = 5\ pF$

$\Omega/2\pi \approx \omega_0/2\pi = 7.06\ GHz$

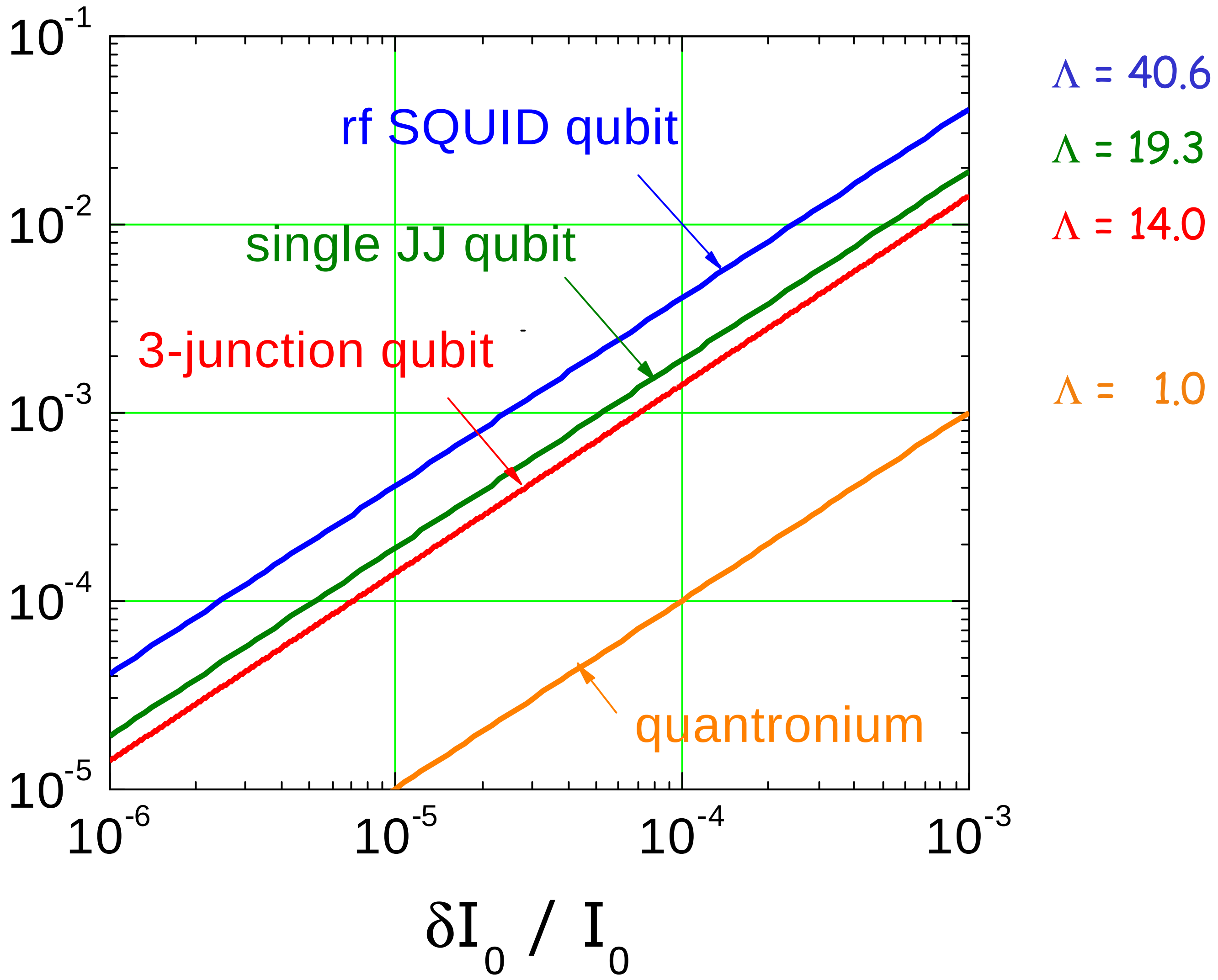


$\Lambda = 19.3$



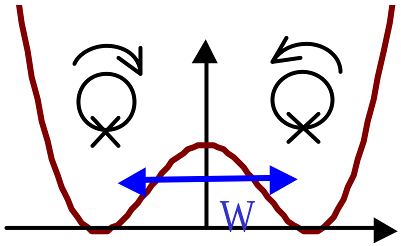


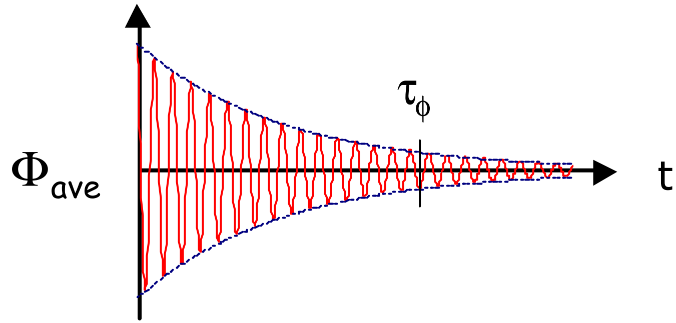
Effect of critical current fluctuations on tunneling frequency



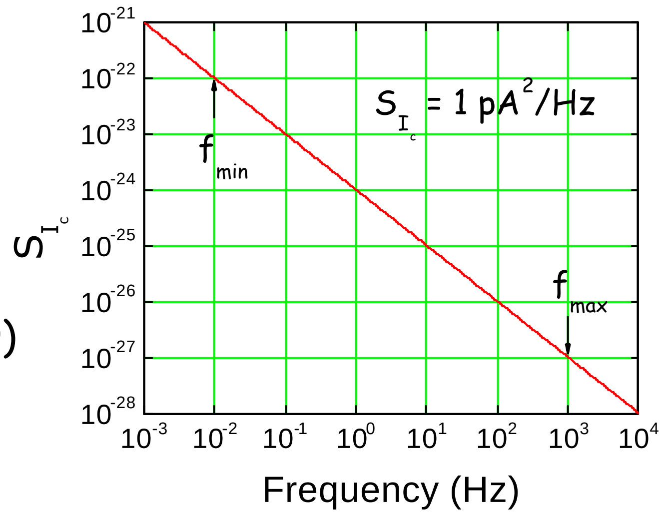
Determining the decoherence time from 1/f noise

- Assume a measurement of the coherent oscillations in a qubit





- Put in 1/f critical current noise with spectrum appropriate to the measurement scheme:



τ_{ss} = single shot flux sampling time (~ 1ms)

N_{ss} = number of flux samples at each time (> 1000)

N_p = number of time points to map oscillations (> 100)

$N = N_{ss} \times N_p$ = total number of measurements (> 10^5)

$N\,\tau_{ss}$ = total measurement time (> 100s)

- Determine decoherence time τ_ϕ limited by dephasing

I. Analytical calculation

(Martinis, Nam, Aumentado, Lang and Urbina - preprint)

Phase shift:

$$\delta\phi(t) = \int_0^t dt' \; \delta\Omega(t') = \int_0^t dt' \left(\frac{d\Omega}{dI_c}\right)\delta I_c(t')$$

Phase noise:

$$\langle\phi^2(t)\rangle \approx \left[\ln\left(\frac{0.4}{f_{min}\,t}\right)\left(\frac{\partial\Omega}{\partial I_c}\right)^2 S_{I_c}(1\text{Hz})\right] t^2 \approx \left(\frac{t}{\tau_\phi}\right)^2$$

Decay of oscillation amplitude:

$$\Phi_{env} \sim \exp\left[-\frac{1}{2}\left(\frac{t}{\tau_\phi}\right)^2\right]$$

Decoherence time:

$$\tau_\phi \sim \left(\frac{1}{\ln(0.4N)}\right)^{1/2}\left(\frac{I_c}{\Omega\Lambda}\right)\left(\frac{1}{S_{I_c}(1\text{Hz})}\right)^{1/2}$$

where $N = f_{min}\tau_{ss}$

For N = 10^5 →

$$\tau_\phi \approx 0.3\left(\frac{1}{\Omega\Lambda}\right)\underbrace{\left(\frac{I_c{}^2}{S_{I_c}(1\text{Hz})}\right)^{1/2}}_{\text{fractional change in } I_c}$$

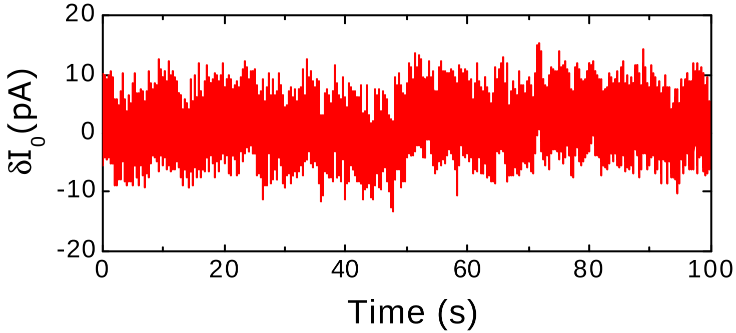
fractional change in I_c

II. Numerical simulation

Assume a 1/f critical current noise spectrum: $S_{I_0}(f) \sim \frac{1}{f}$

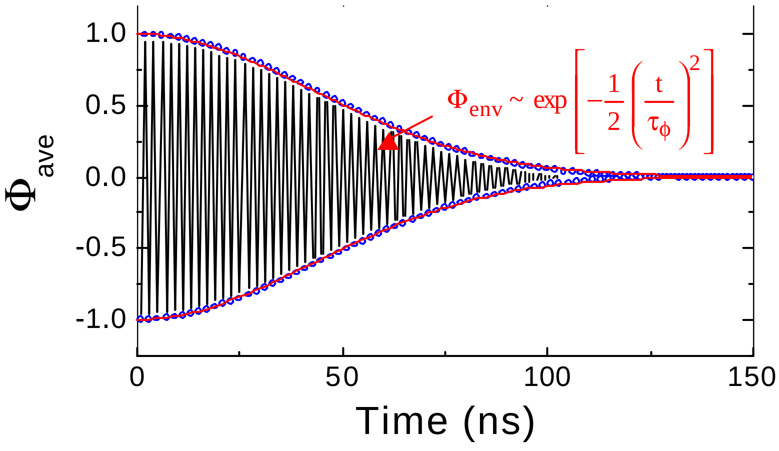
Generate critical current fluctuation distribution: $\delta I_0(t)$

Example: $S_{I_0}(f) = \frac{1\mathrm{pA}^2}{f}$



Calculate flux oscillations averaged over the ensemble of measurements:

$$\Phi_{ave}(t) = \frac{1}{N}\sum_{n=1}^{N} \cos\left[\left(\Omega_0 + \frac{d\Omega}{dI_0}\delta I_0(t + n\,tss)\right)t\right]$$



$I_c = 1\ \mu A$

$\Omega/2\pi = 10$ GHz

$\Lambda = 10$

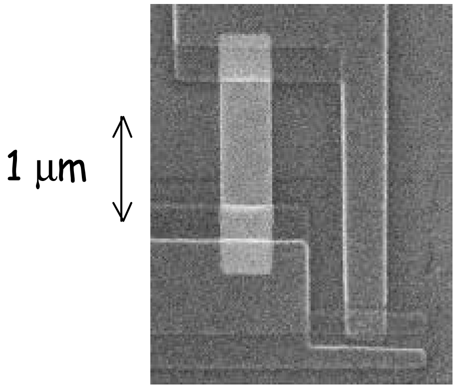
} $\tau_\phi \approx 450$ns

$$\tau_\phi \sim \left(\frac{1}{\Omega\Lambda}\right)\left(\frac{I_0^2}{S_{I_0}(1\mathrm{Hz})}\right)^{1/2}$$

Noise measurements in a finite voltage state

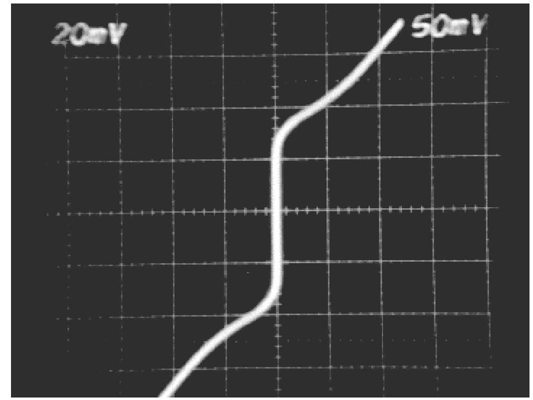
Shunted Al-AlO$_x$-Al Josephson tunnel junctions and dc SQUIDs

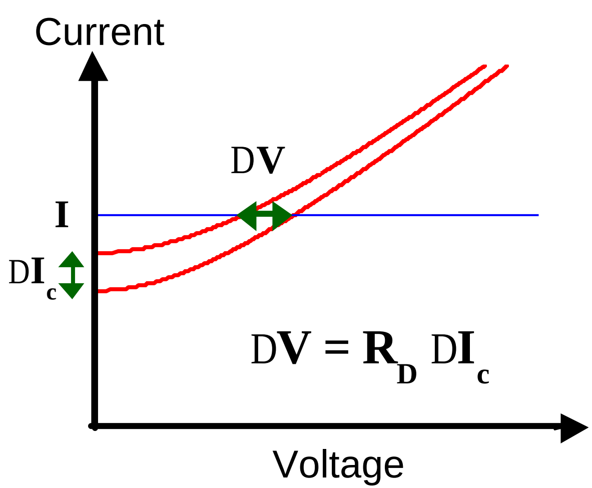
fabricated by electron-beam lithography and shadow-mask evaporation



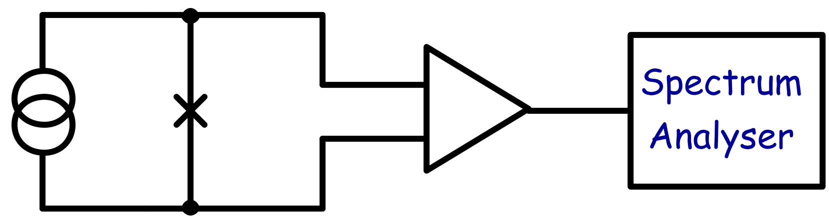
Typical parameters:

- $A = 200\text{nm} \times 400\text{nm}$
- $C = 4$ fF
- $I_c = 1 - 10\ \mu A$
- $R = 5 - 20\ \Omega$
- $R_D(\text{max}) = 40 - 200\ \Omega$

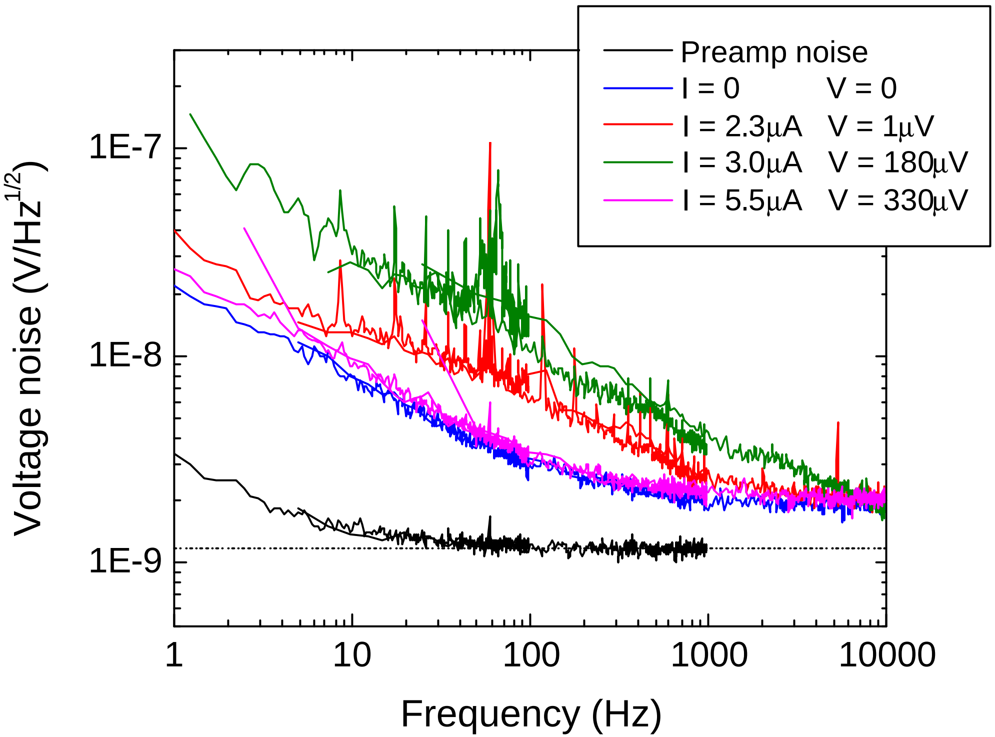


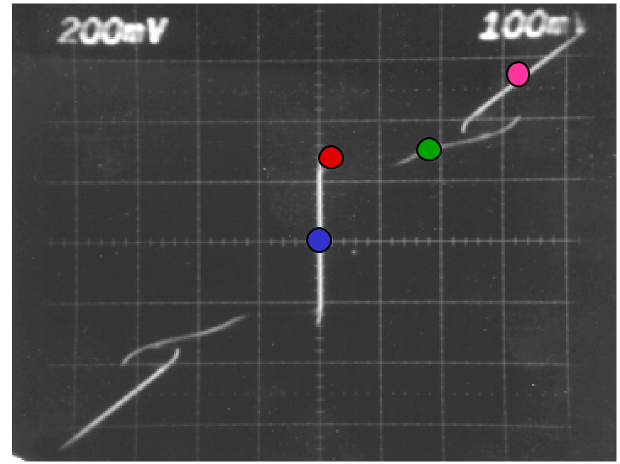


Current bias in finite voltage state →
determine changes in I_c from voltage noise

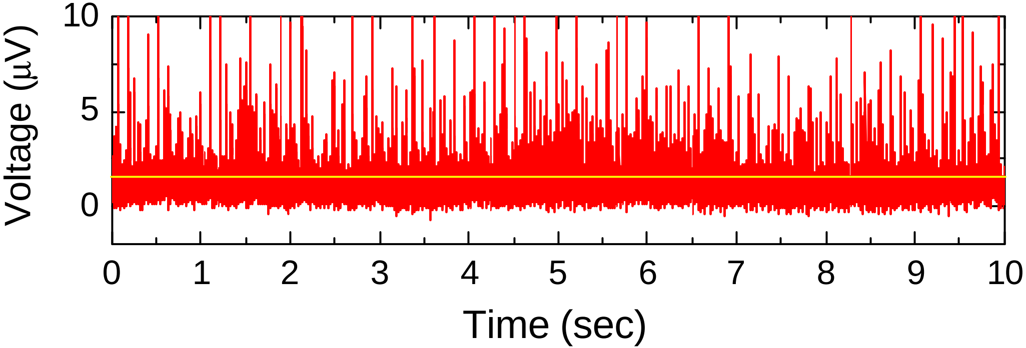


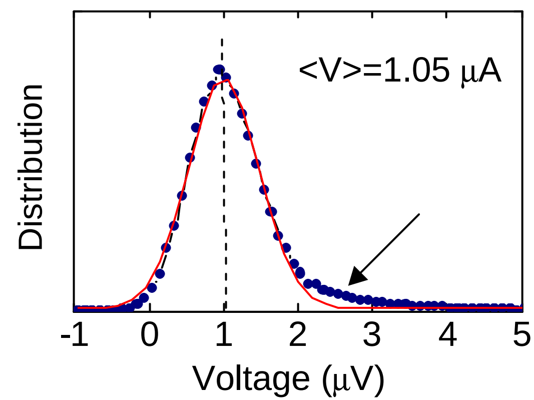
Noise measurements in single junctions



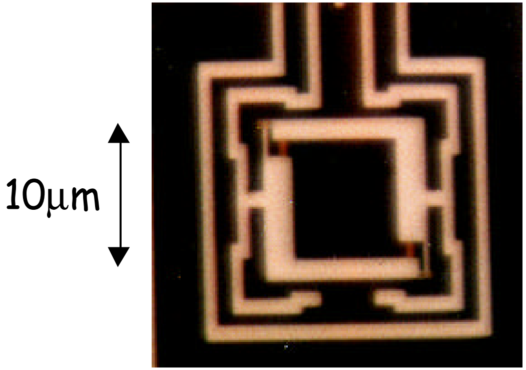


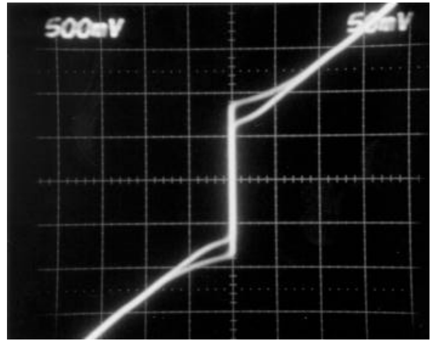
I_c= 2μA R=65Ω



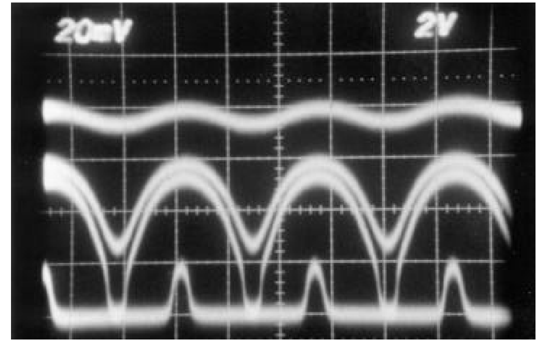


Noise measurements (dc SQUID)

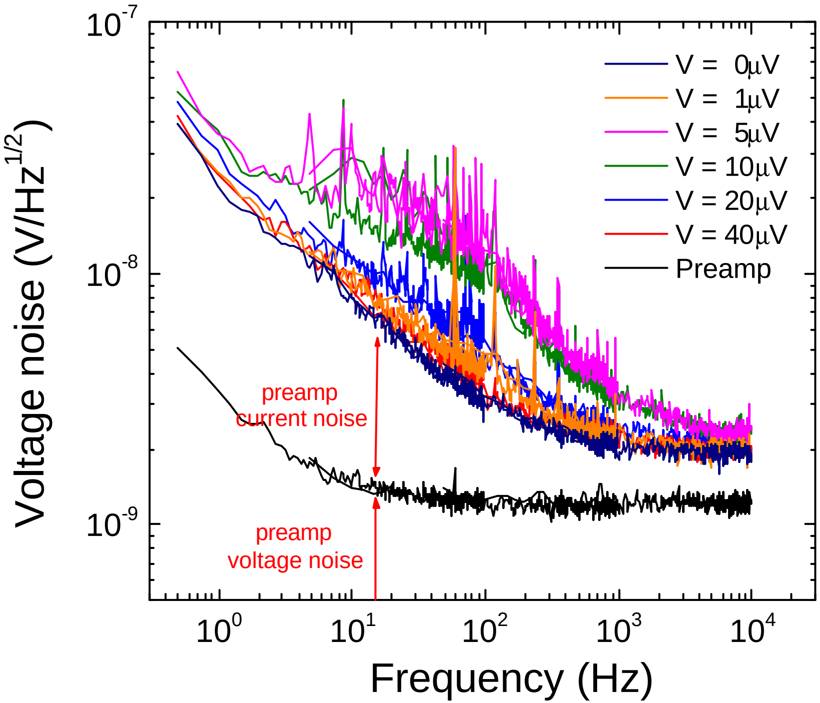




I vs. V

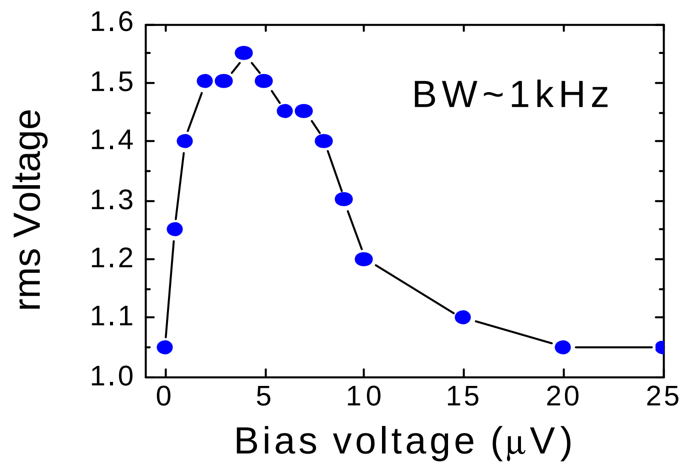


V vs. Φ

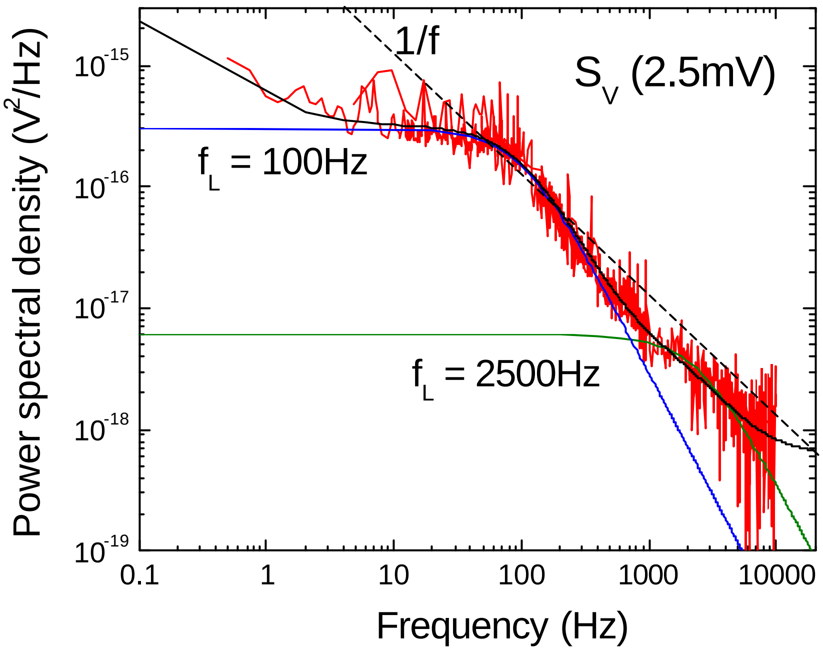


$dV/d\Phi(max) = 600\ \mu V/\Phi_0$

$S_\Phi^{1/2}(100\ Hz) = 3 \times 10^{-5}\ \Phi_0/Hz^{1/2}$



Extracting critical current fluctuations from noise spectra



Spectrum fits to 2 Lorentzians at 100 Hz and 2.5 KHz

$A = 8 \times 10^4\ nm^2$

$S_V = 2 \times 10^{-16}\ V^2/Hz$ $\quad$ $S_V^{1/2} = 1.4 \times 10^{-8}\ V/Hz^{1/2}$

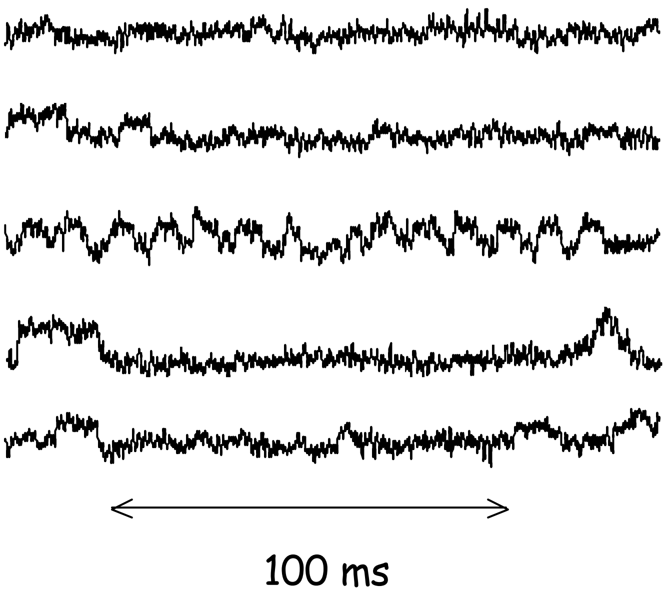
$R_D = 200\ \Omega \Rightarrow S_{Ic}^{1/2} = S_V^{1/2}/R_D = 7 \times 10^{-11}\ A/Hz^{1/2}$

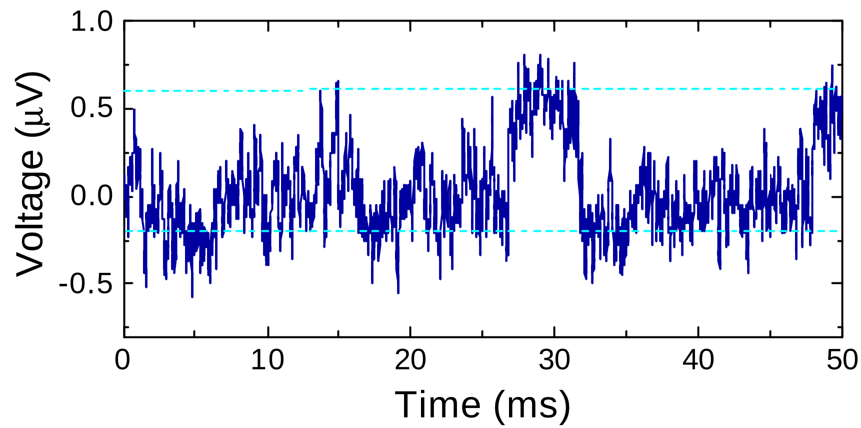
$BW = 100\ Hz \Rightarrow \Delta I_c = 0.7\ nA$ $\quad$ $I_c = 8\ \mu A \Rightarrow$ $(\delta I_c/I_c) = 8.8 \times 10^{-5}$

Assuming that $(\delta A/A) = (\delta I_c/I_c) \Rightarrow \delta A = 7\ nm^2 \Rightarrow$ $r = 1.5\ nm$

Extracting critical current fluctuations from switching noise

Time traces exhibit switching "random telegraph" noise





dc SQUID parameters: $L = 40$ pH $I_c = 8\ \mu A$ $A = 8 \times 10^4\ nm^2$

Bias parameters: $R_D = 40\ \Omega$ $dV/d\Phi = 600 \mu V/\Phi_0$

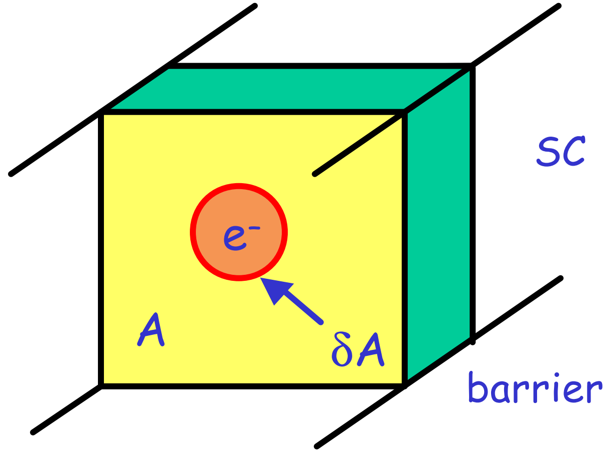
$\Delta V = 0.8\ \mu V$ $\Rightarrow$ $\Delta I_c = 15$ nA $\Rightarrow$ $(\delta I_c/I_c) = 1.9 \times 10^{-3}$

Assuming that $(\delta A/A) = (\delta I_c/I_c)$ $\Rightarrow$ $\delta A = 152\ nm^2$ $\Rightarrow$ $\delta s = 12.3$ nm

1/f noise in the critical current of Josephson junctions

Traps:

oxygen vacancy
dangling bond
structural defect
???



A trapped electron changes local barrier height over a radius of ~1 nm

Change in critical current: $\delta I_0 = (\delta A/A)\, I_0$

For one trap: $S^{(1)}_{I_0}(f) \propto (\delta I_0)^2$

For N independent traps: $S_{I_0}(f) \sim N(\delta I_0)^2 \sim (nA)\left[\left(\frac{\delta A}{A}\right) I_0\right]^2$

assuming a uniform areal trap density n

Thus: $S_{I_0}(f) \sim \frac{I_0{}^2}{A}$

For given junction technology, we expect $A^{1/2} S^{1/2}_{I_0}(f)/I_0$ = constant

Compilation of Josephson junction critical current 1/f noise results

$T = 4.2K \qquad f = 1\ Hz$

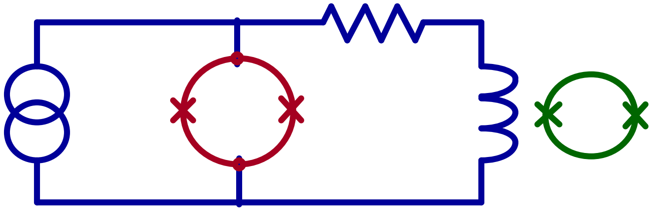
Materials	Area (μm^2)	I_0 (μA)	$S_{I_0}^{1/2}$(1 Hz) ($pA/Hz^{1/2}$)	$A^{1/2}S_{I_0}^{1/2}$(1 Hz)/I_0 [$\mu m(pA/Hz^{1/2})/\mu A$]
Nb-AlOx-Nb [a]	9	9.6	36	11
	8	2.6	6	7
	115	48	35	8
	34	12	41	20
Nb-Ox-PbIn [b]	4	21	74	7
	4	4.6	46	20
	4	5.5	25	9
	4	5.7	34	12
	4	11.4	91	16
Nb-NbOx-PbInAu [c]	1.8	30	184	8
PbIn-Ox-Pb [d]	6	510	6	15
Average				12

a Savo, Wellstood, Clarke [26] b Wellstood[27] c Foglietti *et al*.[28] d Koch, Van Harlingen, Clarke[29]

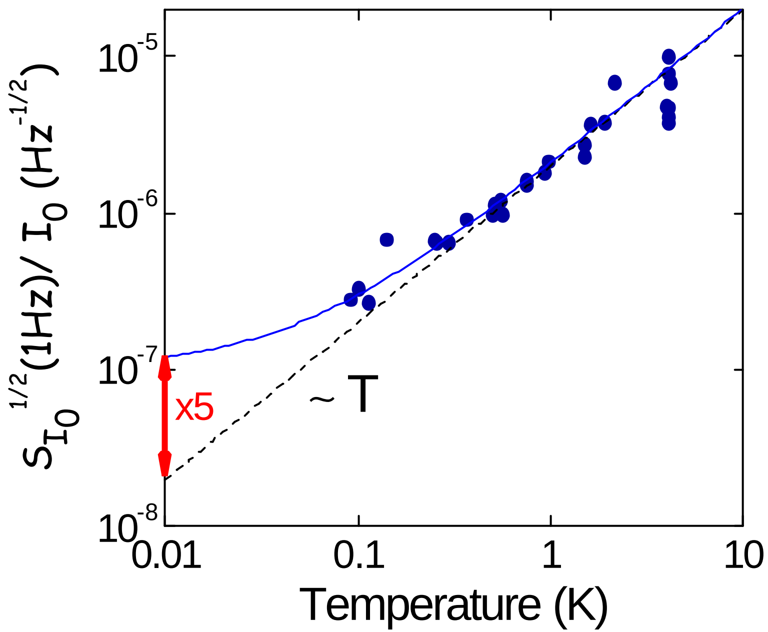
"Universal" value of 1/f noise: $S_{I_0}(1Hz, 4.2K) \approx 144 \frac{(I_0/\mu A)^2}{(A/\mu m^2)} \frac{pA^2}{Hz}$

Temperature dependence of 1/f critical current noise

Only known study: Fred Wellstood, Ph.D. thesis



- SQUIDs biased at low voltage ($eV \ll \Delta$) and at flux $\Phi = 0$ ($dV/d\Phi=0$)
- Measured current fluctuations with a SQUID magnetometer
- Found T^2 dependence down to ~ 0.3K (possible flattening at lower T?)



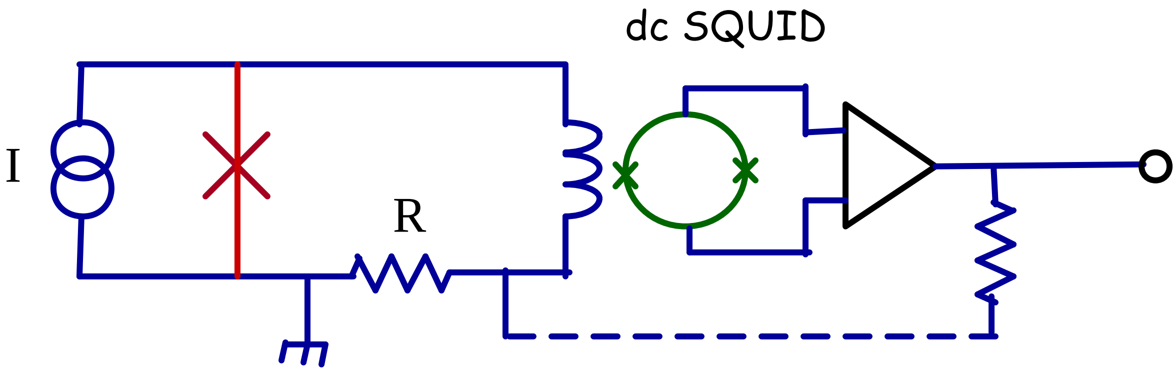
? No known mechanism for T^2 variation for charge traps in the tunneling regime (can be explained by thermal activation in an anisotropic potential)

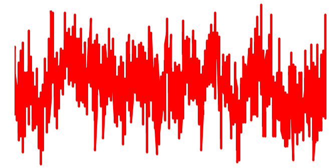
? Source of charge for trapping is unknown --- should be frozen out in SC state

Temperature-dependent 1/f noise (optimistic):

$$S_{I_0}(f,T) \approx \left[144 \frac{(I_0/\mu A)^2}{(A/\mu m^2)} \left(\frac{T}{4.2K} \right)^2 pA^2 \right] \frac{1}{f}$$

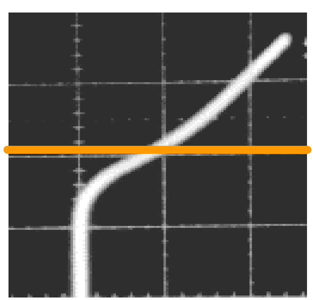
Plan for 1/f noise measurements





Bias junction with current I:

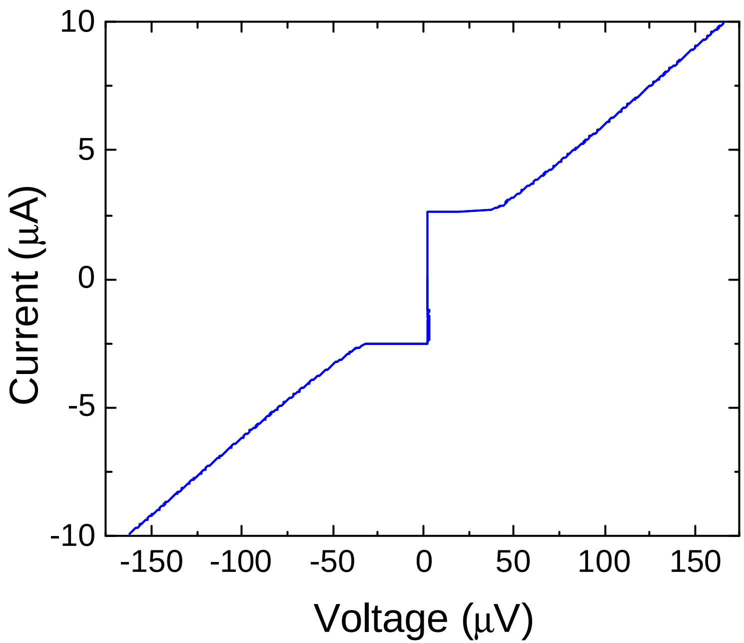
- with feedback, $I_J = I =$ constant → measure IV curve
- without feedback, $V \approx RI \approx$ constant → measure current fluctuations $\delta I \sim \delta I_c$

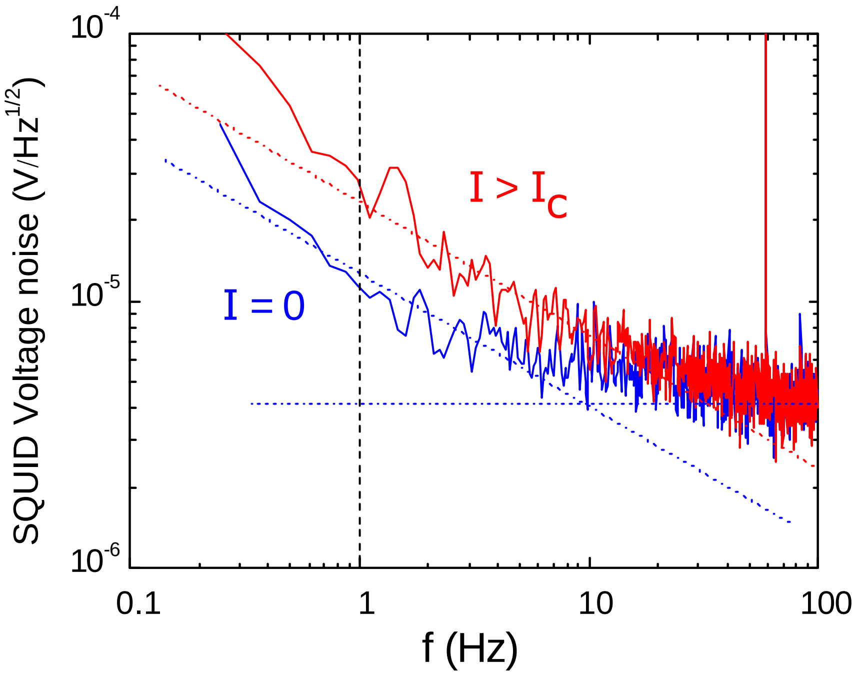


sensitivity set by SQUID noise: $S_{I_c}{}^{1/2} \sim 10 fA/Hz^{1/2}$

Measurements on Nb-Al-AlOx-Nb Josephson tunnel junctions (10μm x 10μm)

(fabricated by John Martinis --- NIST Boulder)





Parameters: $I_c = 2.5\mu A$
$A = 100\mu m^2$
$T = 90mK$

From universal 1/f model:
$S_{I0}(1Hz) = 60\ fA/Hz^{1/2}$

$S_{VSQUID}(1Hz) = 4.5 \times 10^{-5}\ V/Hz^{1/2}$ → $S_{I0}(1Hz) = 180\ fA/Hz^{1/2}$

[larger by x 3]

General expression: decoherence from 1/f critical current noise

1/f critical current noise:

$$S_{I_c}(f,T) \approx \left[144 \frac{(I_c/\mu A)^2}{(A/\mu m^2)} \left(\frac{T}{4.2K} \right)^2 pA^2 \right] \frac{1}{f}$$

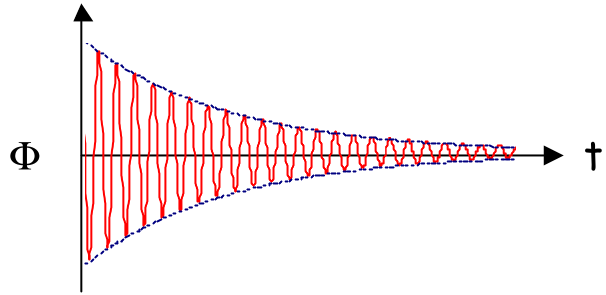
decoherence time:

$$\tau_\phi \approx 0.3 \left(\frac{1}{\Omega\Lambda} \right) \left(\frac{I_c{}^2}{S_{I_c}(1Hz)} \right)^{1/2}$$

$$\tau_\phi(\mu s) \approx 17 \frac{\sqrt{A(\mu m^2)}}{\Lambda\, f_{osc}(GHz)\, T(K)}$$

$$N_{osc} = \frac{\Omega \tau_\phi}{2\pi} \approx 17{,}000 \frac{\sqrt{A(\mu m^2)}}{\Lambda\, T(K)}$$

Number of oscillations before decoherence



Qubit parameters and predicted performance (T=100mK)

Parameters	rf SQUID	3-junction SQUID	single JJ	Quantronium
I_0 (μA)	1.46	0.57	20.0	0.036
L (pH)	240	11	---	---
b_L	1.06	0.019	---	---
A (μm^2)	2	0.05	100	0.11
C (fF)	103	2.6	5000	5.4
$W/2p$ (GHz)	2.52	1.14	7.06	36.2
L	40.6	14.0	19.3	1.0
t_f (μs) calc	2.4	2.5	12	1
t_f (μs) meas	---	0.200	0.050	0.500
$W\,t_f/2p$ calc	6300	2800	82,000	56,000
$W\,t_f/2p$ meas	---	240	340	29,000

Conclusion: Predicted decoherence times from 1/f noise are <u>longer</u> than what has been obtained by measurements (<< 1μs) → may not be the limiting mechanism but may become a problem in the future

Conclusions/Plans

. 1/f noise is a serious problem for superconducting flux qubits, and perhaps all solid state qubits --- may limit coherence times

- Characterization of 1/f critical current noise in qubit junctions
  vs. size and technology (Nb trilayer, Al shadow, …)
  temperature dependence (T < 100mK)
  voltage dependence ($V < 2\Delta/e$)

- Materials approaches to reduce noise: (with Jim Eckstein, UIUC)
  superconducting electrode material (Nb vs. Al vs. Pb vs...)
  tunneling barrier morphology (amorphous vs. epitaxial vs. defect doped),

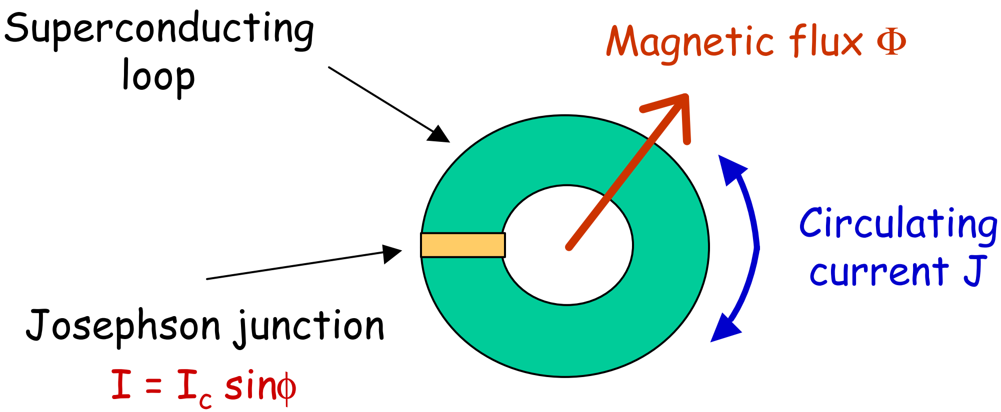
- Flux noise calculations: effects of 1/f flux noise on decoherence → introduces chial asymmetry/breaks degeneracy (with Tony Leggett, UIUC)

- Novel junction designs: SFS π-junctions --- decoherence concerns due to low resistance and 1/f magnetic domain noise (with Valery Ryazanov, ISSP)

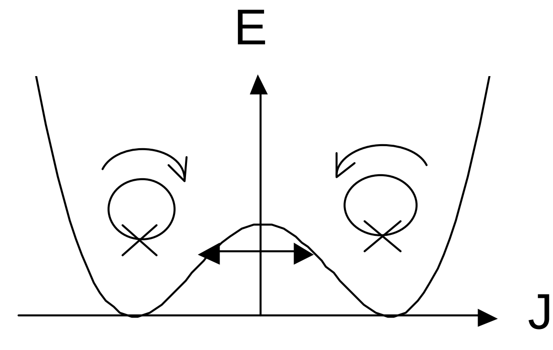
- Measurement schemes: "spin-echo" pulse sequences to reduce effects of low frequency noise (cancels 1/f noise below pulse interval frequency)

p-Josphson junctions for Quantum Computing

Basic Qubit = rf SQUID



Qubit states correspond to clockwise and counterclockwise currents

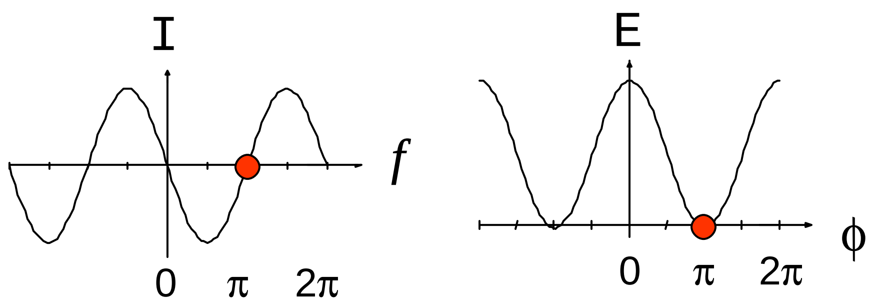


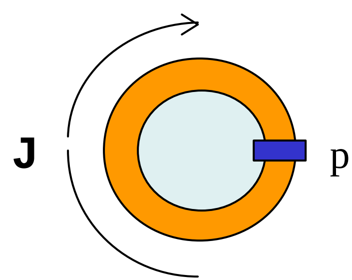
$\Phi = 1/2\ \Phi_0$

<u>Our approach</u>: utilize π-Josephson junctions in superconducting flux qubit

p-junction

Negative I_c → minimum energy at π

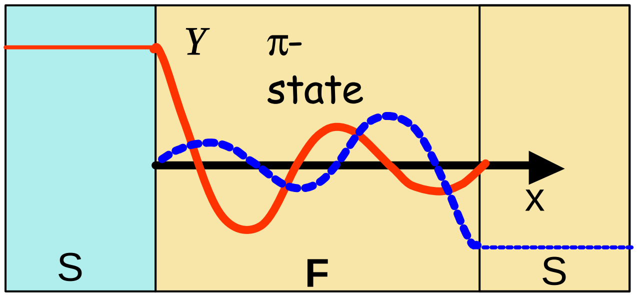


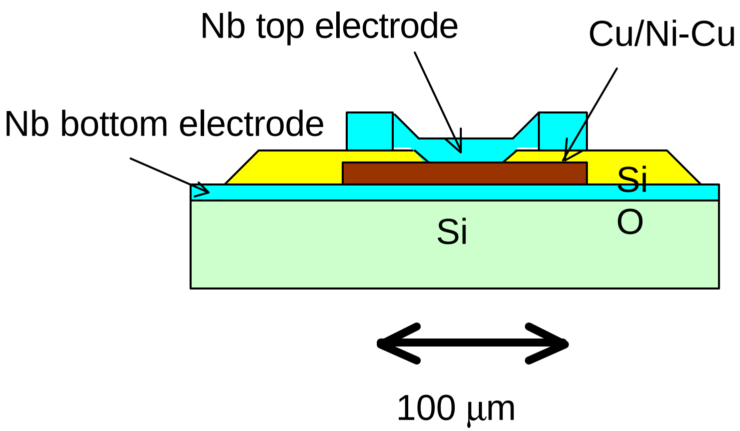


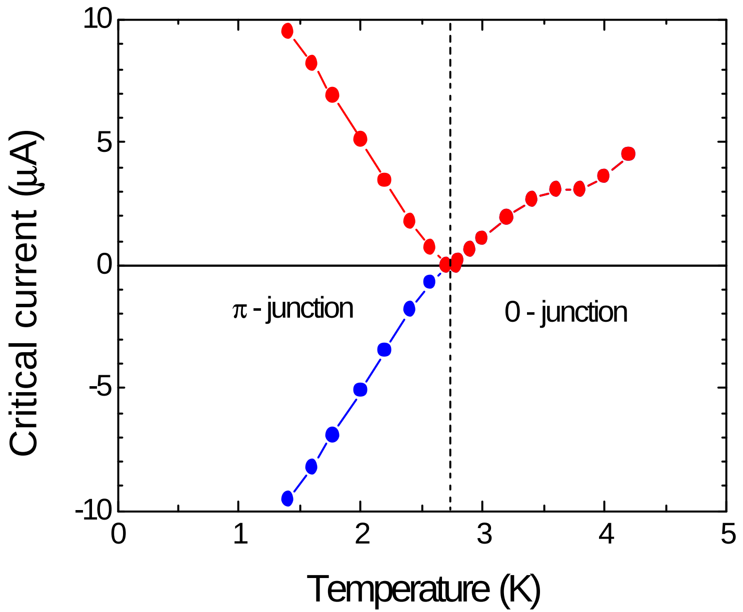
Spontaneous circulating current in rf SQUID

SFS Josephson junctions

<u>Principle</u>: FM Exchange field produces oscillations of the superconducting order parameter. For certain thickness of the FM-layer, the order parameter is of the opposite sign on two sides of the junctions, i.e. it is shifted by π.





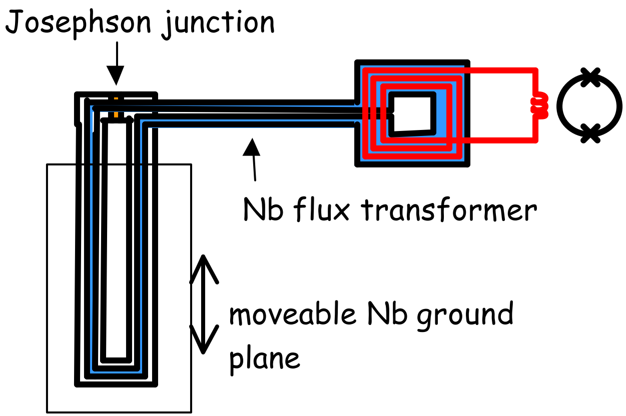


The critical current of SFS Josephson π-junctions changes sign as a function of temperature [Ryazanov et al.]

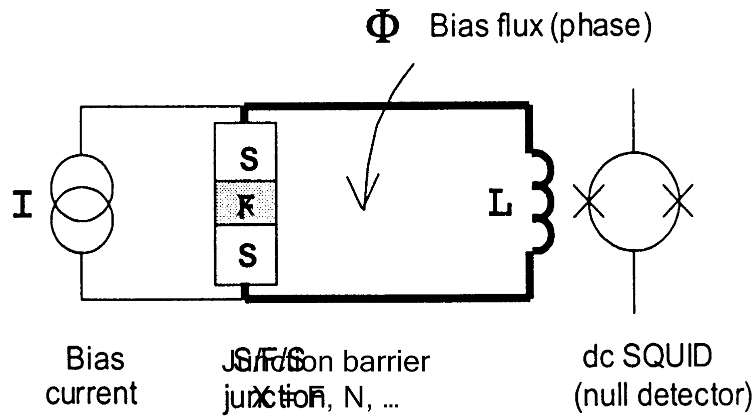
Ongoing research projects/plans

1. Verify π-junction behavior via phase-sensitive tests

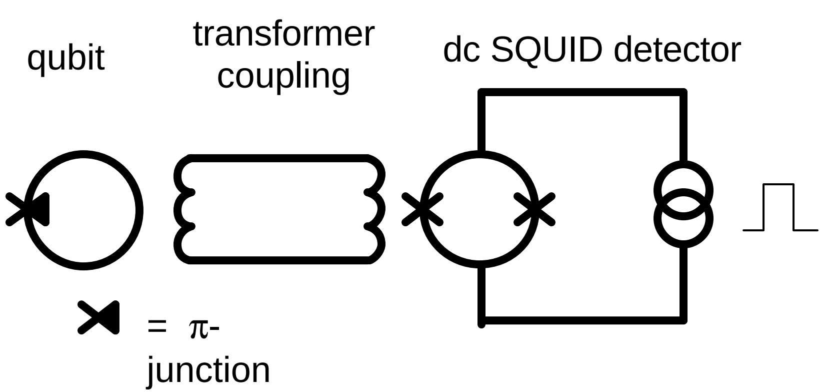
Trombone experiment: measure spontaneous flux for phase shift of π

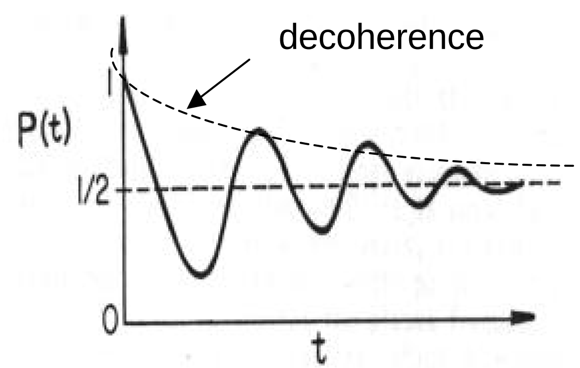


Current phase-relation experiment: map out I(ϕ) by SQUID interferometry

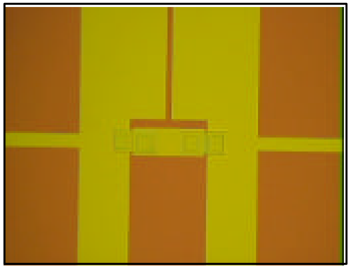


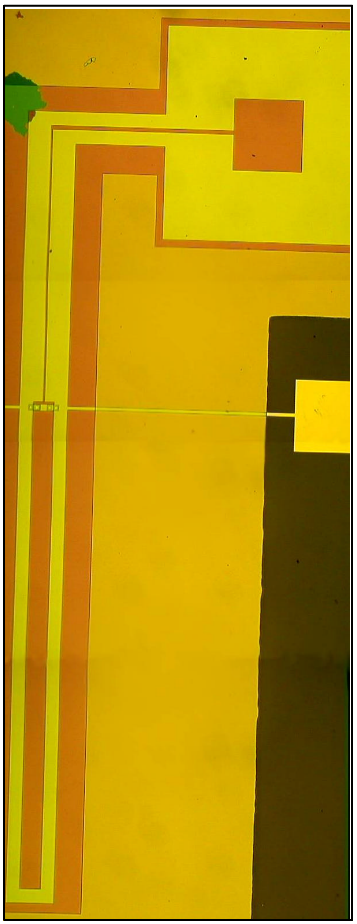
2. Observe coherent quantum oscillation in a flux qubit incorporating π-junctions.





Trombone Current Injection Experiments: determination of b_L





$$b_L = \frac{2pI_C L}{\Phi_0}$$