Aharonov-Bohm conductance through a single-channel quantum ring

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Outline:

\textit{single-channel quantum-ring interferometer}

- Aharonov-Bohm resonances
- correlated electrons on the ring:
  \textit{Persistent-Current Blockade}  
  \rightarrow novel type of Coulomb oscillations
- dephasing vs charge quantization:
  suppression of plasmon-induced dephasing
  \textit{Zero-Mode Dephasing}

PRL '10
Aharonov-Bohm interferometer

\[ I = G(\phi)V \]

Aharonov-Bohm effect:

\[ G(\phi + 1) = G(\phi) \]

\[ \phi = \Phi/\Phi_0 \]
\[ \Phi_0 = hc/e \]

Key parameter: phase breaking rate \( 1/\tau_\phi \)
**AB conductance: what is known**

*Multi-channel quantum ring (quasi-1D)*:
two types of conductance oscillations with the periods

\[ \Delta \phi = 1, \quad \Delta \phi = 1/2 \]

AB oscillations are suppressed by dephasing

\[
\frac{1}{T_\varphi} \propto g^n T^m
\]

In all cases

\[ n, m > 0 \]

- interaction constant
- temperature

quasi-1D: *Ludwig, Mirlin ‘04*

**Dephasing** is due to thermal fluctuations of electric potential created by other electrons *(Nyquist bath)*

*Altshuler, Aronov, Khmelnitskii ‘82*
Single-channel quantum ring:
role of e-e interaction is dramatically enhanced

\[ G(\phi) \]
for low temperature \( T << \Delta \) (level spacing):
- no dephasing \( \rightarrow \) sharp AB resonances affected by Coulomb blockade

Jagla & Balseiro ‘93
Kinaret, Jonson, Shekhter, Eggert ‘98
Pletyukhov, Gritsev, Pauget ‘06
Eroms, Mayrhofer, Grifoni ‘08

We focus on the opposite limit \( T >> \Delta \):
- Interference is not destroyed by thermal averaging:
  conductance shows sharp anti-resonances
- AB dephasing differs qualitatively
  from the multi-channel case
Formulation of the problem: Parameters

(spinless) electrons in a single-channel ballistic ring of length $L$

$$E_F \gg T \gg \Delta \gg \Gamma$$

$\Delta$ - level spacing
$\Gamma$ - tunneling rate

Interaction: Luttinger liquid model

$g \ll 1$
Hamiltonian

\[ H = H_{\text{ring}} + H_{\text{tun}} + H_{\text{leads}} \]

\[ H_{\text{ring}} = \sum_{\mu = \pm} \int_{0}^{L} dx \left( -i\mu v\psi^{\dagger}_{\mu} D_{x} \psi_{\mu} + \frac{1}{2} V_{0} \hat{n}_{\mu} \hat{n}_{-\mu} \right) \]

\[ D_{x} = \partial_{x} - 2\pi i\phi/L \quad \phi = \Phi/\Phi_{0} \quad \hat{n}_{\mu} = \psi^{\dagger}_{\mu} \psi_{\mu} : \]

\[ H_{\text{tun}} = t_{0} \left[ \psi^{\dagger}_{L} \psi(0) + \psi^{\dagger}_{R} \psi(L/2) \right] + \text{h.c.} \]

\[ H_{\text{leads}} \quad \rightarrow \quad \text{structureless density of states in the leads} \]
Dephasing in the almost closed geometry

naive expectation: \(1/\tau_\varphi \sim g^2 T\) is wrong!!

Open-geometry
OK!

Almost closed ring

Physics: size and charge quantization

dephasing is suppressed
Noninteracting case: Landauer approach

\[ G \propto \Gamma^2 \langle |A_R + A_L|^2 \rangle_T \]

\[ A_R = \sum_{n=0}^{\infty} e^{i(kL+2\pi\phi)(1/2+n)} \]
\[ A_L = \sum_{n=0}^{\infty} e^{i(kL-2\pi\phi)(1/2+n)} \]

\( T=0 \Rightarrow \) resonances for sharp dependence on both \( k \) and \( \phi \)

\[ kL + 2\pi\phi = 2\pi l \]
\[ kL - 2\pi\phi = 2\pi m \]
\( T \gg \Delta \quad \Rightarrow \quad A_R, A_L \) oscillate rapidly with changing energy

\[
A_R A_L^* = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e^{i(kL+2\pi \phi)(1/2+n)} e^{-i(kL-2\pi \phi)(1/2+m)}
\]

\[
\langle e^{ikL(n-m)} \rangle_T \rightarrow 0, \quad \text{for } n \neq m \quad \Rightarrow \quad \text{only } n=m \text{ terms contribute, no dependence on } k
\]

but (!) the resonant dependence on \( \phi \) survives

\[
\phi = 1/2 \quad \Rightarrow \quad A_R^n = -A_L^n \quad \Rightarrow \quad G = 0
\]

destructive interference for any energy

\( \text{Büttiker, Imry, Azbel '84; Gefen, Imry, Azbel '84} \)
non-interacting case: narrow AB anti-resonances at \( \phi = 1/2, 3/2, \ldots \)

Expectation \( \Rightarrow \) interaction leads to the broadening of the anti-resonances: width \( 1/\tau_\phi \)

\[
\frac{G(\phi)}{G(0)} = \frac{\cos^2(\pi \phi)[1 + (\Gamma/\Delta)^2]}{\cos^2(\pi \phi) + (\Gamma/\Delta)^2} \approx \frac{(\phi - 1/2)^2}{(\phi - 1/2)^2 + (\Gamma/\Delta)^2}
\]

for \( |\phi - 1/2| \ll 1 \)
Evolution of AB conductance with increasing interaction strength $g$

\[ g \ll \frac{\Gamma}{\Delta} \left( \frac{\Delta}{T} \right)^{1/2} \]

\[ \frac{\Gamma}{\Delta} \left( \frac{\Delta}{T} \right)^{1/2} \ll g \ll \frac{\Gamma T}{\Delta^2} \]

\[ \frac{\Gamma T}{\Delta^2} \ll g \]
Main results:

1. Interaction splits the AB resonances

\[ G(\phi)/G(0) \]

- fine structure resolved for \( g \geq \Gamma T / \Delta^2 \)

- width of the “Zug”

- width of a single peak

2. Interaction-induced dephasing rate does not depend on \( g \)

\[ \frac{1}{\tau_\phi} = \Gamma \frac{T}{\Delta} \]
Two types of excitations in a single-channel ring

1) **Bosonic excitations with** $q \neq 0$:
   right- and left-moving plasma waves

\[
\varphi_R = \varphi + \theta, \quad \varphi_L = \varphi - \theta
\]
\[
\rho = \left(\frac{1}{\pi}\right) \frac{\partial \varphi}{\partial x}, \quad j = \left(\frac{v_F}{\pi}\right) \frac{\partial \theta}{\partial x}.
\]

2) **Bosonic excitations with** $q=0$ : Zero-mode excitations

\[
\hat{Q} = \hat{N}_R + \hat{N}_L
\]
**total charge of the ring**

\[
\hat{J} = \hat{N}_R - \hat{N}_L
\]
**circular (persistent) current**

$N_{R,L}$ - numbers of right- and left-movers in the ring
Zero-mode energy

\[ \epsilon_{N_R, N_L} = \frac{\Delta}{4} \left[ \frac{(N_R + N_L - 2N_0)^2}{K} + K(N_R - N_L - 2\phi)^2 \right] \]

Splitting of AB resonances by the ZM interaction

\[ G_{\text{int}}(\phi) \propto \text{Re}(A_R A_L^*) \]

\[ A_{R,L} \propto |A_{R,L}| e^{i\alpha_{R,L}} \]

\[ \alpha_R = (\epsilon_{N_R+1,N_L} - \epsilon_{N_R,N_L})L/2u \]

\[ \alpha_L = (\epsilon_{N_R,N_L+1} - \epsilon_{N_R,N_L})L/2u \]

interference part of the conductance

phases acquired by a tunneling electron along two interfering paths
\[ \alpha_R - \alpha_L \approx g(N_R - N_L) \Rightarrow \delta \phi \approx g(N_R - N_L) \]

Splitting is due to the quantization of circular current \( \hat{J} = \hat{N}_R - \hat{N}_L \)

\[
\frac{G(\phi)}{G(0)} \approx \frac{1}{Z} \sum_{N_R, N_L} e^{-\epsilon_{N_R, N_L}/T} \frac{[\phi - 1/2 - gJ]^2}{[\phi - 1/2 - gJ]^2 + (\Gamma/\Delta)^2}
\]

without dephasing

\[ \delta J \sim \sqrt{T/\Delta} \Rightarrow \delta \phi_T = g\delta J = g \sqrt{T/\Delta} \]

width of the Zug
Absence of the plasmon mechanism of dephasing

Fluctuating potential created by the plasmon thermal bath

\[ U(x, t) = U_R(x - ut) + U_L(x + ut) \]

Properties of the plasmon bath in the ring:

1) Closed geometry:
   \( U_{R,L} \) are periodic functions of \( x \) with the period \( L \) \( \Rightarrow \) they are also periodic functions of \( t \) with the period \( L/u \)

2) Hartree-Fock cancellation for a short range potential:
   Right-movers do not interact with \( U_R \), and left-movers do not interact with \( U_L \)
Phase coherence is restored periodically

\[ e^{i(\alpha_R - \alpha_L)} \]

\[ \mathcal{R} = \int_0^t dtU_L[x(t) + ut] = \int_0^t dt \ U_L(2ut) \]
\[ \mathcal{L} = \int_0^t dtU_R[x(t) - ut] = \int_0^t dt \ U_R(-2ut) \]

\[ U_{R,L} \] are random on the short time scale but repeat themselves periodically with the period \( L/2u \)

\( g^2 T \)
Dephasing is due to circular-current fluctuations

\[ \phi_{N_R,N_L} = \frac{1}{2} + g(N_R - N_L) \]

positions of the resonances

\[ N_{R,L} \rightarrow N_{R,L}(t), \quad J(t) = N_R(t) - N_L(t) \]

\[ V_{eff}(t) = g J(t) \Delta \]

dephasing potential

\[ e^{i(\alpha_R - \alpha_L)} = e^{i \int_0^t V_{eff} \, dt} = e^{ig \Delta \int_0^t J \, dt} \]
Time scale of the current fluctuations

Population of one level fluctuates on a time scale $1/\Gamma$

$J(t) = N_R(t) - N_L(t)$

$T/\Delta$ - number of active levels $>>1$

$N_{R,L} \rightarrow N_{R,L}+1$  

Shorter time scale: $\delta t = \frac{1}{\Gamma} \frac{\Delta}{T}$
Quasiclassical approach

$N(t)$

$N_0$ $N_0+1$ $N_0+2$ $N_0+3$

δt

$t$

$\delta t = \frac{1}{\Gamma} \frac{\Delta}{T}$

$e^{S} = \left\langle e^{ig\Delta \int_{0}^{t} dt [N(t) - N_0]} \right\rangle = \exp \left\{ \left[ \frac{\sin(g\Delta t)}{g\Delta} - t \right] \frac{\Delta}{\delta t} \right\} \approx \exp(-t/\delta t)$

for $t \gg 1/g\Delta$

$\Gamma \varphi = \Gamma \frac{T}{\Delta}$

- much faster than the tunneling
- does not depend on the interaction

telegraph noise
Dephasing by telegraph noise

\[ e^S = \left\langle e^{ig\Delta \int_0^t dt(N_R - N_L)} \right\rangle = \left\langle e^{ig\Delta \int_0^t dt N_R} e^{-ig\Delta \int_0^t dt N_L} \right\rangle \]

\[ = \left\langle e^{ig\Delta \int_0^t dt N} \right\rangle \times \left\langle \text{c.c.} \right\rangle \]

\[ N_R = n_1 + n_2 + \cdots + n_N + \cdots \]

\[ n_N = 0, 1 \]

\[ T/\Delta \]

\[ e^S = \prod_N \left\langle e^{ig\Delta n_N t} \right\rangle \prod_N \left\langle \text{c.c.} \right\rangle \]

\[ n_N = 0, 1 \quad \rightarrow \quad \text{telegraph noise} \quad \rightarrow \quad \text{dephasing} \]
Dephasing action

\[ e^S = \prod_N (1 - f_N) e^{-\Gamma t f_N} + e^{ig\Delta t} e^{-\Gamma t (1 - f_N)} \]
\[ \times \prod_N (1 - f_N) e^{-\Gamma t f_N} + e^{-ig\Delta t} e^{-\Gamma t (1 - f_N)} \]

\[ e^S = \exp \left[ -\frac{2T}{\Delta} (1 - \cos g\Delta t) \right] \exp \left\{ -\Gamma_\varphi \left[ t(1 + \cos g\Delta t) - \frac{2 \sin g\Delta t}{g\Delta} \right] \right\} \]

\[ e^{S_0} \quad e^{S_{\text{deph}}} \]

here: linear in \( \Gamma \) terms only

\[ \Gamma_\varphi = \Gamma \frac{T}{\Delta} \]

zero mode dephasing
Quantum beats in the dephasing action

\[ e^{S_0} = \frac{1}{Z} \sum_{N_R, N_L} e^{ig\Delta t} e^{-\epsilon_{N_R, N_L}/T} \]

coherent oscillations in a closed ring

\[ e^{-\Gamma \varphi t} \]

slow envelope

1/g\Delta
Persistent-current blockade

Sum over the winding number:

\[ t_n = \frac{2\pi(n + 1/2)}{\Delta} \]

\[
\frac{G_{\text{int}}(\phi)}{G(0)} \approx -\frac{2\pi\Gamma}{\Delta} \sum_{n=0}^{\infty} \cos[2\Delta(\phi - 1/2)t_n] e^{-\Gamma t_n - S(t_n)}
\]

\[
\frac{G(\phi)}{G(0)} \approx \frac{1}{Z} \sum_{N_R, N_L} e^{-\epsilon_{N_R, N_L}/T} \left[ 1 - \frac{\Gamma\Gamma\varphi}{\Delta^2(\phi - 1/2 - gJ)^2 + \Gamma^2} \right]
\]

\[
\Gamma\varphi = \Gamma \frac{T}{\Delta}
\]

Quantum properties of Luttinger liquid

\[
\Gamma \rightarrow \Gamma \left( \frac{T}{E_F} \right)^{g^2/2}
\]
Renormalization of the tunneling rate

\[ \frac{d\Gamma}{d\Lambda} = g\Gamma^2 - \frac{g^2\Gamma}{2} \]

\( \Gamma \) in units of \( \Delta \) (tunneling transparency of the contact)

\[ \Lambda = \ln \left( \frac{E_F}{T} \right) \]

non-trivial geometry, junction of 3 wires (Das, Rao, Sen '04)

renormalization of the bulk density of states

\( \Gamma_0 \ll g \) 

\( g/2 \)

we are here

renormalization of the density of states dominates
Tunneling Hamiltonian (point contact): small $\gamma$

Perturbative renormalization 
($g \ll 1$, $\gamma \ll 1$):

\begin{align*}
g &= 0 \\
r &= 1 - 2\gamma \\
g_2 &= g
\end{align*}

fermion loop $\Rightarrow$ opposite sign
Summary of the main results

height of n-th resonance:
\[
\left( \frac{G_{int}}{G_{cl}} \right)_n = \exp(-n^2 \Delta/T) \left( \frac{\Delta}{T} \right)^{3/2}
\]

width of the resonances:
\[
\delta \phi = \frac{\Gamma \varphi}{\Delta}
\]

interference part of the conductance:
\[
\frac{G_{int}(\Phi)}{G(0)} \simeq - \left\langle \frac{\Gamma \Gamma \varphi}{[\phi - 1/2 - \alpha J]^2 \Delta^2 + \Gamma^2} \right\rangle_{\text{Gibbs}}
\]

Zero-mode dephasing:
\[
\Gamma \varphi = \Gamma T / \Delta
\]
Conclusions

• Electron-electron interactions \(\Rightarrow\) profound effect on transport through a single-channel ring

• Main phenomenon: persistent-current blockade

• PCB-induced splitting of AB antiresonances

• Zero-mode dephasing due to fluctuations of PCB

• PCB is robust (spin, curvature, asymmetry...)