Satellite Workshop
"Strongly correlated electrons in nanoscale devices"
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Cotunneling of Cooper pairs in the Bloch transistor with dissipative electromagnetic environment

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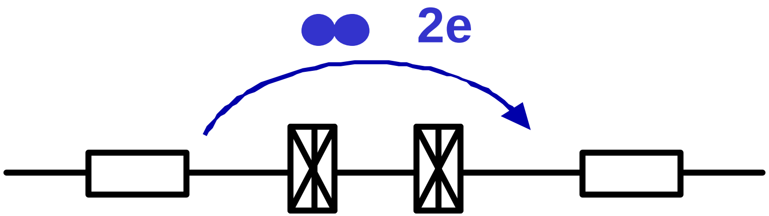


Outline

1. Introduction:

   Cooper pair (CP) tunneling in a single junction

   Sequential CP tunneling in a transistor

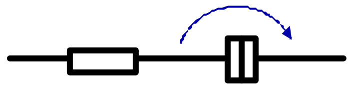
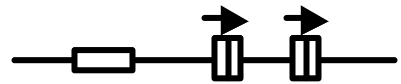
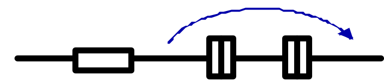
2. Theory of CP cotunneling

3. Experiment

4. Possible application of the effect of suppression of the CP cotunneling

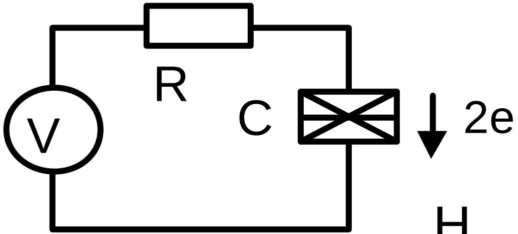
5. Conclusion

Single charge transport in circuits with finite environmental e/m impedance

	Single junction 	**Double junction:** *Sequential tunneling* 	**Double junction:** *Cotunneling* 
Electrons/ quasiparticles e	**Theory:** 1. Averin and Likharev (1986) 2. Nazarov/Devoret (1990) 3. Ingold and Nazarov (1991) **Experiment:** 1. Chalmers (90-91) 2. Delft TU (90-91)	**Theory:** Ingold and Nazarov (1991) **Experiment:** Chalmers (1995)	**Theory:** Odintsov, Bubanja and Schön (1992) **Experiment:** PTB (2000)
Cooper pairs 2e	**Theory:** 1. Averin, Nazarov and Odintsov (1990) 2. Ingold and Nazarov (1991) **Experiment:** 1. Kuzmin et al. (1991)	**Theory:** 1. Ingold and Nazarov (1991) 2. Wilhelm, Schön and Zimányi (2001) **Experiment:** 1. Kycia et al. (2002) 2. Lu et al. (2002)	**?**

Cooper pair tunneling rate and I-V of single junction at R ¹ 0

$E_c = e^2/2C$; $E_J = (\Phi_0/2\pi)I_c^0$



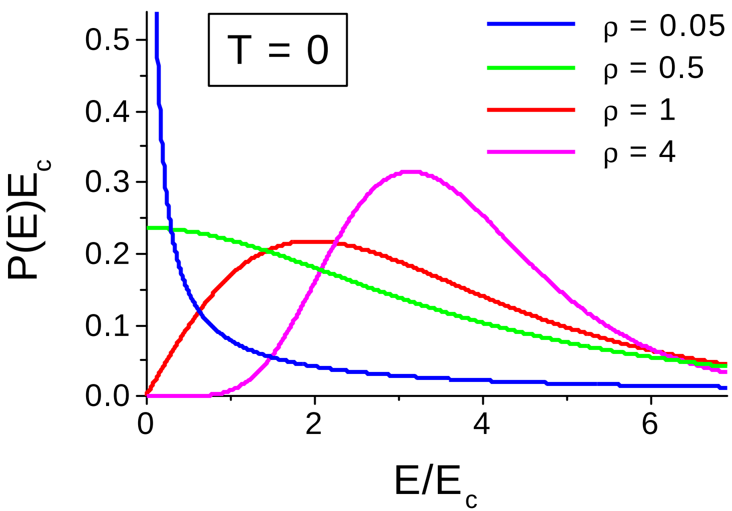
Regime of incoherent tunneling of pairs

For $(E_J/E_c)(R_Q/R)^{1/2} \ll 1$ the rate:

$$H_{Jos} = -\frac{E_J}{2}\left(e^{if} + e^{-if}\right)$$

$$G_{\pm} = \frac{p}{2h} E_J^2 P(\pm 2eV)$$

Function P(E) describes the environment



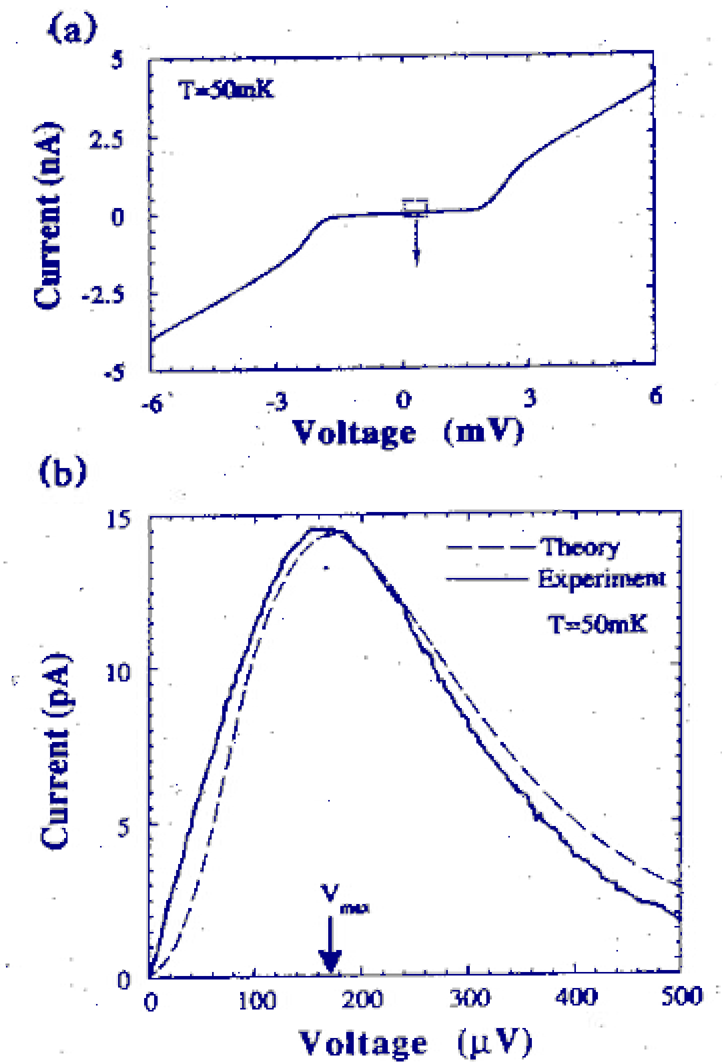
Characteristic parameter:

$$r = \frac{R}{R_Q}$$

$R_Q = h/4e^2 \approx 6.45\ k\Omega$

Averin et al. (1990); Ingold and Nazarov (1992); Falci et al. (1991).

Experiment with a single junction



L.S. Kuzmin et al., PRL 67, 1161 (1991).

I-V curve:

$$I = 2e\,[\Gamma(V) - \Gamma(-V)]$$

$$G = \frac{\pi}{2\hbar} E_J^2 P(2eV)$$

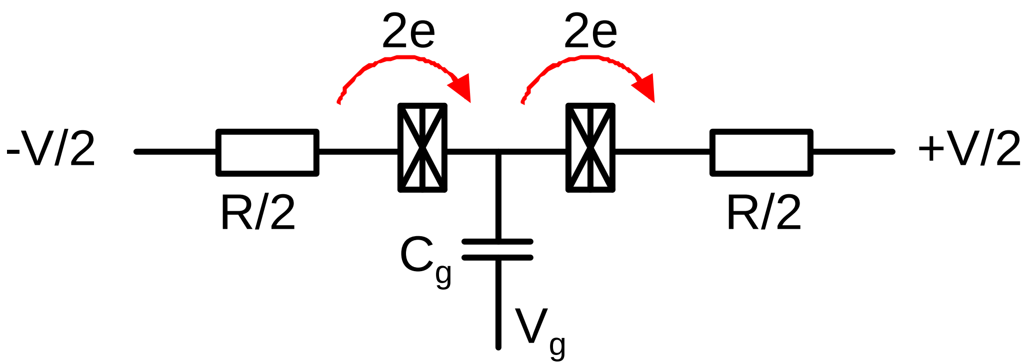
$E_J/E_c \sim 0.01$

$R = 22k\Omega$

2-junction system...

Sequential tunneling of pairs in the superconducting (Bloch) transistor

at $t=t_1,t_3,...$ at $t=t_2,t_4,...$



$t_1 < t_2 < t_3 < ...$

At $C_gV_g = 0$ (mod 2e) - blocked, $\Gamma \propto \exp(-4E_c/k_BT)$

At $\color{red}{\mathbf{C_gV_g = e}}$ (mod 2e) - open

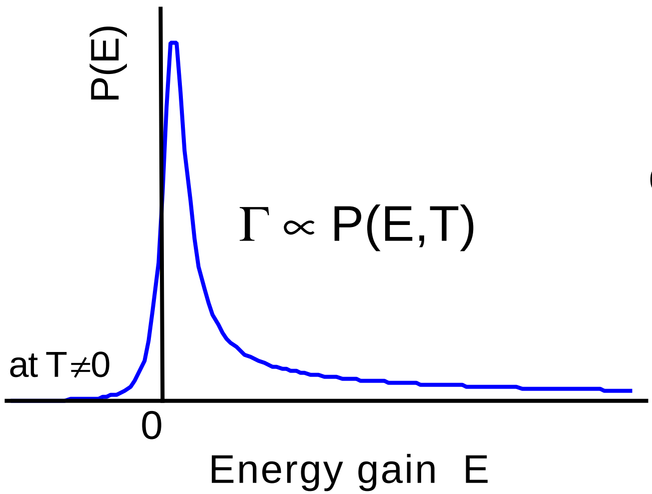
<u>Theory</u>: Wilhelm, Schön and Zimányi (2001)
<u>Exp</u>.: Kycia et al. (2002), Lu et al. (2002)

Generally, operation is similar to SE
but

Two comments:

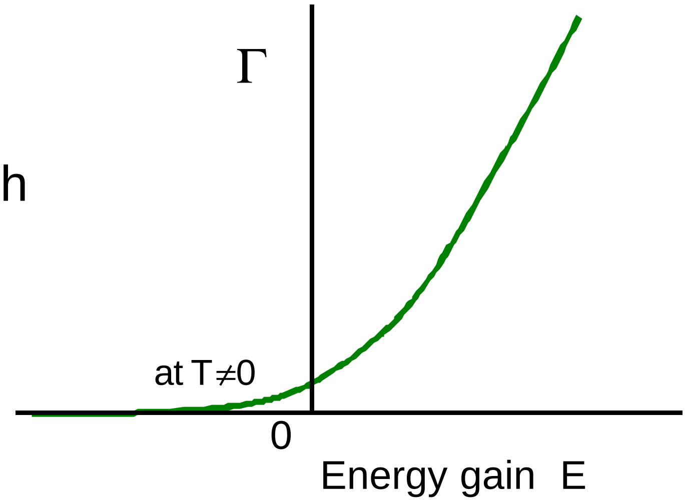
(1)

Bloch transistor (2e)

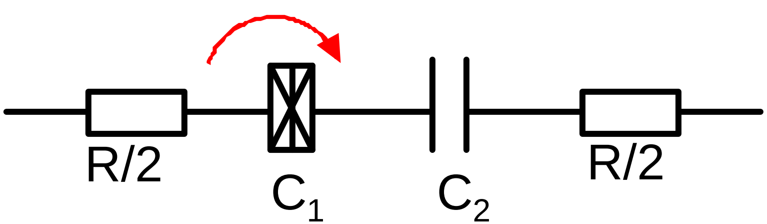


compare with

Normal SET (e)



(2) Effect of the environment is weaker!

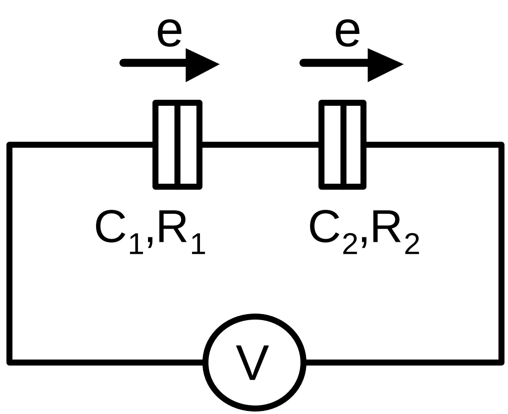


$C_{ser} = C_1C_2/(C_1+C_2)$

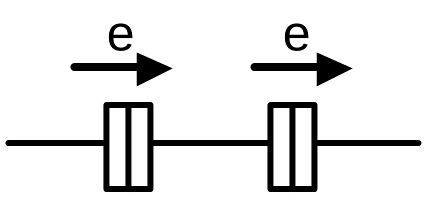
Effective $\rho_{eff} = \kappa^2\rho$,

where $\kappa^2 = (C_{ser}/C_{2,1})^2 =$ **1/4**

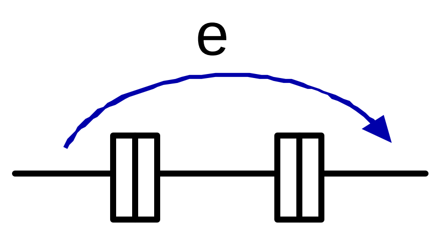
In normal SET the regime of simultaneous tunneling of two electrons across the junctions is also possible!



cotunneling of electrons

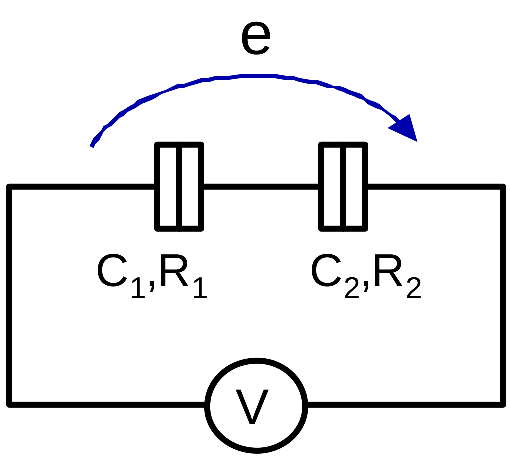


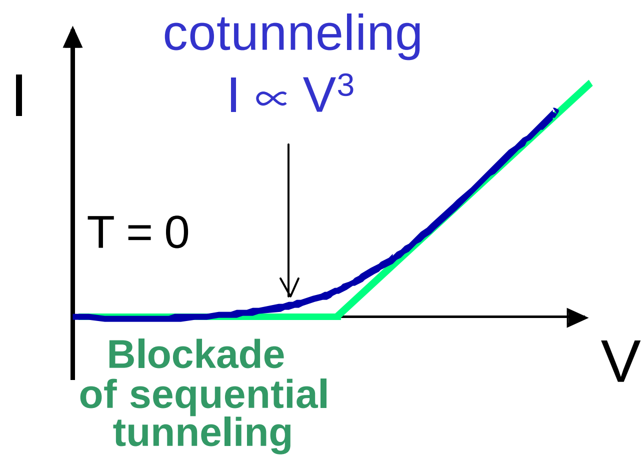
eqv.



Single electron cotunneling

(Averin and Odintsov 1989; Averin and Nazarov 1990; for R≠0 Odintsov et al. 1992)





The electron cotunneling in normal metallic SET devices is usually the **inelastic** process, because it creates the "e-h" excitations in the island.

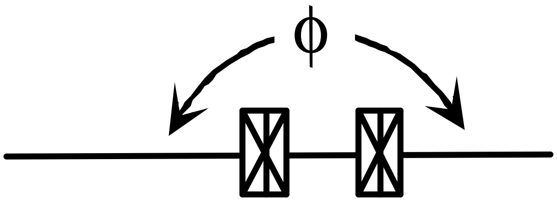
$$I = \frac{\hbar}{12\pi e^2 R_1 R_2}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)^2\left[(eV)^2+(2\pi k_B T)^2\right]V\ ,$$

$E_{1,2}$ - changes of energy.

Is a Cooper pair cotunneling possible and what shall we understand under the term “cotunneling of pairs”?

(1)

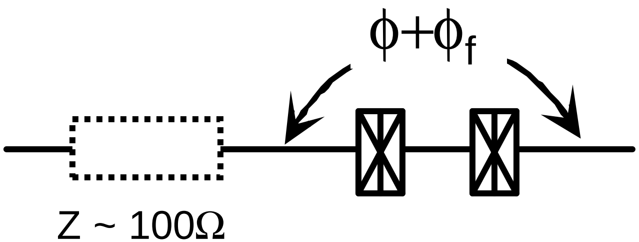
R = 0
(theo)



The through supercurrent $I_s(\phi)$, V=0.

(2)

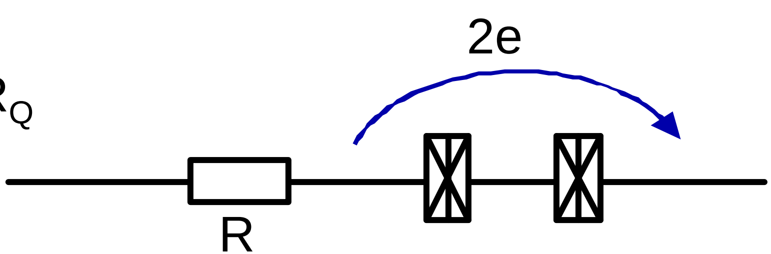
R = 0
(exp)



Phase diffusion, $\langle I_s(\phi)\rangle \neq 0$, V- small.

(3)

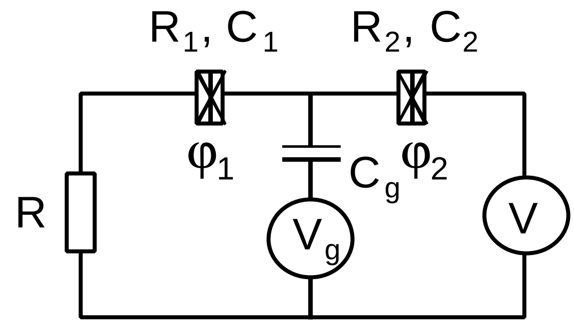
$R \sim R_Q$



Phase fluctuations are large, so the language of tunneling of discrete charges is now adequate, $I = 2e\Gamma$.

our case

Theory



Assumptions:

$C_g << C_{1,2}$; $C_1 \sim C_2 = C$

$E_{J1,J2} = (\Phi_0/2\pi) I_{c1,c2} \leq E_c = e^2/2(C_1+C_2+C_g)$

$eV, k_BT << 2E_c, 2\Delta_{Al}$

(contribution of qps to the net current is negligibly small!)

$$H = \underbrace{H_{Jos} + H_{charge}}_{H_{trans}} + H_{env} + H_{int},$$

$$H_{Jos} = -E_{J1}\cos\varphi_1 - E_{J2}\cos\varphi_2 = -E_J^{trans}(\phi)\cos[\varphi + \gamma(\phi)]$$

where

$$E_J^{trans} = (E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\phi)^{1/2}, \quad \tan\gamma = \frac{E_{J1} - E_{J2}}{E_{J1} + E_{J2}}\tan\frac{\phi}{2},$$

$\phi = \varphi_1 + \varphi_2$; $\varphi = (\varphi_1 - \varphi_2)/2$; $[\varphi, Q] = 2ei$, Q is the island charge.

When $E_{J1} \approx E_{J2}$ and $C_g \ll C_{1,2}$ the variables φ and ϕ are decoupled!

1. Quantum mechanics with the island variable φ

   [theory Likharev and Zorin, 1985].

$$H_{trans} = E_0[l\,(f\,), Q_0], \quad l\,(f\,) = E_J^{trans}(f\,)/E_c \; ? \; 2l_0 \; ? \; 2(E_{J1} + E_{J2})/E_c$$

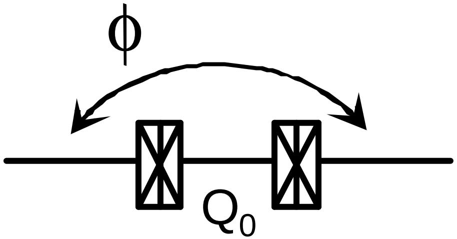
$Q_0 = C_g V_g$ is the gate charge

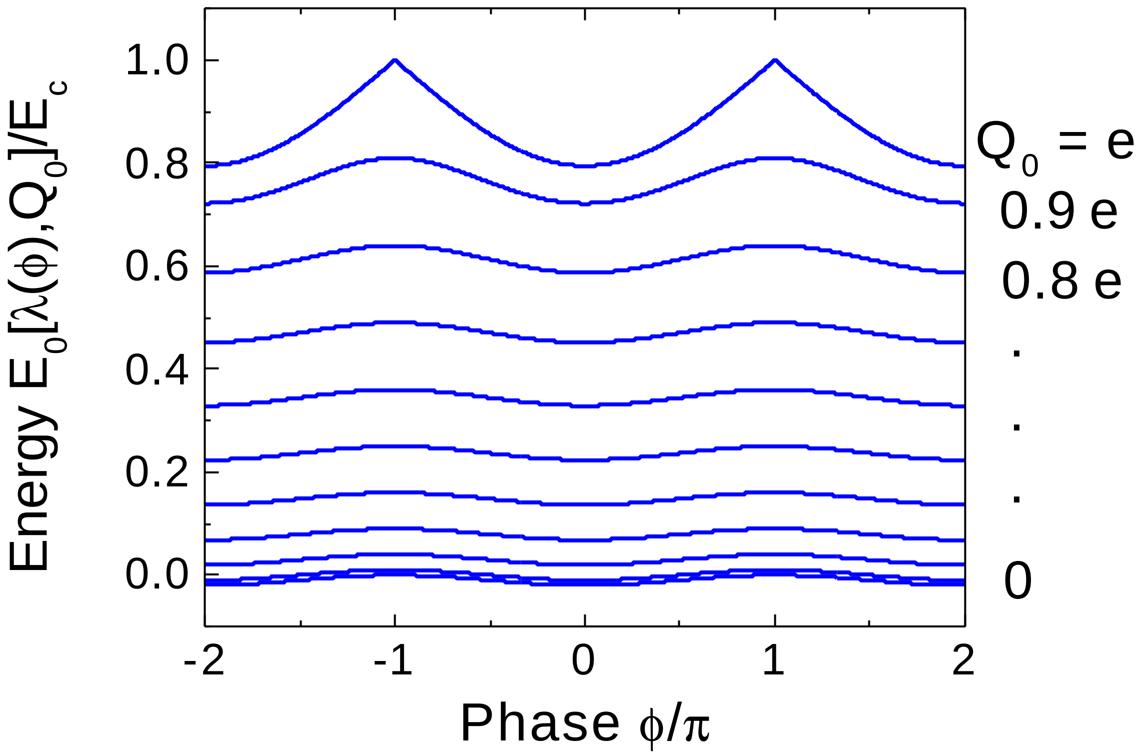
2. Perturbation theory for the outer variable ϕ.

$$G = \frac{p}{2h} E_0^2 \, P(E)$$

Ground state energy $E_0(\phi,Q_0)$

Bloch transistor (Q_0 fixed):

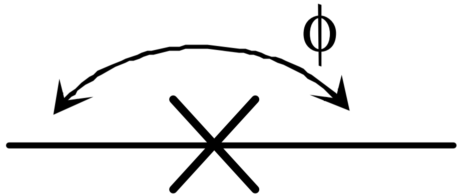




I_s ...

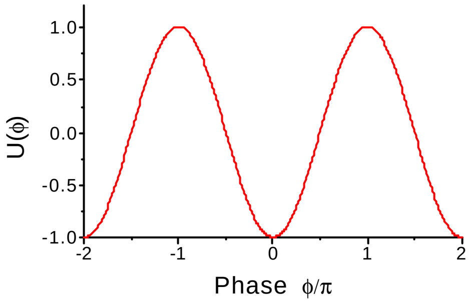
For comparison:

Josephson coupling energy



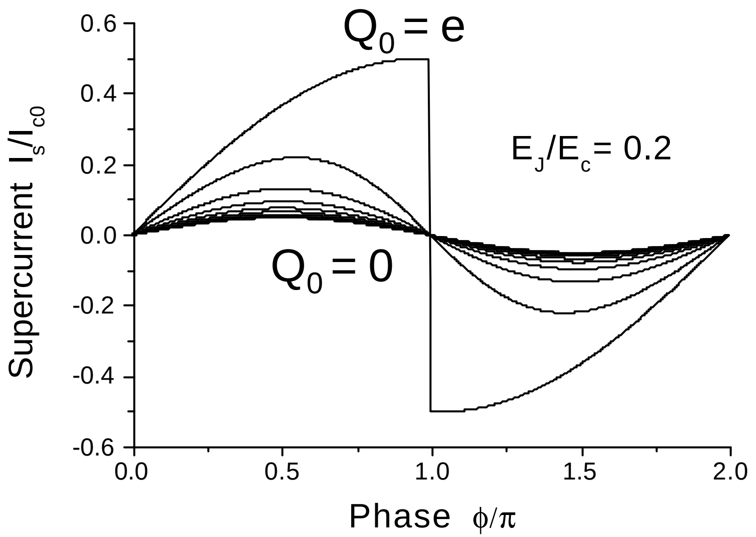
Single junction

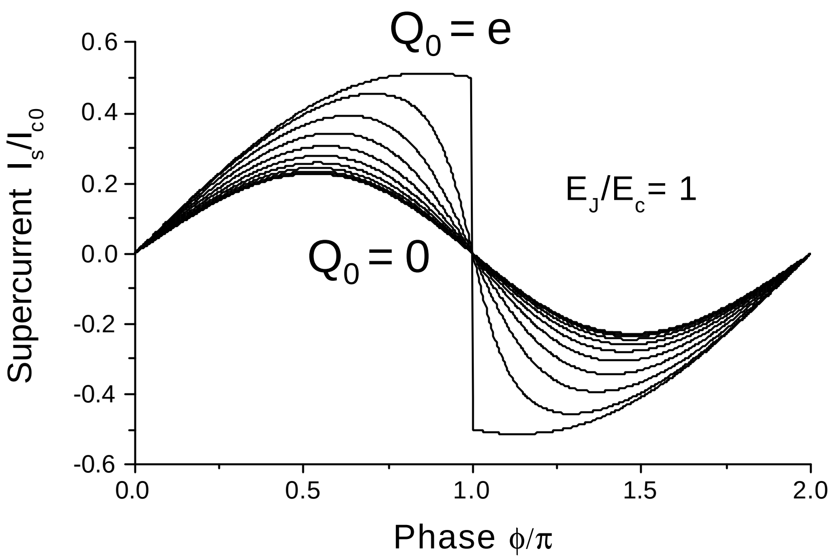
$U = - E_J \cos\phi$



Yields $I_s \propto \sin\phi$

Supercurrent in the Bloch transistor versus total phase difference





[A.Z. 1997]

Compare with the dependence $I_s = I_c \sin\phi$ in a single tunnel junction.

Fourier series:

$$H_{trans}(\varphi, Q_0) = -\sum_{k=0}^{\infty} E_J^{(k)}(Q_0)\cos(k\varphi), \qquad I_c^{trans}(Q_0) = \max_{\varphi}\left\{\frac{\partial H_{trans}}{\partial \varphi}\right\}$$

$$E_J^{(k)}(Q_0) = \begin{cases} \approx E_J^{(1)}\delta_{1,k}, & \text{when } Q_0 \approx e \bmod(2e), \\ \left(\dfrac{4}{\pi}\right)\dfrac{(-1)^{k-1}}{(2k)^2-1}E_J, & \text{when } E_{J1} = E_{J2} \equiv E_J. \end{cases}$$

Operators **$\exp(\pm ik\varphi)$** describe tunneling of charge **2ek** across the Bloch transistor in either direction

The net cotunneling current:

$$I_{cot} = 2e\sum_{k=1}^{\infty} k\left[\Gamma_k(2keV) - \Gamma_k(-2keV)\right]$$

For the case of ohmic resistor
(similar to the result for single junction, Ingold et al. 1994):

$$I_{cot}(Q_0,V) = \Phi_0 \underbrace{\frac{\left(\pi I_c^{trans}(Q_0)\right)^2}{4E_c}}_{\text{Gate dependence}} e^{-2\gamma\rho}\rho^{2\rho}\left(\frac{2E_c}{\pi^2 k_B T}\right)^{1-2\rho}\underbrace{\frac{\left|\Gamma\left(\rho - i\frac{eV}{\pi k_B T}\right)\right|^2}{\Gamma(2\rho)}\sinh\left(\frac{eV}{k_B T}\right.}_{\text{Transportvoltage dependence}}$$

This formula is valid if $I_{cot} \ll \rho I_c^{trans}$

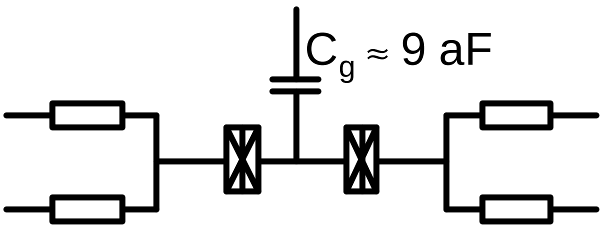
The gate dependence is explicitly given by:

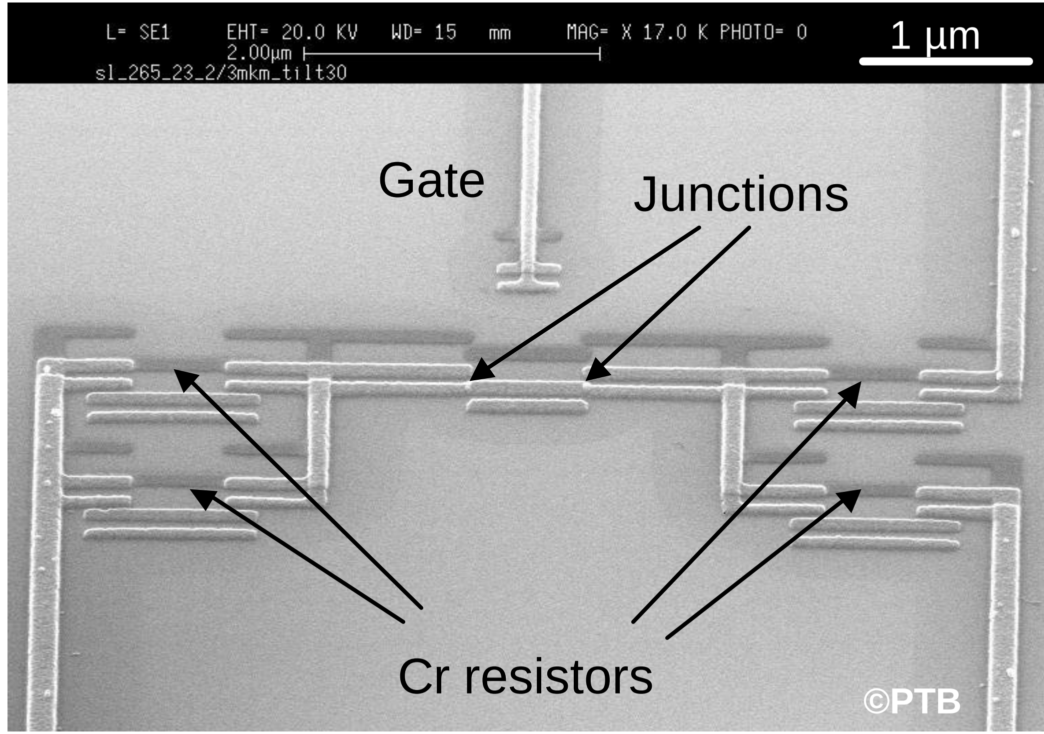
$$I_c^{trans}(Q_0) = \max_{\varphi}\left(\frac{\partial H_{trans}}{\partial \varphi}\right) = \frac{\lambda_0}{4}\left(1-\left(\frac{Q_0}{e}\right)^2\right)^{-1} I_c^0, \quad E_J \ll E_c;\; Q_0 \ll e.$$

experiment

Layout of the plain Bloch transistor samples

Method: 3-angle evaporation
(Cr, Al <+ oxidation> , Al)





<u>Typical parameters</u>

Junctions: Al/AlO$_x$/Al; $C_\Sigma \approx 500$ aF, $E_c = e^2/2C_\Sigma \approx 150$ µeV; $\Delta_{Al} \approx 200$ µeV;

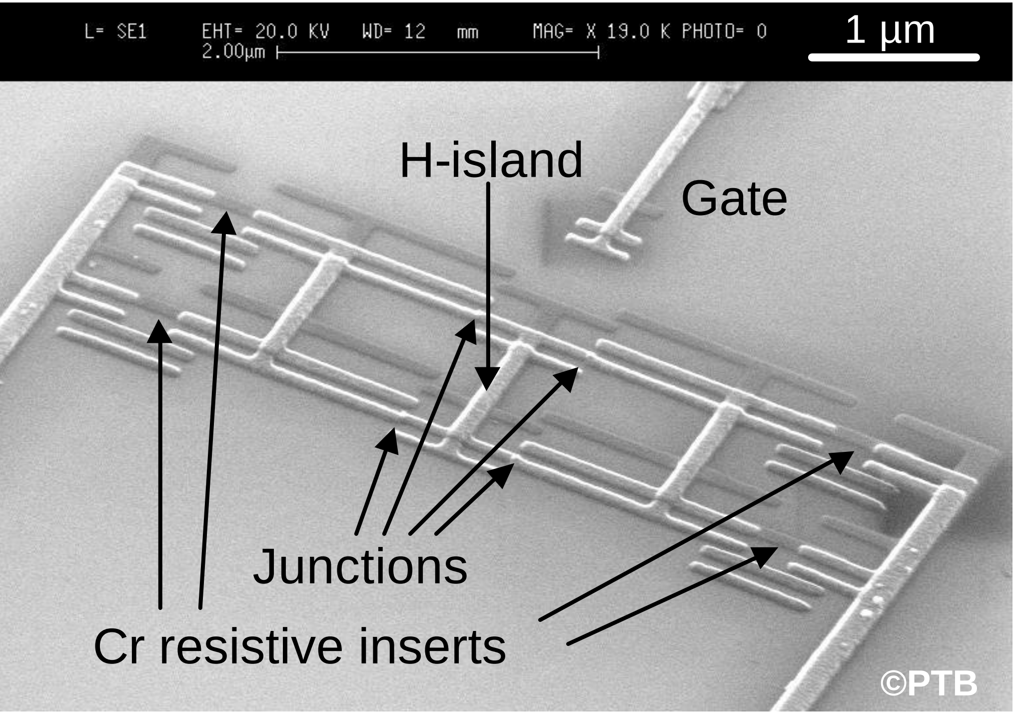
$E_{J1,J2} = E_J = (\Phi_0/2\pi)I_c^0 \approx 30$ µeV; $I_c^0 \approx 16$ nA;

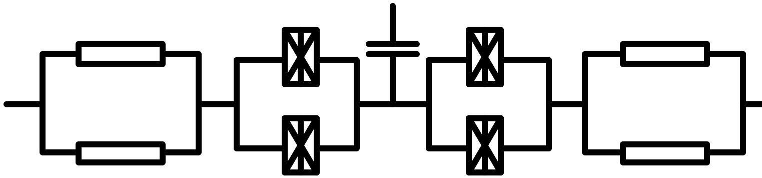
E_J/E_c » 0.2

Resistors: **R = 2 - 20 kW**; w = 100 nm, h = 7 nm, l = 0.3–3 µm; material - Cr;

$R_{\bullet}$ = 0.55 –0.7 kΩ

Samples of SQUID configuration

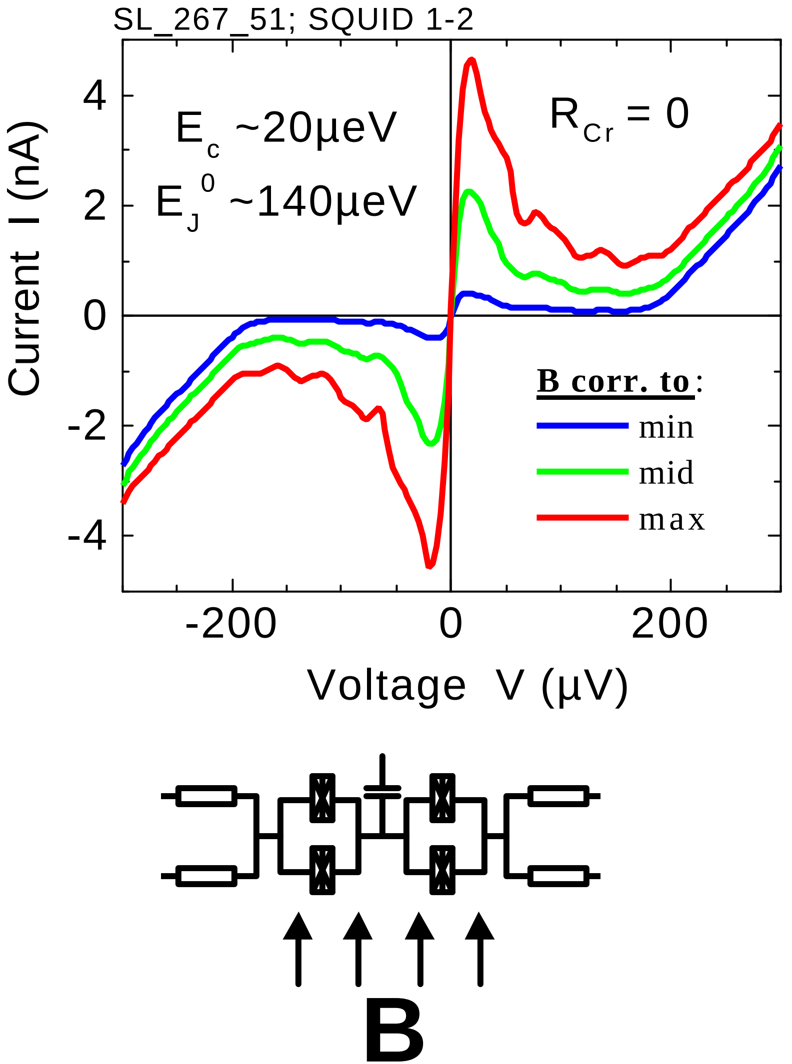




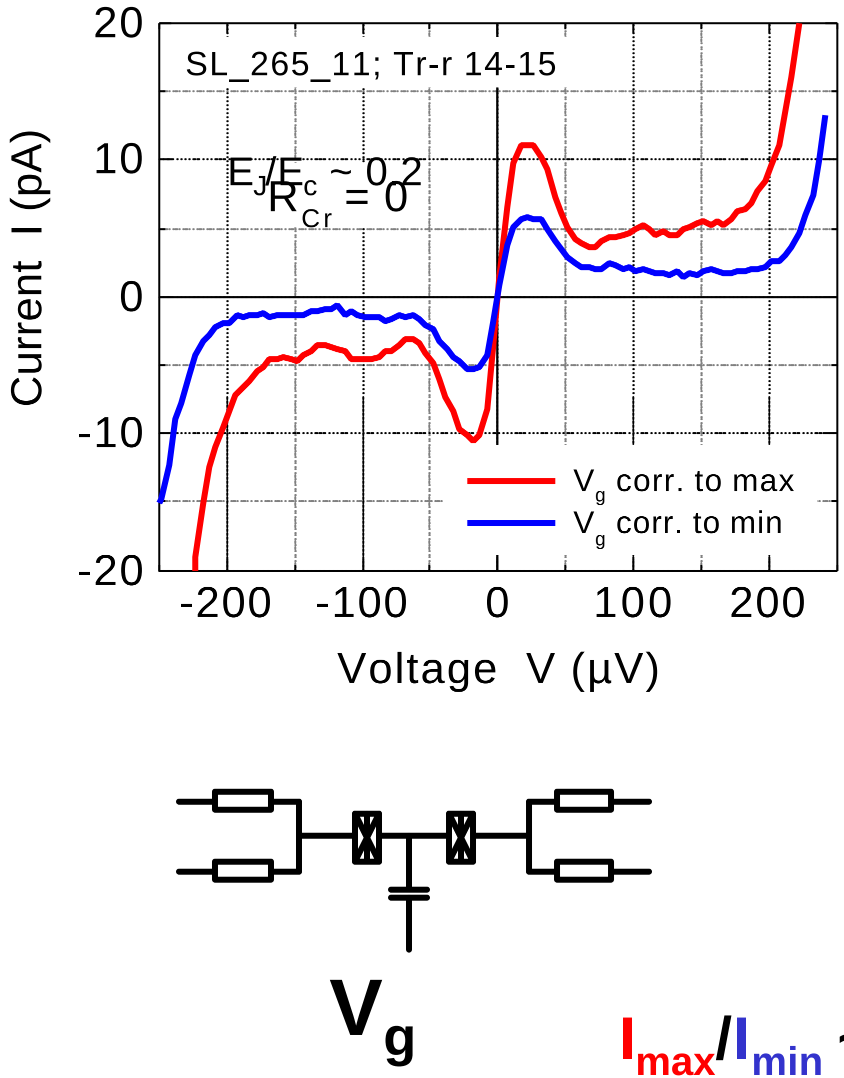
The E_J/E_c ratio can be varied by applying a perpendicular magnetic field

(Possible) disadvantage: Cr-film shadow always falls on the Al island making it “not clean”.

Effect of weak magnetic field



Effect of gate



I_{max}/I_{min} ~ 2

Range of the gate charge variation

Although all samples had $E_c \sim 150\ \mu eV < \Delta_{Al} = 200\ \mu eV$, the I-V curves exhibited only 1e-periodic dependence on the gate charge = $C_g V_g$:

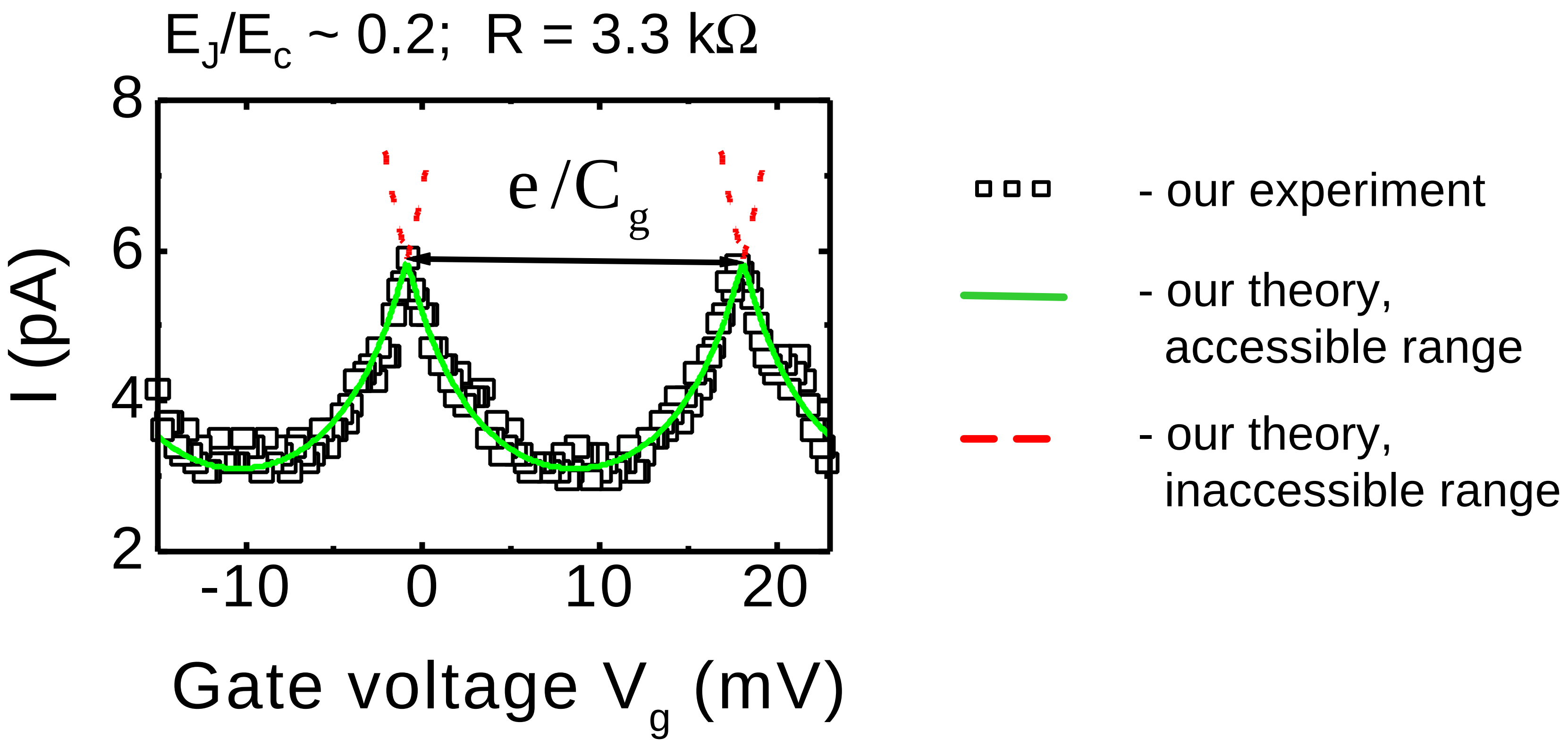
quasiparticle poisoning

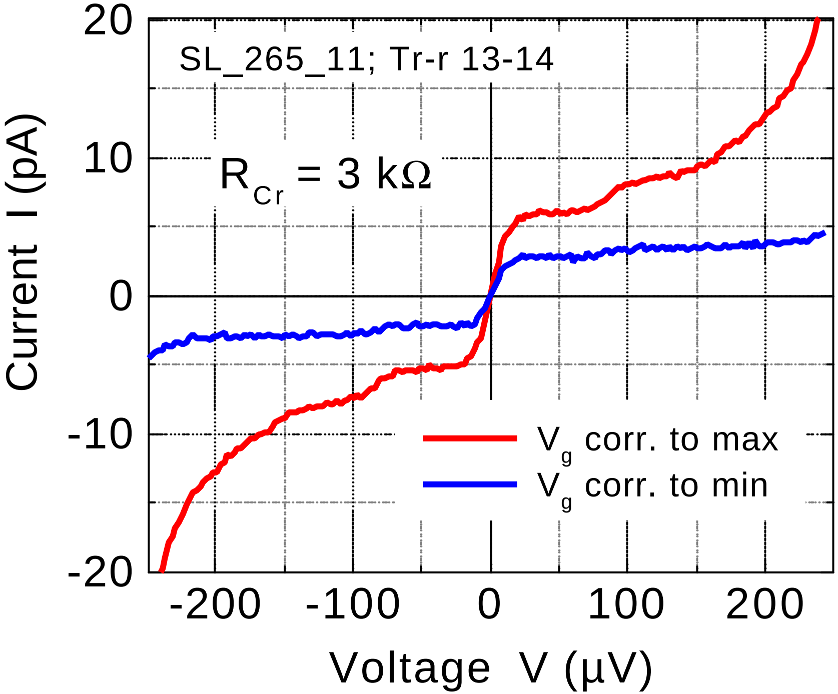
Possible reason: E_c was not MUCH less than Δ_{Al} and possible contamination of the island.

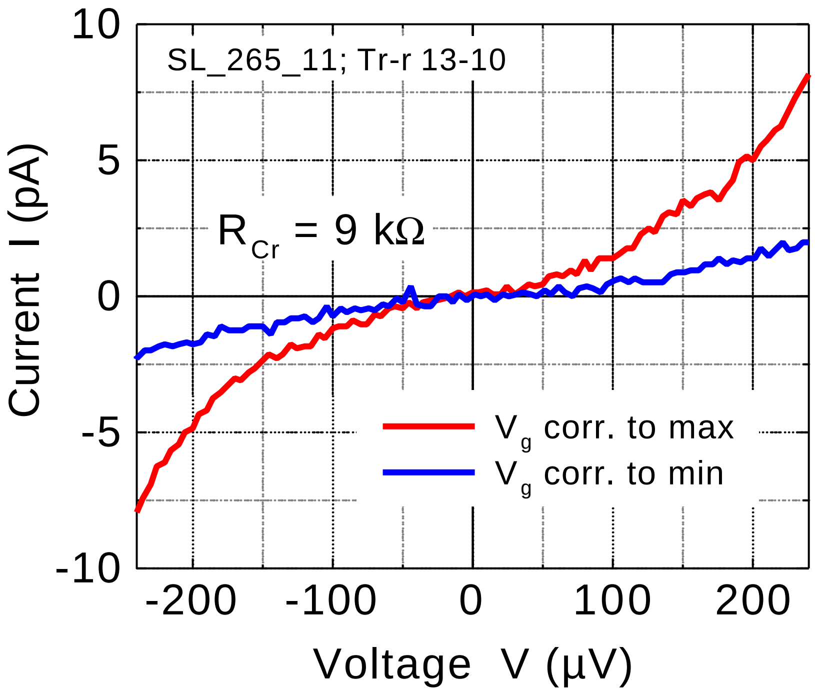
$$- e/2 \ ?\ Q_0 \ ?\ e/2 \quad \text{and} \quad \frac{I_{max}}{I_{min}} = \frac{(I_{cot})_{max}}{(I_{cot})_{min}} = \left[\frac{I_c^{trans}(\pm e/2)}{I_c^{trans}(0)} \right]^2 = \frac{16}{9} \text{ » } 2.$$

Sequential tunneling is exponentially suppressed: at $Q_0 = \pm e/2$, $I_{seq} \propto \exp(-2E_c/k_B T)$.

Gate dependence of cotunneling current



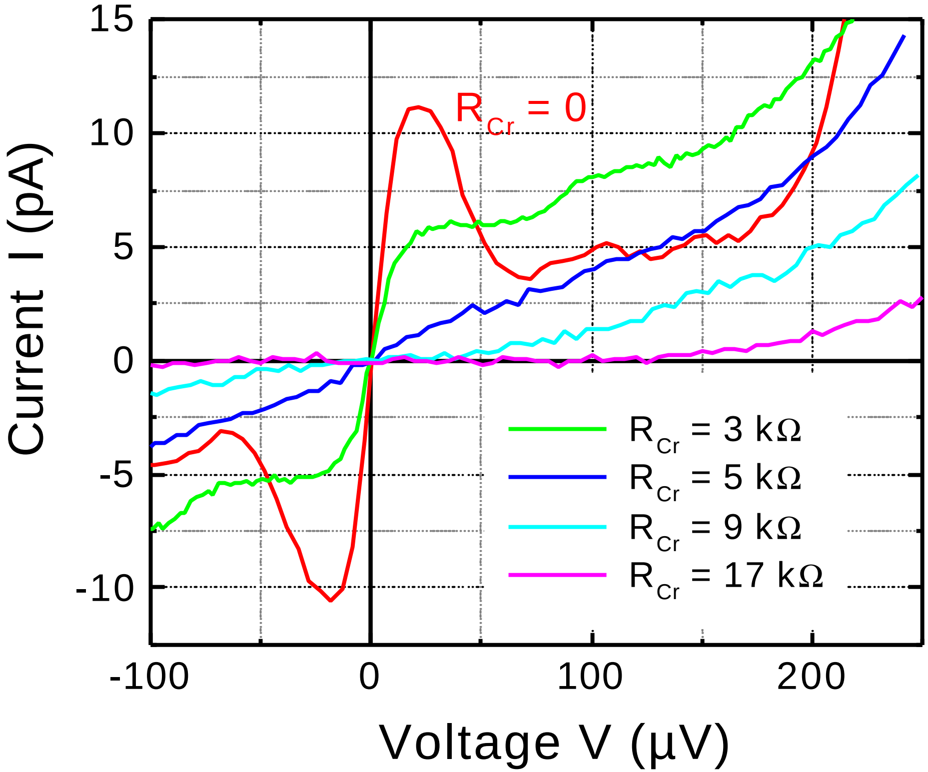




Suppression of current at large R...

Suppression of CP current at larger values of R

Gate voltage values corresponds to maximum of current (presumably, $C_g V_g = Q_0 = \pm e/2$)

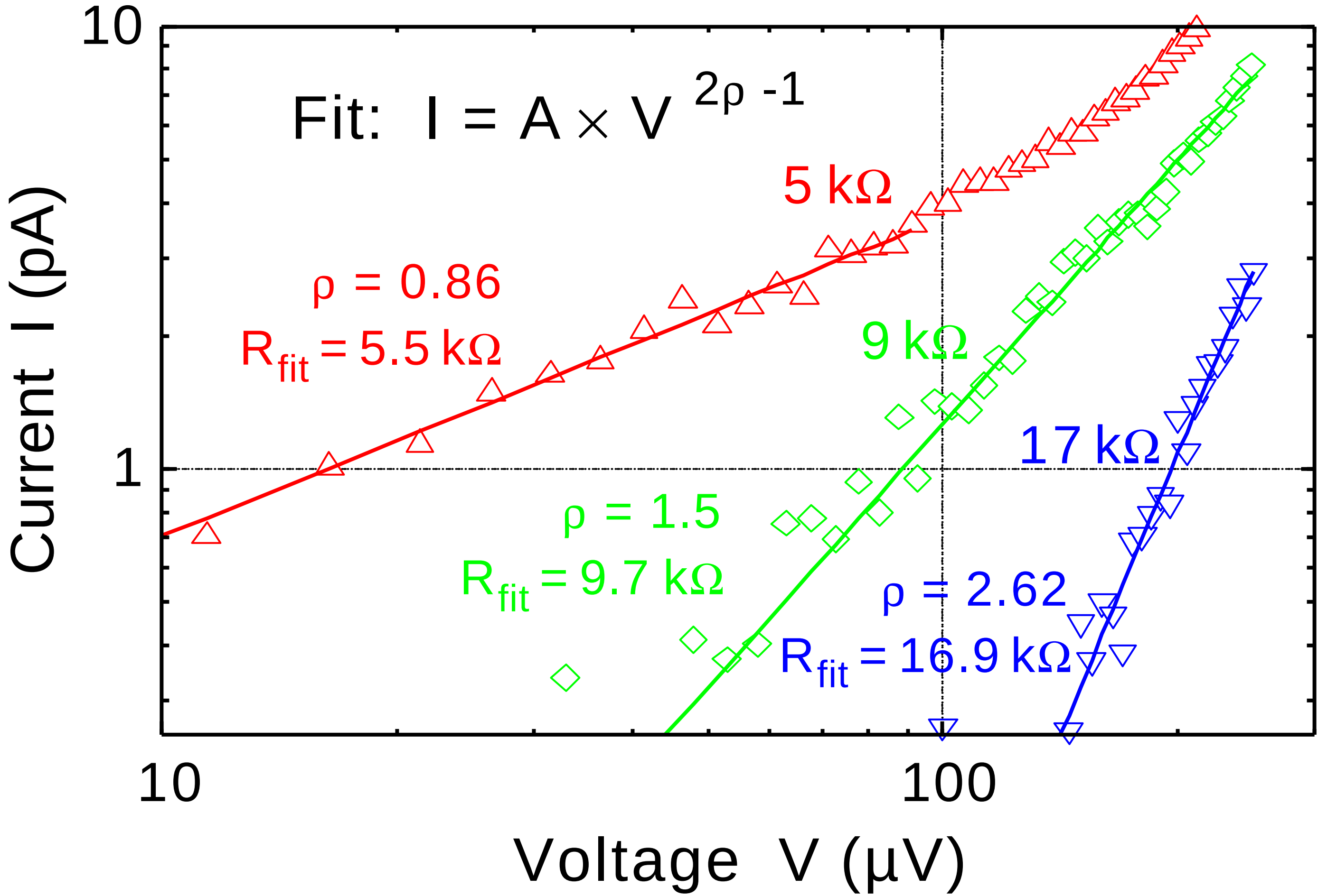


For ohmic environmental impedance:

$$I_{cot} = I_0 \left(\frac{V}{V_0}\right)^{2\rho-1}, \quad \text{where} \quad I_0 = \frac{\pi\rho F_0 e^{-2\gamma}(I_c^{tr})^2}{32\Gamma(2\rho)E_c} \quad \text{and} \quad V_0 = \frac{2e}{\pi\rho C}.$$

$\gamma = 0.577\ldots$ Euler's constant, $T = 0$.

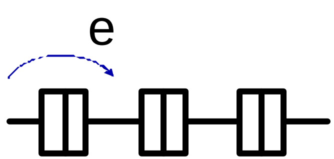
Comparison with experimental data



Possible application of the effect of suppression of Cooper pair cotunneling

In **SET pump** the speed of operation is determined by errors because of:

1. missing of tunneling events
2. enhanced electron temperature in the islands
3. co-tunneling
4. ...



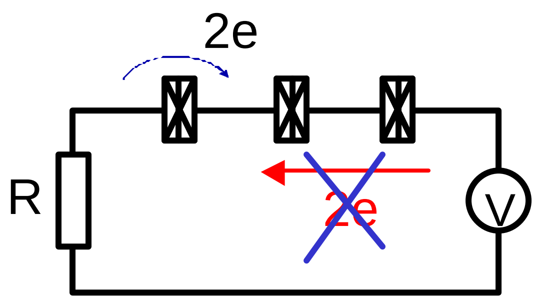
I = ef ~ few pA

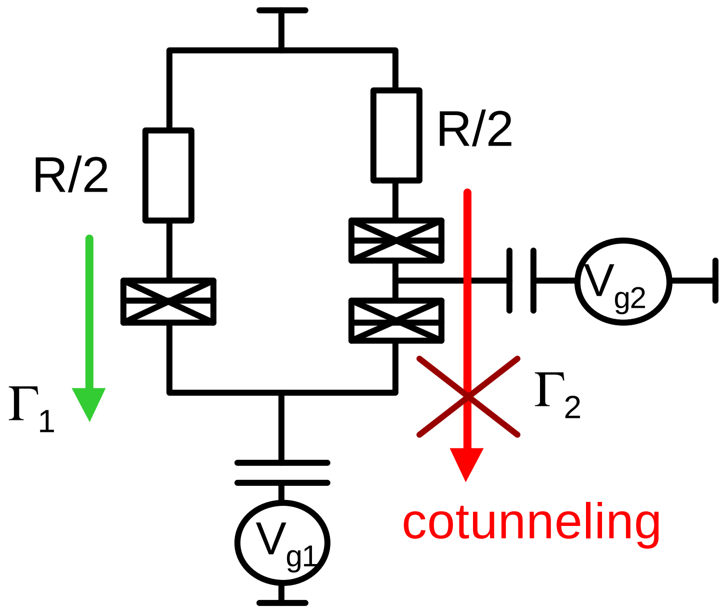
is too small for using as the current standar

<u>The idea is:</u>

To realize a high-speed regime of single charge tunneling using Cooper pairs instead of electrons.

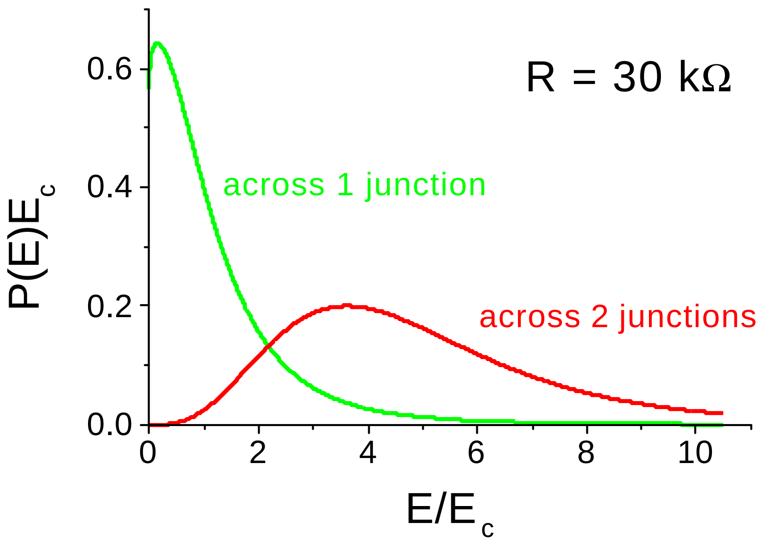
R should efficiently suppress the cotunneling!





Finite dissipation in the leads ($R > R_Q$) makes the rates Γ_1 and Γ_2 very unequal ($\Gamma_1 >> \Gamma_2$).

Incoherent tunneling



One can operate such CP pump even at $E_J/E_c \sim 1$, i.e. when Landau-Zener tunneling is negligibly small.

Conclusion

1. Theory of CP cotunneling is developed
2. Experimentally, the regime of CP cotunneling was realized in the range of gate charges from -e/2 to e/2 for the wide range of on-chip resistors.
3. Possible application of suppression of CP current at large R is outlined

Preprint: cond-mat/0305113

Problems:

(a) Extention of the gate charge range up to from -e to e.

(b) Stable 2e-periodicity in the transistors at small bias voltage.