

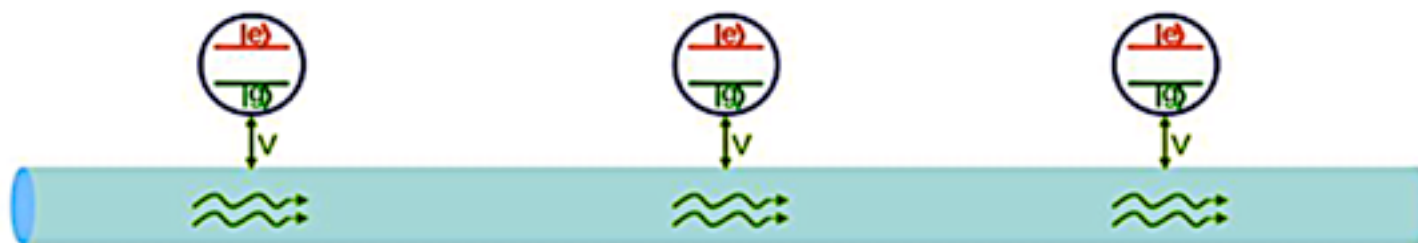


Waveguide QED: Quantum Transport of Strongly Correlated Photons

Harold Baranger, *Duke University*,
with Huaixiu Zheng and Dan Gauthier



Topic: **1D bosons** [photons, plasmons, cold atoms, ...] interacting with **two-level systems** (2LS) [atom, qubit, quantum dot, ...]



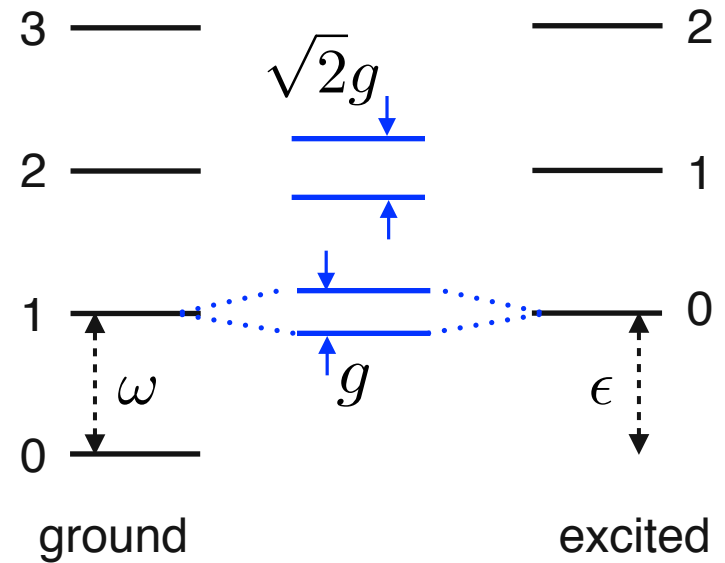
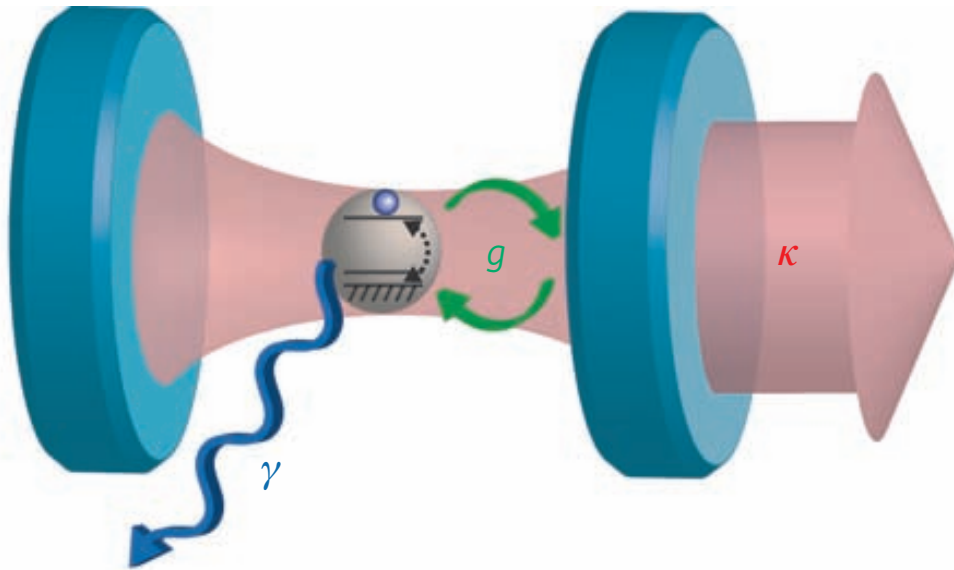
Outline:

- **motivation**:
quantum optics, quantum information, phases of “interacting” light
- **experimental** systems: optical, superconducting qubits, nanowires, ...
- **simple model** → surprising result, 2 photon correlated state
- our results: **photon blockade** using a 4 level system; **2 qubits**
- conclusions and speculations...

Motivation: Quantum Optics

Quantum Optics:

Two level atom in a cavity: cavity QED



Jaynes-Cummings model:
$$H_{JC} = \hbar\omega a^\dagger a + \frac{\epsilon}{2}\sigma^z + g(\sigma^+ a + a^\dagger \sigma^-)$$

strong coupling shifts the modes of the cavity

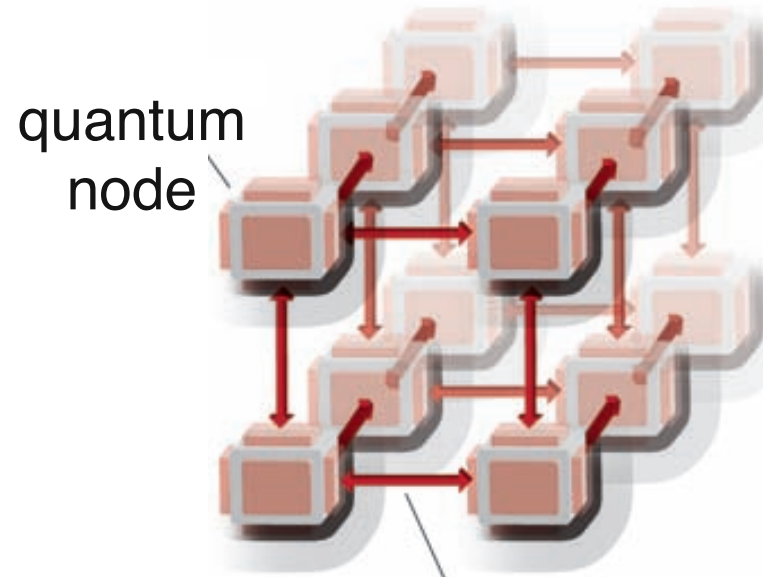
→ no longer an evenly spaced harmonic osc. ladder

→ nonlinear spacing produces photon blockade

~ effective interaction between photons

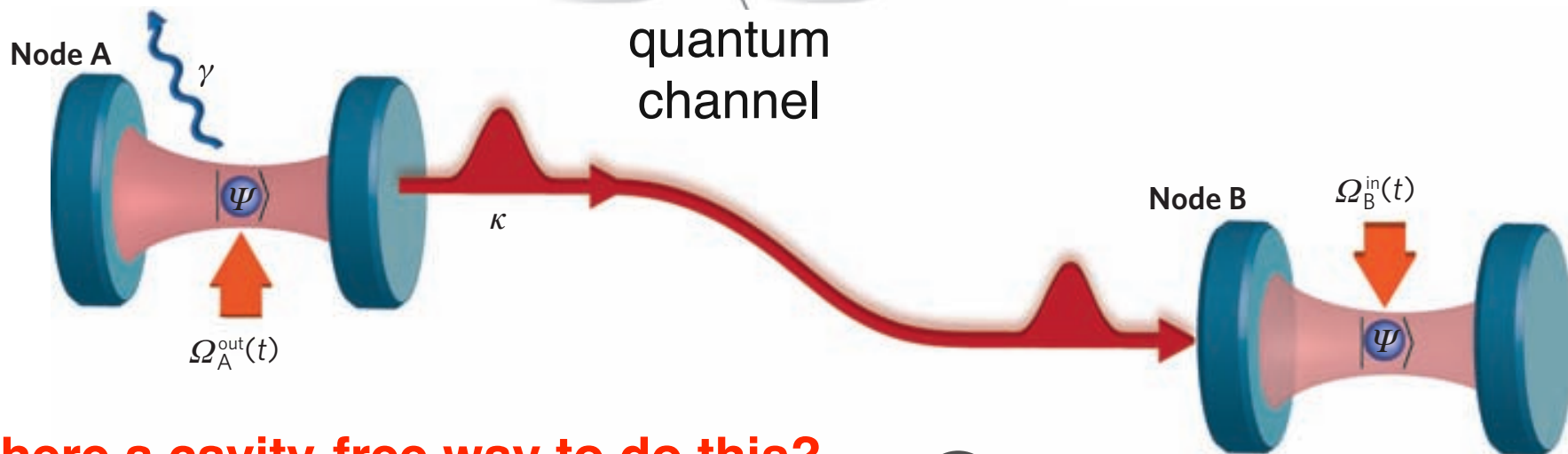
Does coupling to a **continuum** of modes bring in something new?

Motivation: Flying Qubits for Quantum Networks

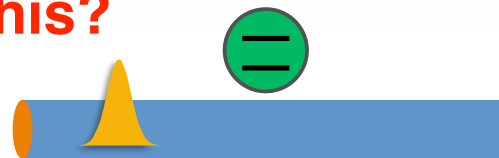


from H. J. Kimble,
“Quantum Internet”,
Nature 2008

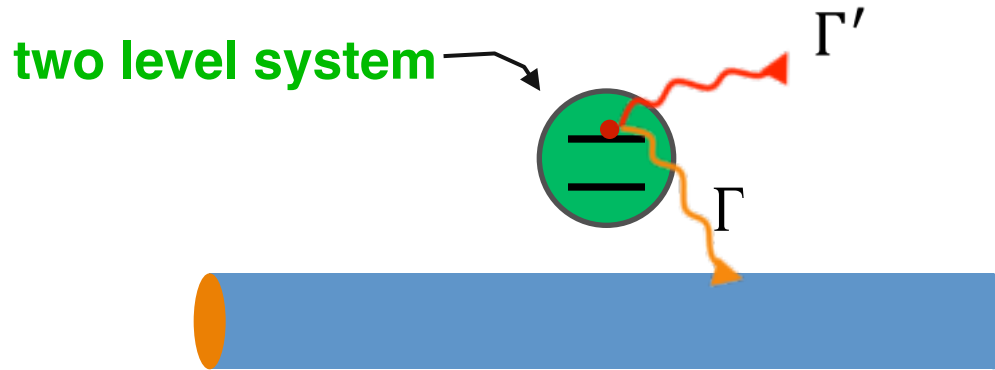
Scheme being
developed based
on cavity QED:



Is there a cavity-free way to do this?



1D Waveguide QED: Strong Coupling



❖ Γ' : decay rate to modes other than waveguide

❖ $\Gamma \propto 1/R^3$: strong confinement, R

❖ Strong coupling: $P = \Gamma/\Gamma' \gg 1$

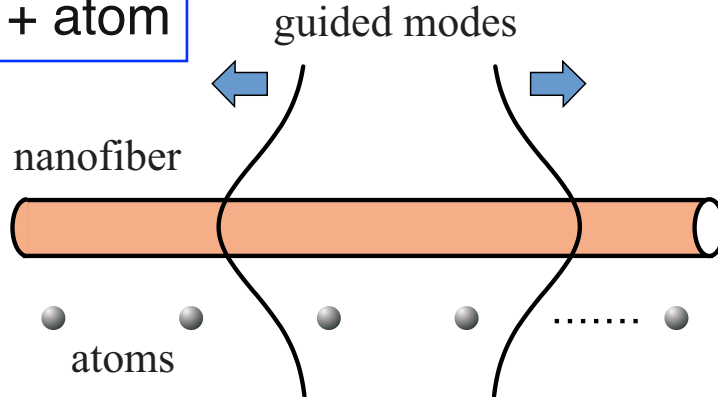
Dipole coupling $-e\mathbf{r} \cdot \mathbf{E} \rightarrow (|g\rangle\langle e| + |e\rangle\langle g|) (a + a^\dagger)$

$$H = \sum \hbar\omega_k a_k^\dagger a_k + \epsilon|e\rangle\langle e| + \sum V \left(a_k^\dagger |g\rangle\langle e| + a_k |e\rangle\langle g| \right)$$

“rotating wave approximation”: neglect 2 terms in the dipole coupling;
valid when $\Gamma/\epsilon \ll 1$

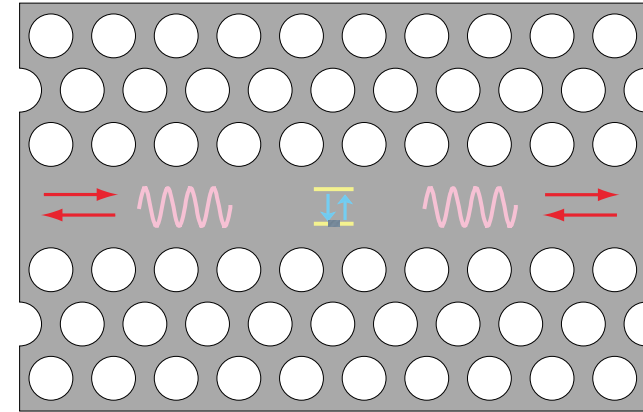
Waveguide QED: Variety of Potential Physical Systems

nanofiber + atom



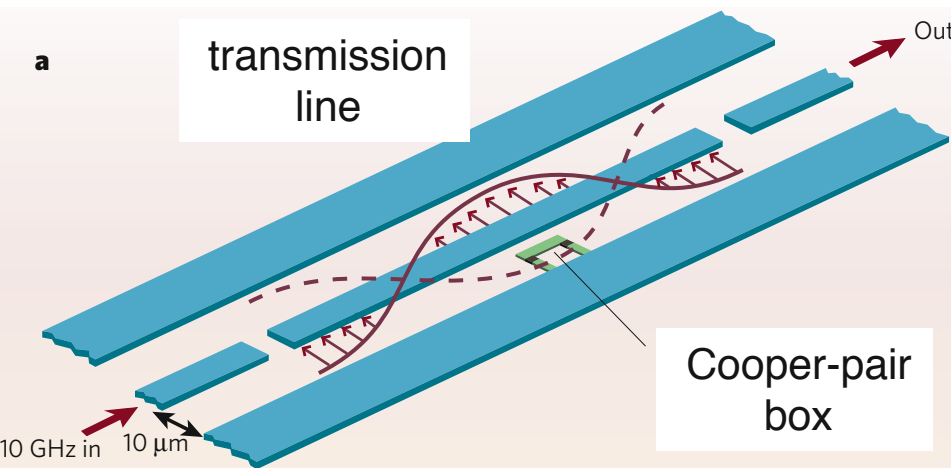
[from LeKien & Hakuta, PRA 2008]

photonic crystal + defect



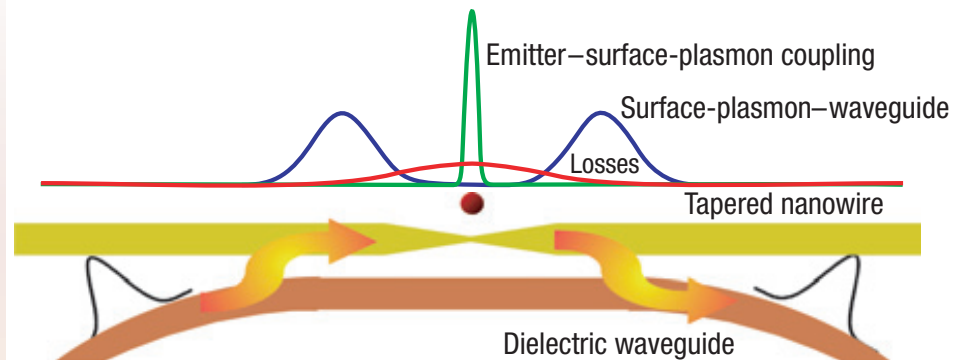
[from Shen & Fan, PRL 2007]

superconducting qubit + stripline



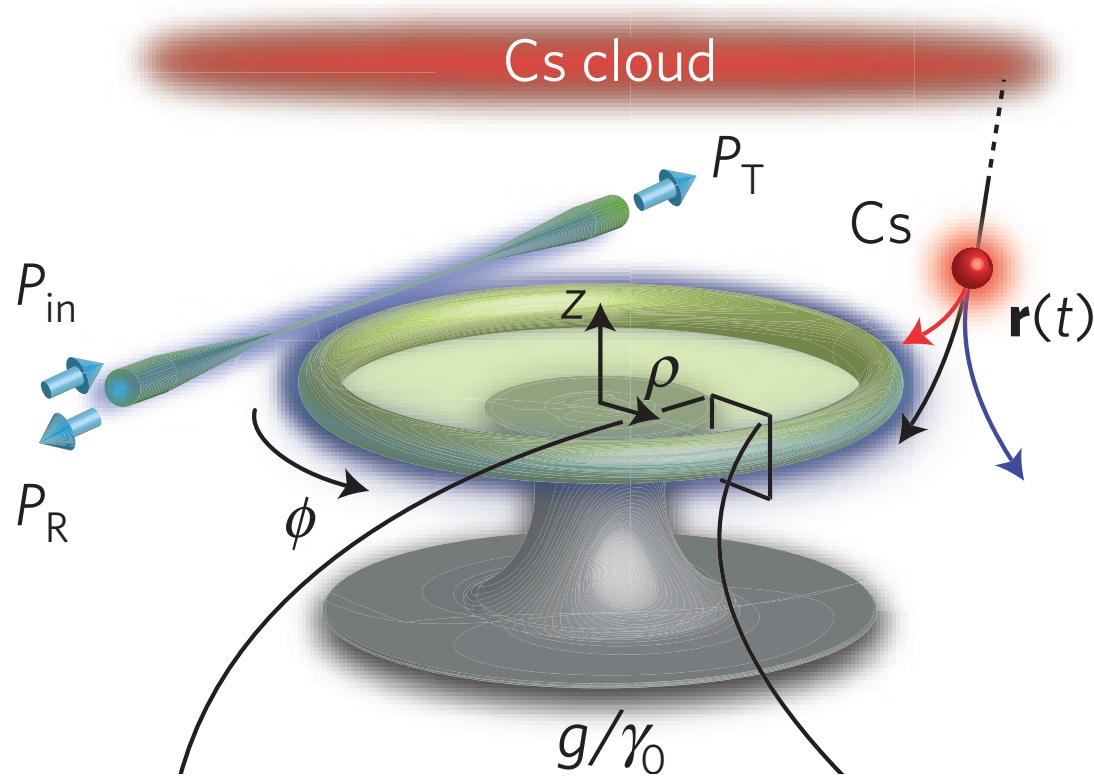
[Schoelkopf & Girvin, Nature 2008]

plasmonic wire + quantum dot



[Chang, Sorensen, Demler & Lukin, Nat.Phys. 2007]

Experimental Systems: Example 1: Atom + Dielectric

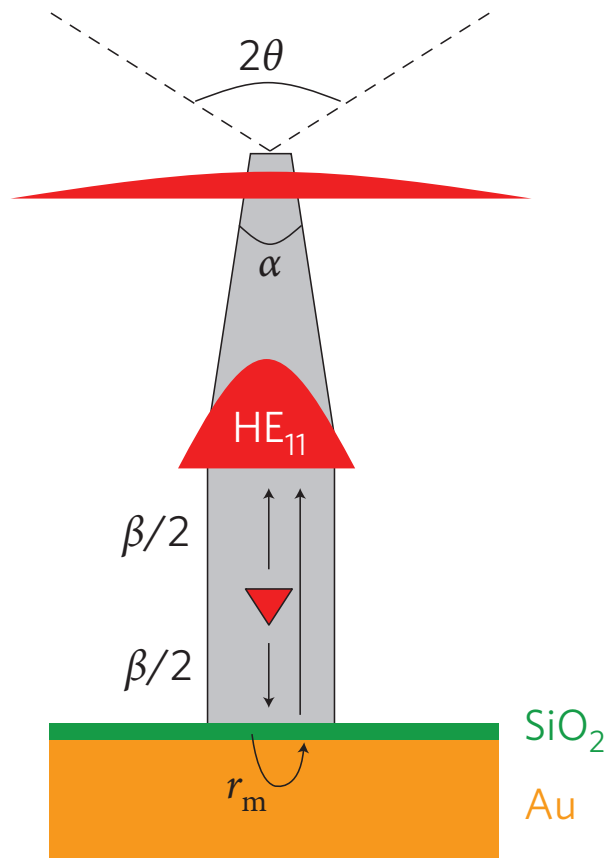


Observed strong interaction of single atom and single photon.

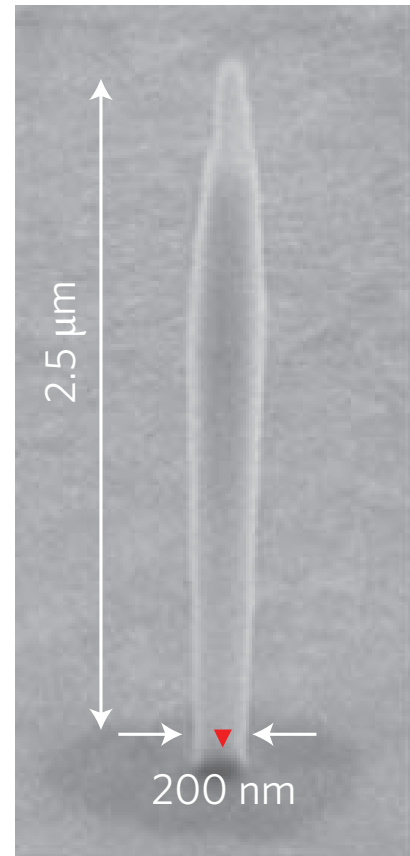
[from Alton, et al. (Kimble group) N.Phys. 2010]

Experimental Systems: Example 2: Quantum Dot + Nanowire

InAs quantum dot embedded in GaAs nanowire:



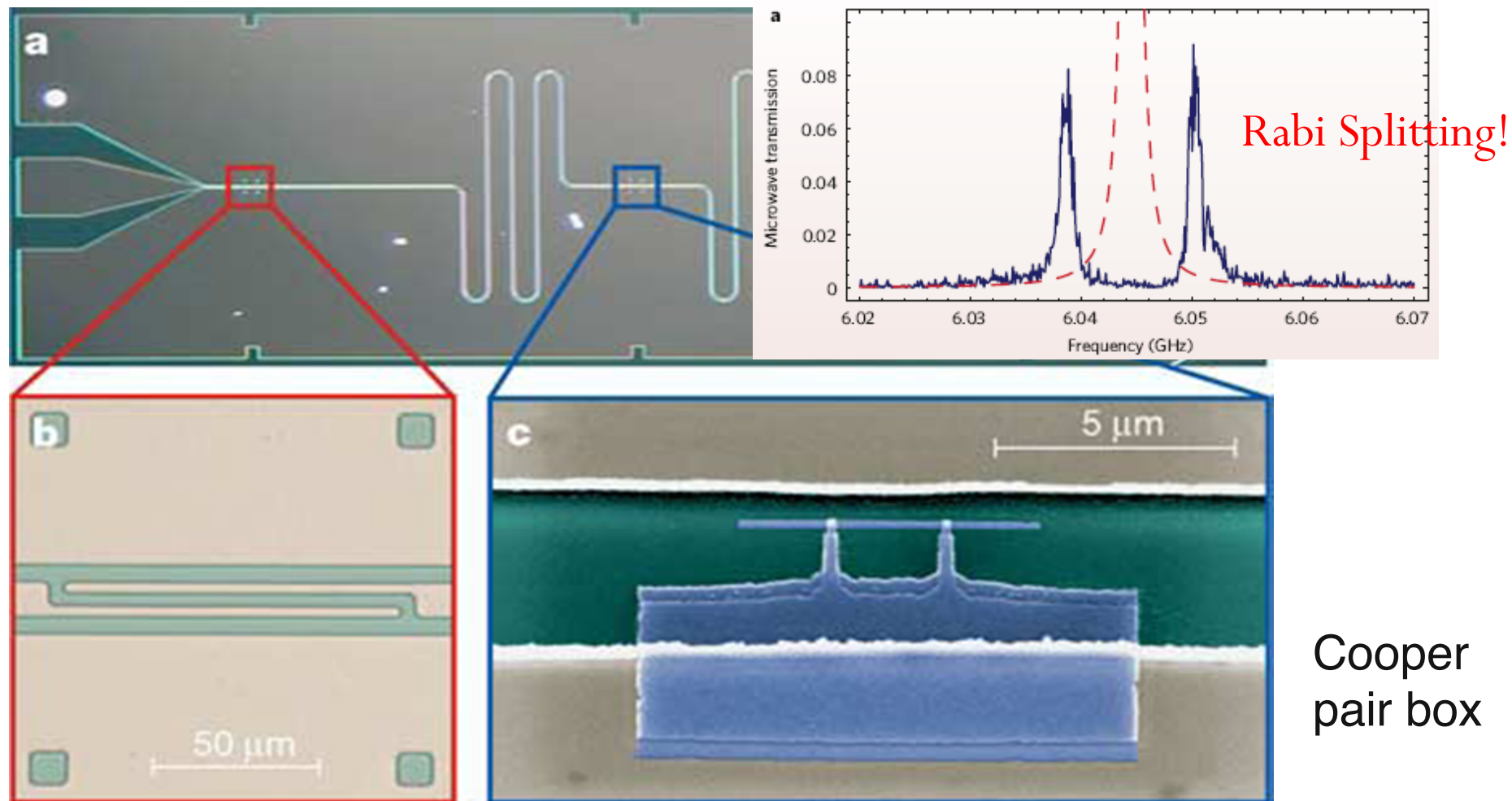
b



P=9

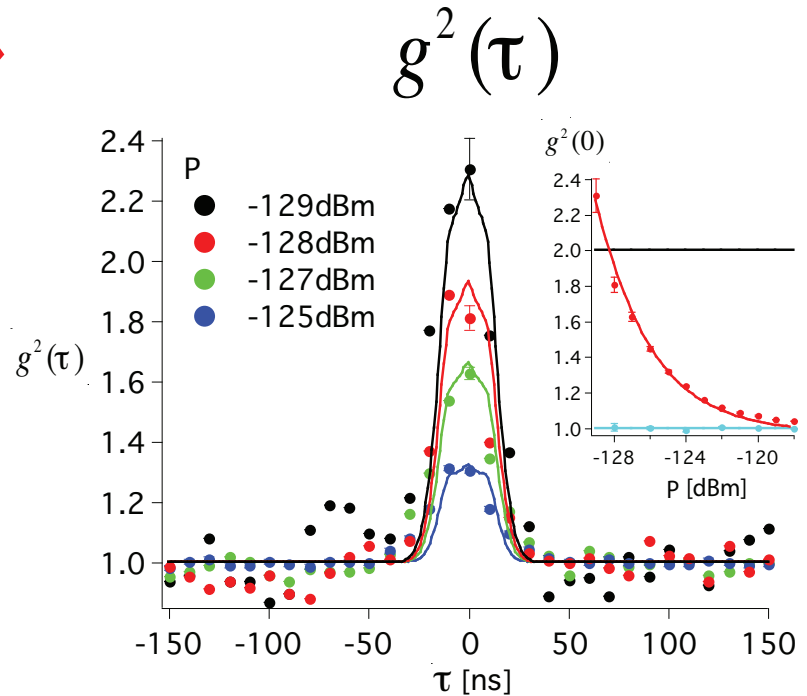
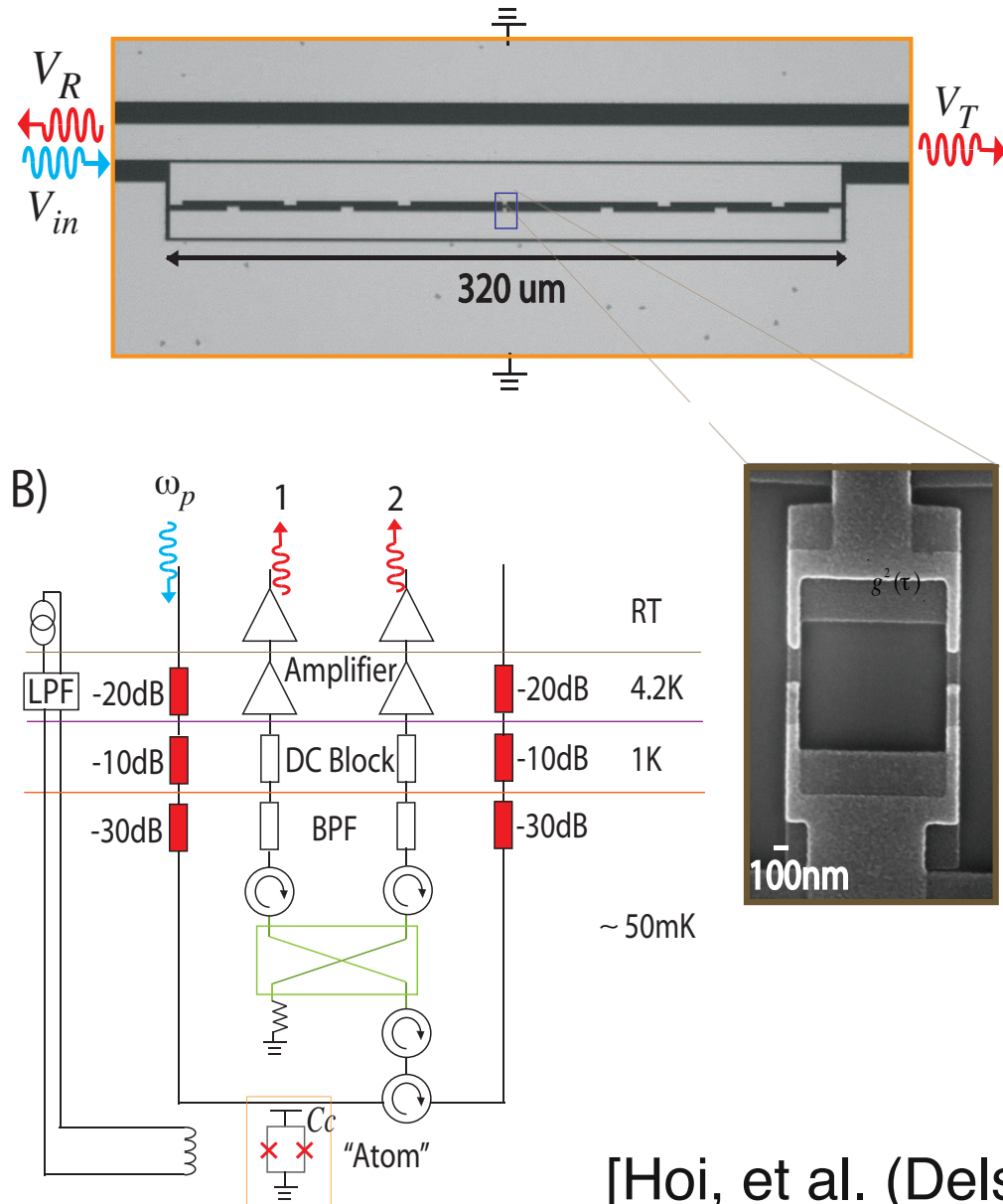
[from Claudon, et al. (Grenoble CEA group),
N.Photonics 2010 & PRL 2011]

Experimental Systems: Example 3: Superconducting Qubit



[from Wallraff, et al. (Yale group) Nature 2004]

Experimental Systems: Example 3: Superconducting Qubit



$P > 20$

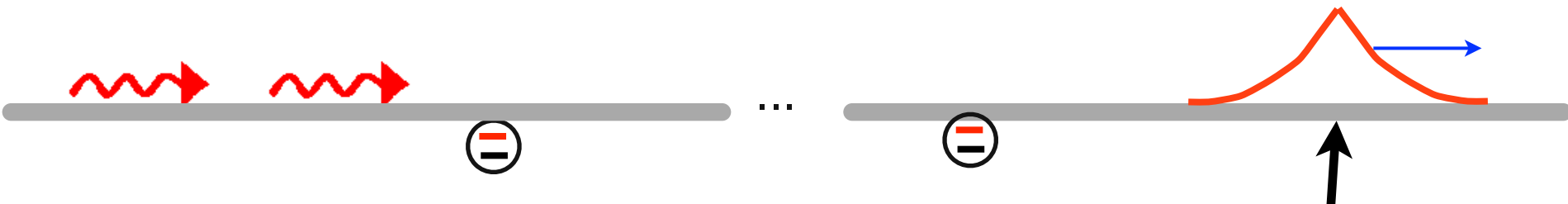
[Hoi, et al. (Delsing/Wilson group) arXiv 2012)]

2 Photon States: Correlation (handwaving...)



Scattering by 2LS depends on whether it is in excited state.

Example result (Shen&Fan, PRL-PRA 2007):



inject 2 photons in uncorrelated resonant plane wave states

part of outgoing wave: the 2 photons are exponentially correlated

* connected to stimulated emission (bosons!)

Strong coupling → 1 photon near 2LS strongly influences others
→ **generates a strong correlation** between the photons

Waveguide QED: Waveguide + Two-Level System (2LS)

1D continuum
(bosons)

+

2 level system
(described by Pauli matrices)

+

coupling

$$H = -i\hbar c \int dx \left[a_R^\dagger(x) \frac{d}{dx} a_R(x) - a_L^\dagger(x) \frac{d}{dx} a_R(x) \right] \\ + (\epsilon - i\Gamma') | \uparrow \rangle \langle \uparrow | \\ + V \left[\sigma^+ a_R(0) + \sigma^- a_R^\dagger(0) + \sigma^+ a_L(0) + \sigma^- a_L^\dagger(0) \right]$$

caveat: strong coupling but not “ultra-strong”

→ rotating wave approximation, so neglect $\sigma^+ a_R^\dagger(0)$
(⇒ makes exact solution straight forward)

1. number of “excitations” conserved (no “Kondo effects”...)
2. L-R symmetry ⇒ make even and odd combination
⇒ only even mode couples to 2LS (and it is chiral...)
3. bosonic statistics of $a_R^\dagger$ is crucial!

Waveguide QED (2LS): 2-Photon Scattering State

1. Scattering wave states for incoming even plane waves

$$n=1 \quad |\psi_1\rangle = \int dx \, g_1(x) a_e^\dagger(x) |0, g\rangle + e_1 |0, e\rangle \quad \text{can satisfy with plane waves}$$

n=2

$$|\psi_2\rangle = \int dx_1 dx_2 \, g_2(x_1, x_2) a_e^\dagger(x_1) a_e^\dagger(x_2) |0, g\rangle + \int dx \, e_2(x) a_e^\dagger(x) |0, e\rangle$$

no plane wave solution: can't make $g_2(x_1, x_2)$ and $e_2(x)$ consistent

$$g_2(x_1, x_2) \sim \text{Sym} \left\{ e^{i(k_1 x_1 + k_2 x_2)} + B e^{i(k_1 + k_2)x_2} e^{-\Gamma|x_2 - x_1|} \right\}$$

correlated state

need a continuum for this

2. Transform back to Left-Right basis

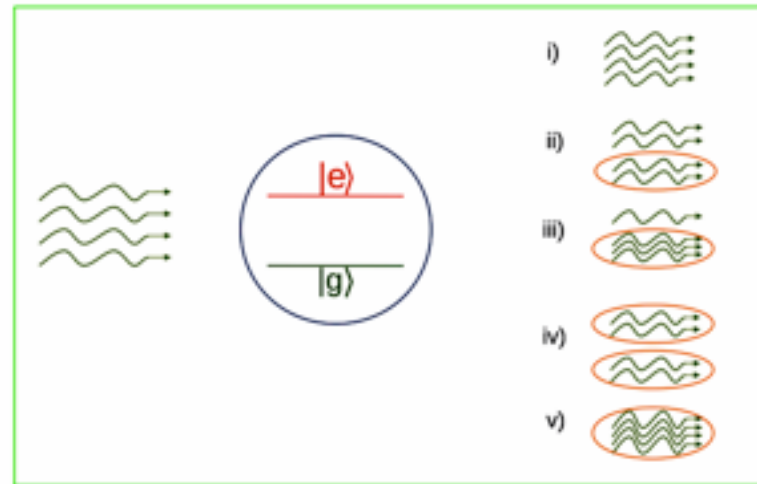
3. Scattering of wave-packet with definite photon number: Fock state
use a Gaussian wave-packet-- spectral width σ

4. Scattering of coherent state: mean photon number

Waveguide QED (2LS):

$n=4$

variety of more
complicated correlations:

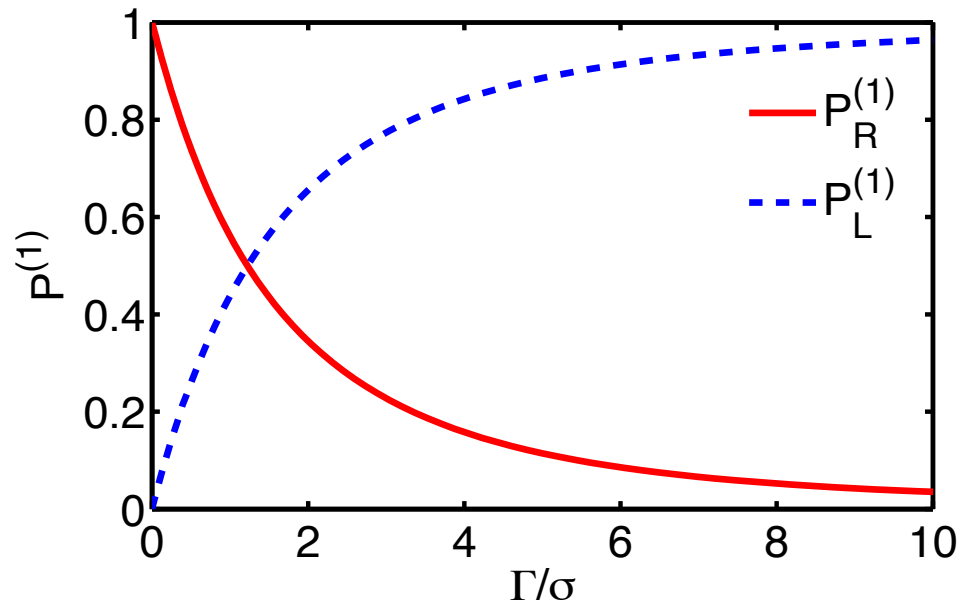


1-photon transmission:

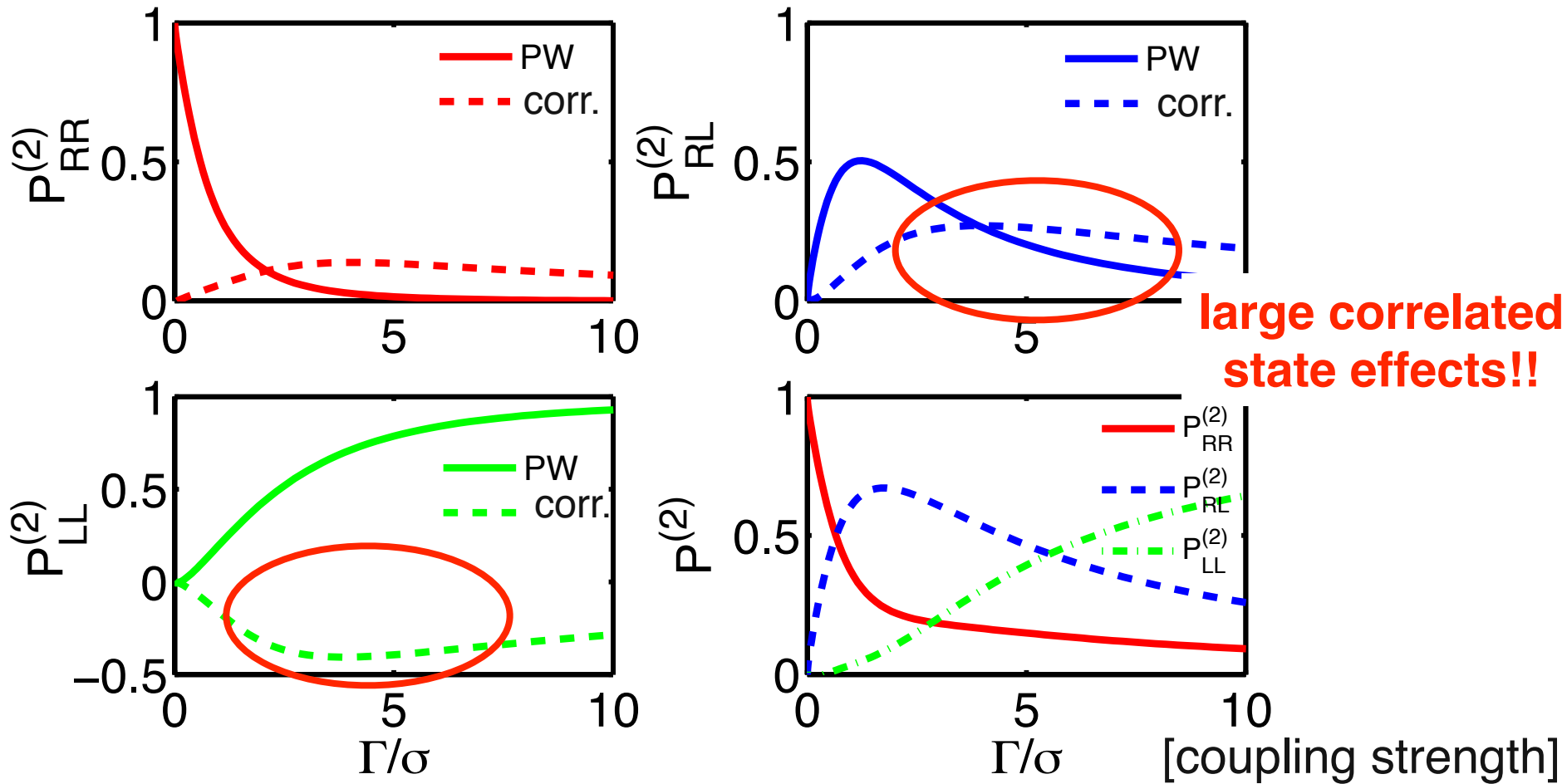
incoming photon at
resonance with the 2LS;
Gaussian wave-packet

decay rate from 2LS
into waveguide:

$$\Gamma \equiv 2V^2/c$$



Waveguide QED (2LS): 2-Photon Transmission



PW = "plane wave"

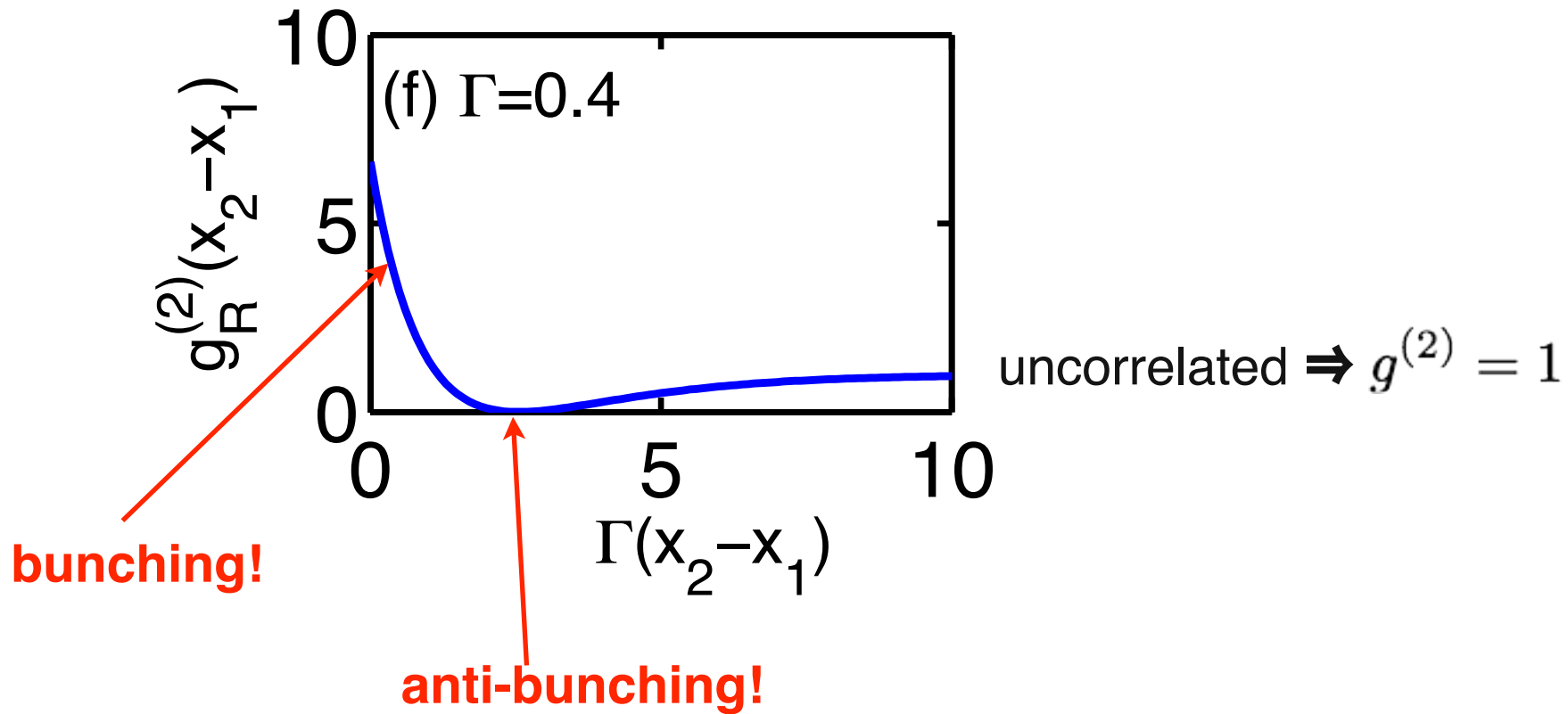
corr. = correlated state + interference part

RR = right-right

RL = right-left (1 photon transmitted, 1 reflected)

Waveguide QED (2LS): Two-point Correlation

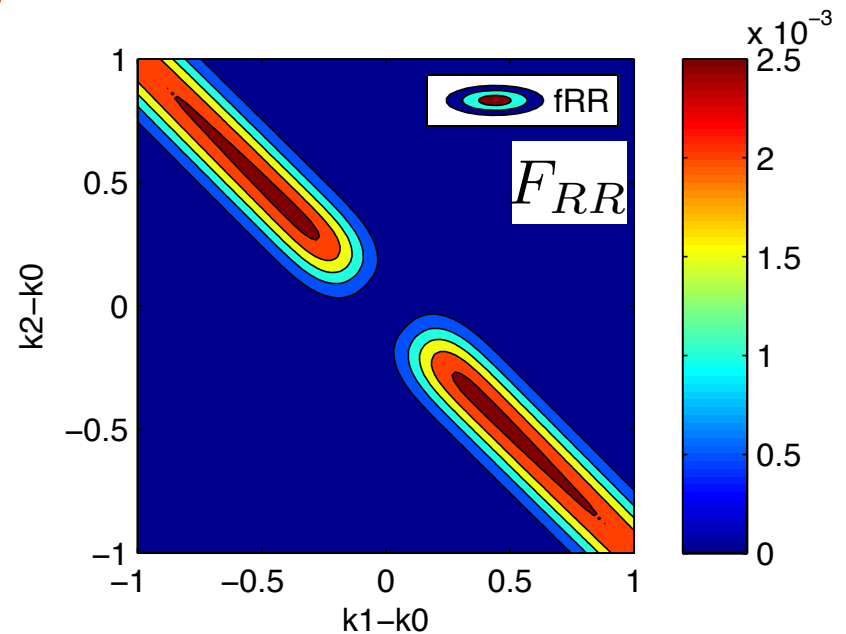
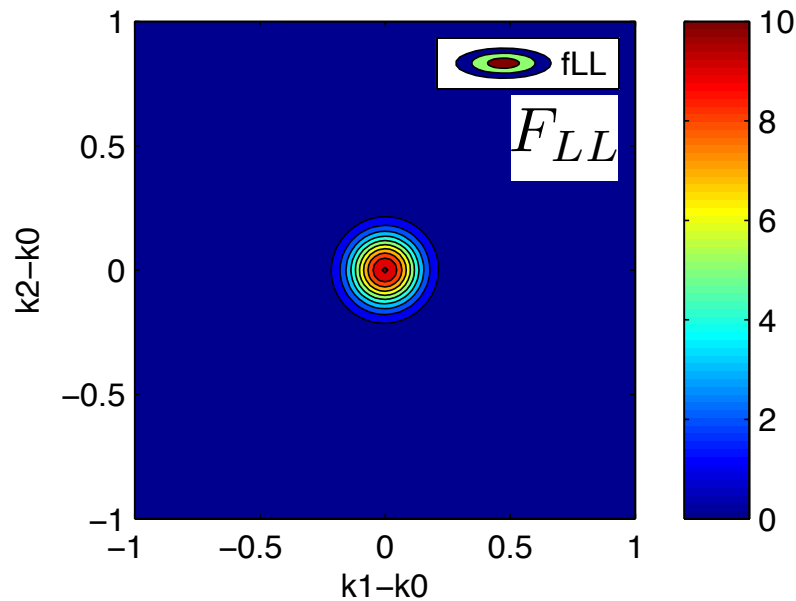
$$g_R^{(2)}(x_2 - x_1) = \frac{\langle \text{out}_\alpha | a_R^\dagger(x_2) a_R(x_2) a_R^\dagger(x_1) a_R(x_1) | \text{out}_\alpha \rangle}{\langle \text{out}_\alpha | a_R^\dagger(x_1) a_R(x_1) | \text{out}_\alpha \rangle^2}$$



Spectral Entanglement: Two-Photon State

$$|\psi^{(2)}\rangle = \int d\omega_1 d\omega_2 \left[f_{RR}(\omega_1, \omega_2) a_R^\dagger(\omega_1) a_R^\dagger(\omega_2) \right. \\ \left. + f_{RL}(\omega_1, \omega_2) a_R^\dagger(\omega_1) a_L^\dagger(\omega_2) \right. \\ \left. + f_{LL}(\omega_1, \omega_2) a_L^\dagger(\omega_1) a_L^\dagger(\omega_2) \right] |\emptyset\rangle,$$

$$F_{\alpha\beta=RR,RL,LL}(\omega_1, \omega_2) = |f_{\alpha\beta}(\omega_1, \omega_2)|^2$$



System parameters:
 $\Gamma=9, \Gamma'=1, \delta=0, \sigma=0.1$

Spectrally entangled photon pair
 (possible large-alphabet
 quantum communication)

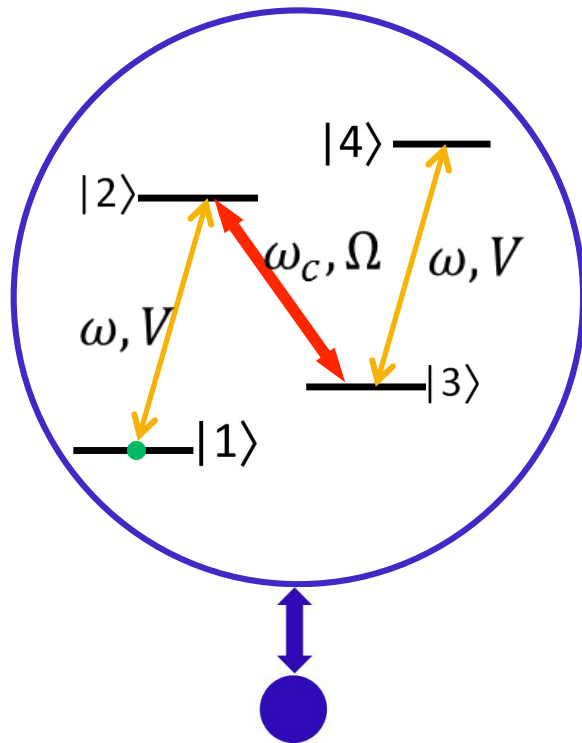
[Huaixiu Zheng]

Waveguide QED: Four-Level System (4LS)

Consider a more complicated local quantum system:

Why??

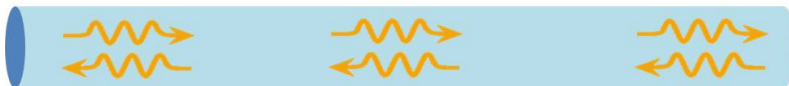
1. would like “classic” effect of interacting photons, such as photon blockade
2. want to change number statistics-- non-classical light



classical controllable field couples levels 2 and 3:
made in optics and supercond. qubits



[Majer et al., PRL 2005]



[Huaixiu Zheng]

Waveguide QED: Four-Level System (4LS)

1D continuum $H = \int dx (-i)\hbar c \left[a_R^\dagger(x) \frac{d}{dx} a_R(x) - a_L^\dagger(x) \frac{d}{dx} a_L(x) \right]$

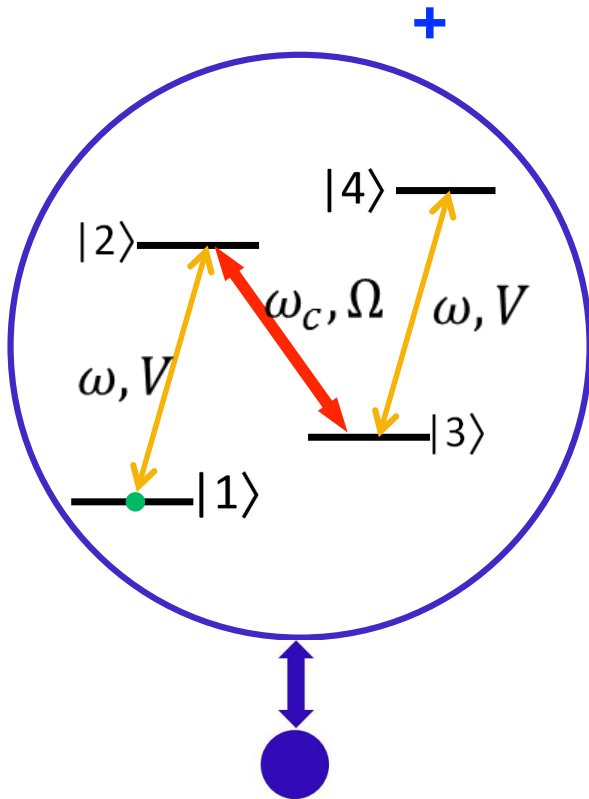
+

coupling $+ \int dx \hbar V \delta(x) \{ [a_R^\dagger(x) + a_L^\dagger(x)] (|1\rangle\langle 2| + |3\rangle\langle 4|) + \text{h.c.} \}$

+

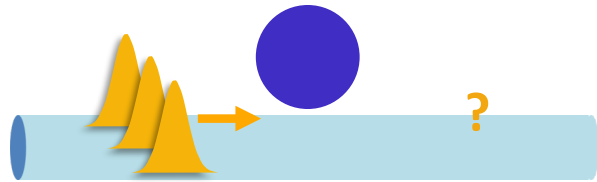
$$+ \sum_{j=2}^4 \hbar \left(\epsilon_j - \frac{i\Gamma_j}{2} \right) |j\rangle\langle j| + \frac{\hbar\Omega}{2} (|2\rangle\langle 3| + \text{h.c.})$$

4 level system



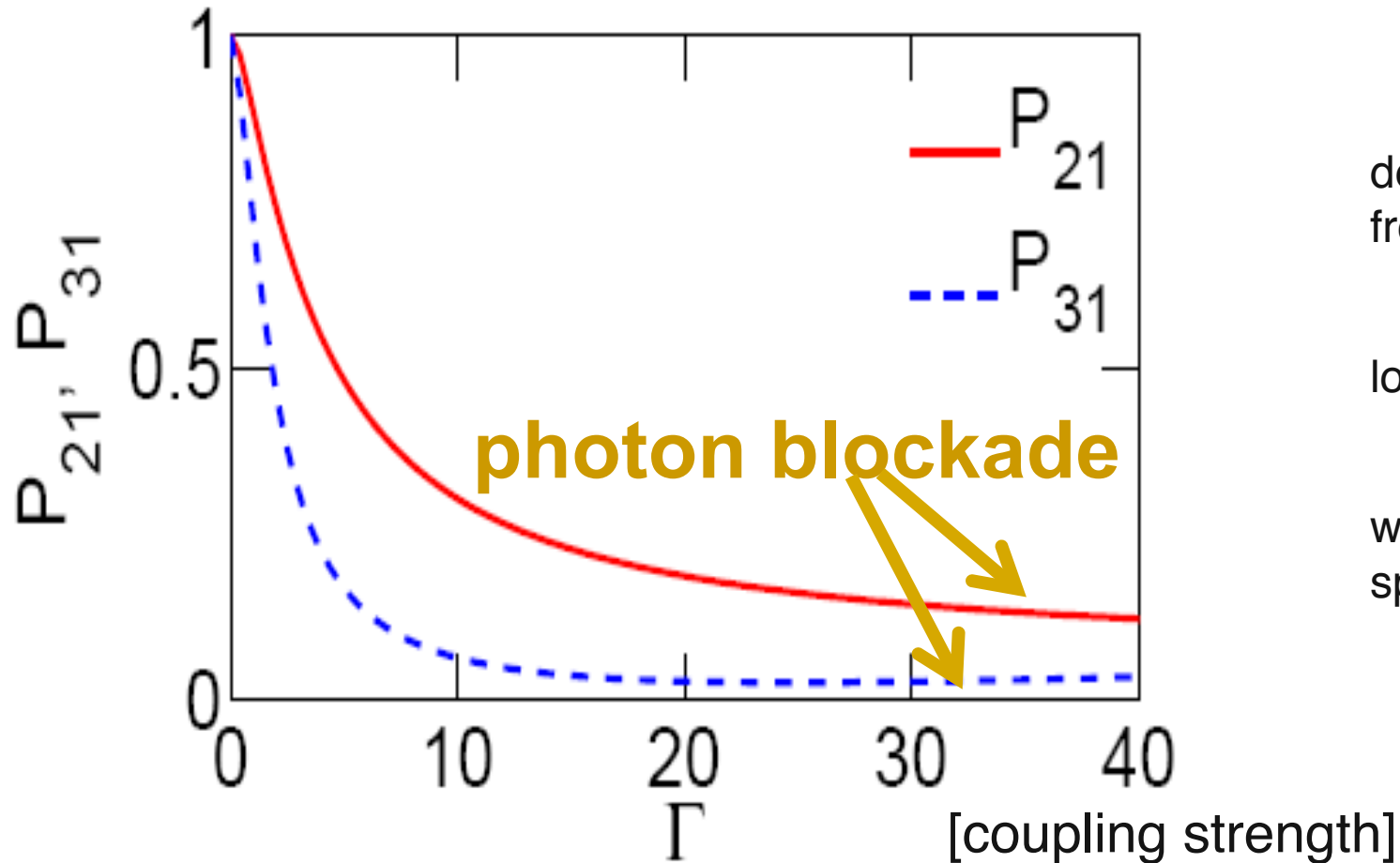
- ❖ Coupling rate to WG: $\Gamma \equiv 2V^2/c$
- ❖ Loss rate: $\Gamma' \equiv \Gamma_2 = \Gamma_4$
- ❖ Rotating wave approx.: $\Gamma \ll \epsilon_2$
- ❖ Metastable level 3: loss rate $\Gamma_3 = 0$

Multi-Photon: Cavity-free Photon Blockade



true 2-photon transmission compared
to 2 independent photons:

$$P_{21} \equiv \frac{T_2}{T_1^2}$$



detuning
from 4LS:
 $\delta = 0$

loss in 4LS:
 $\Gamma' = 1$

wavepacket
spectral width:
 $\sigma = 0.2$

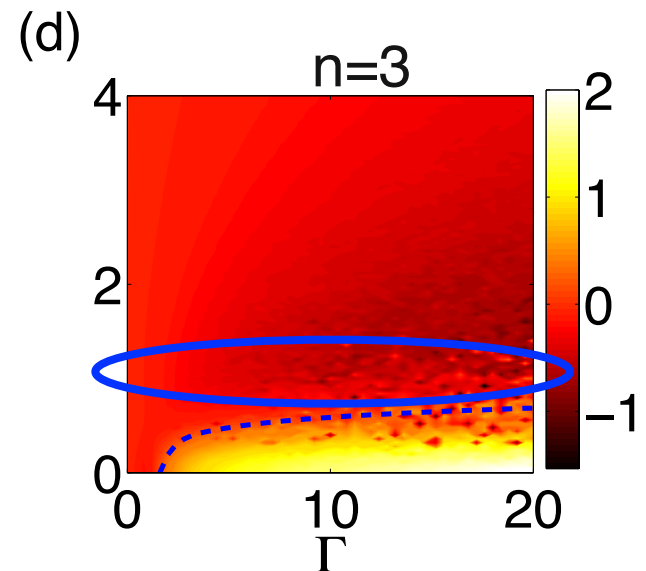
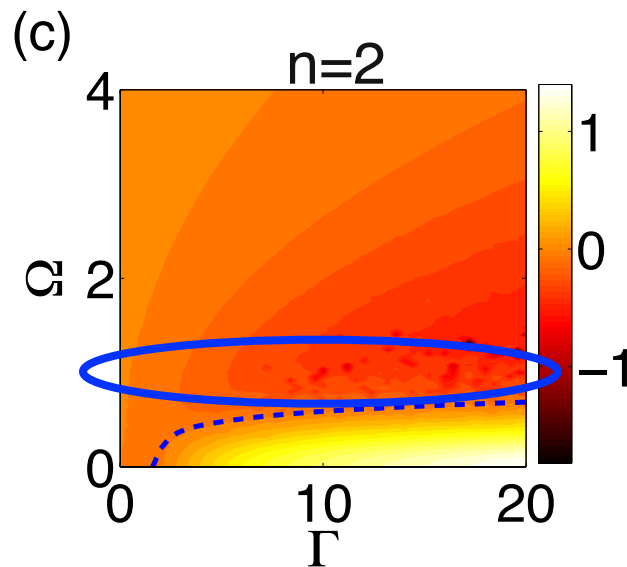
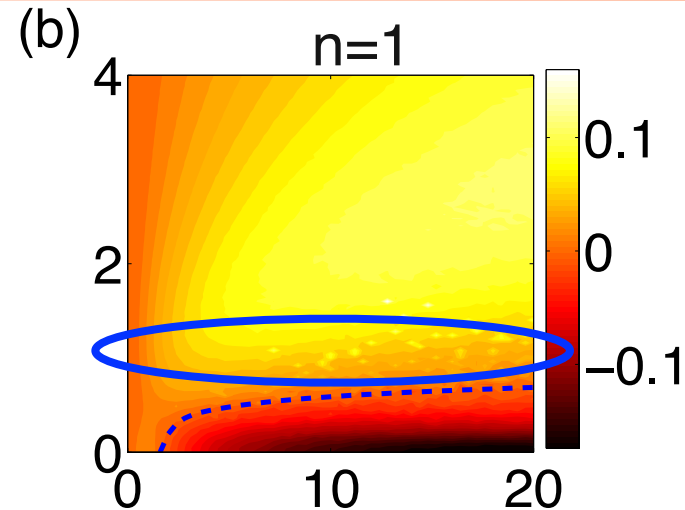
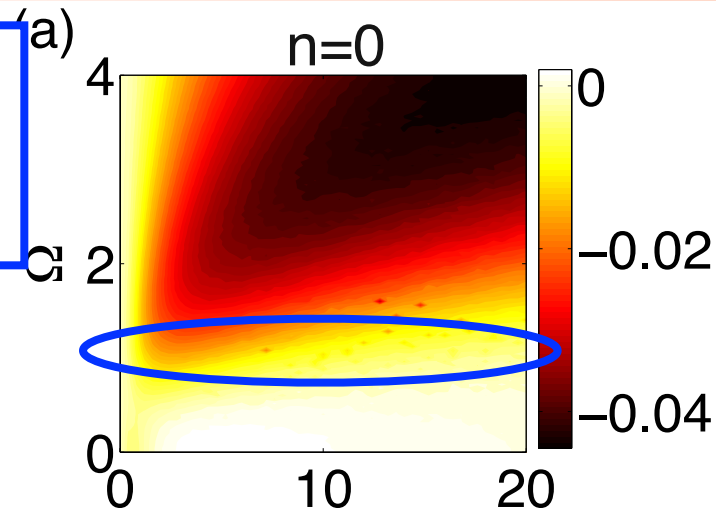
[Huaixiu Zheng]

Toward a Single Photon Source

Number statistics
of transmitted
pulse??

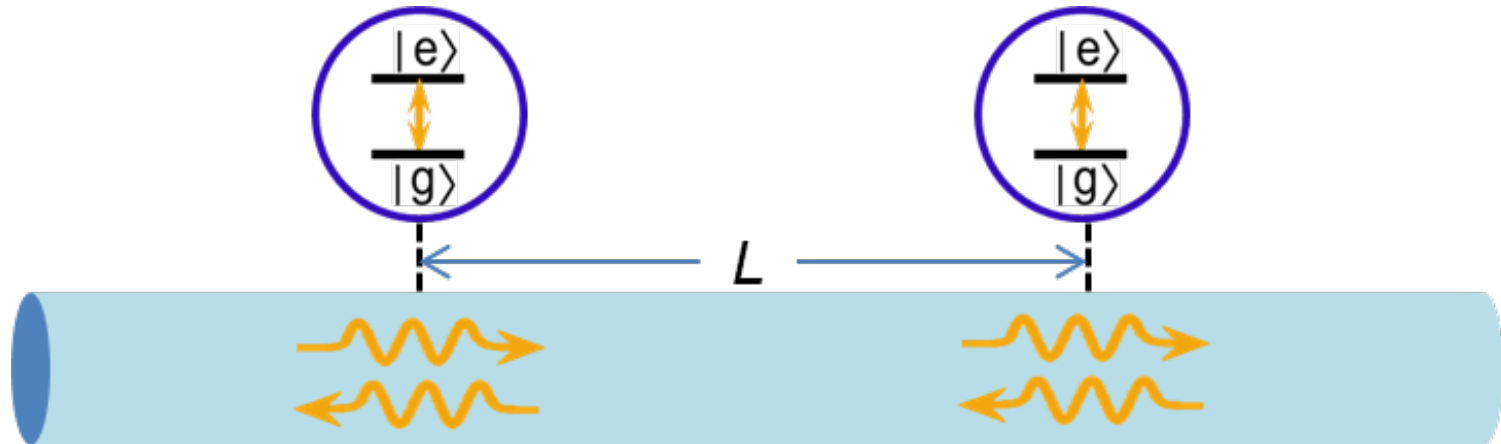
Inject coherent
state pulse with
1 photon on avg.

$$\log_{10} \left[\frac{P_n}{P_{n,\text{Poisson}}} \right]$$



sub-Poissonian statistics: **multi-photon states are suppressed**
single photon source important for secure quantum communication

Two-Qubit Problem



- Minimal system for scalable quantum networks
- Relatively unexplored for 1D waveguide case
- So far, no analytic solution found---in sharp contrast to single-qubit case

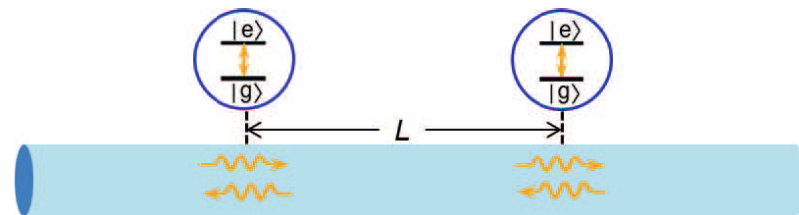
Numerical Green Function Method I

- Original Hamiltonian

$$H = \sum_{j=1,2} \hbar \omega_j \sigma_j^\dagger \sigma_j + H_{wg}$$

$$+ \sum_{j=1,2} \sum_{\alpha=R,L} \int dx \hbar V_j \delta(x - a_j) [a_\alpha^\dagger(x) \sigma_j + \text{h.c.}],$$

$$H_{wg} = \int dx \frac{\hbar c}{i} \left[a_R^\dagger(x) \frac{d}{dx} a_R(x) - a_L^\dagger(x) \frac{d}{dx} a_L(x) \right],$$



- Hamiltonian with bosonic sites

$$H = H_0 + V, \quad V = \sum_{j=1,2} \frac{U}{2} d_j^\dagger d_j (d_j^\dagger d_j - 1),$$

$$H_0 = \sum_{j=1,2} \hbar \omega_j d_j^\dagger d_j + H_{wg}$$

$$+ \sum_{j=1,2} \sum_{\alpha=R,L} \int dx \hbar V_j \delta(x - a_j) [a_\alpha^\dagger(x) d_j + \text{h.c.}],$$

$U \rightarrow \infty$



Numerical Green Function Method II

- Non-interacting scattering eigenstates

$$H_0|\phi\rangle = E|\phi\rangle$$

- Full interacting solution

$$|\psi\rangle = |\phi\rangle + G^R(E)V|\psi\rangle,$$
$$G^R(E) = \frac{1}{E - H_0 + i0^+}.$$

- Photon-photon correlation (g2)

$$g_2(t) = \frac{\langle\psi| a_\alpha^\dagger(x) a_\alpha^\dagger(x+ct) a_\alpha(x+ct) a_\alpha(x) |\psi\rangle}{\langle\psi| a_\alpha^\dagger(x) a_\alpha(x) |\psi\rangle \langle\psi| a_\alpha^\dagger(x+ct) a_\alpha(x+ct) |\psi\rangle}$$

$\alpha = R, L$

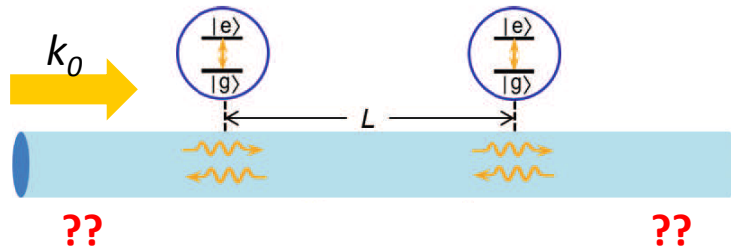
R: Right-going transmitted field

L: Left-going reflected field

Note: $g_2 = 1$ for product state or coherent state (classical)

Photon-Photon Correlation: Quantum Beats

- Weak incident coherent state with $ck_0 = \omega_1 = \omega_2$



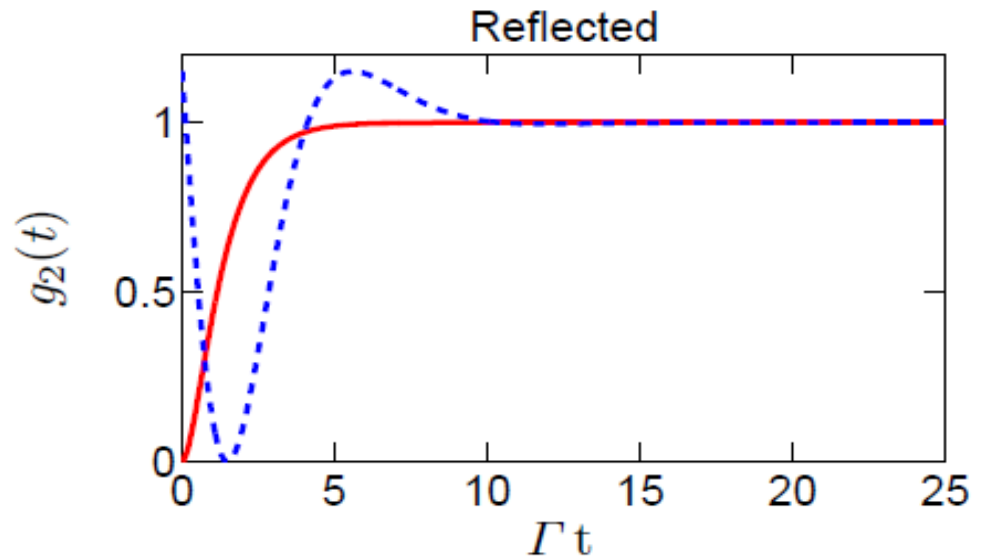
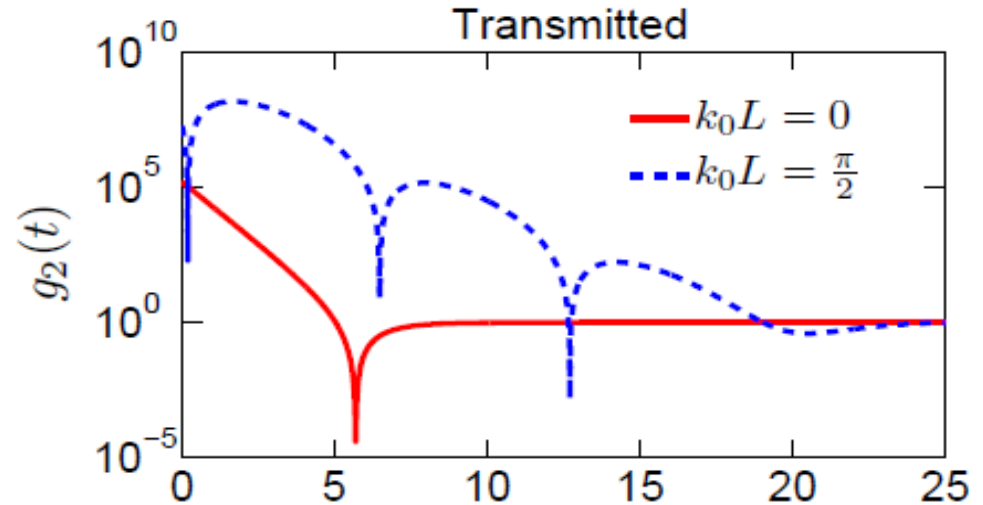
- $k_0L = 0$

Single-qubit behavior!

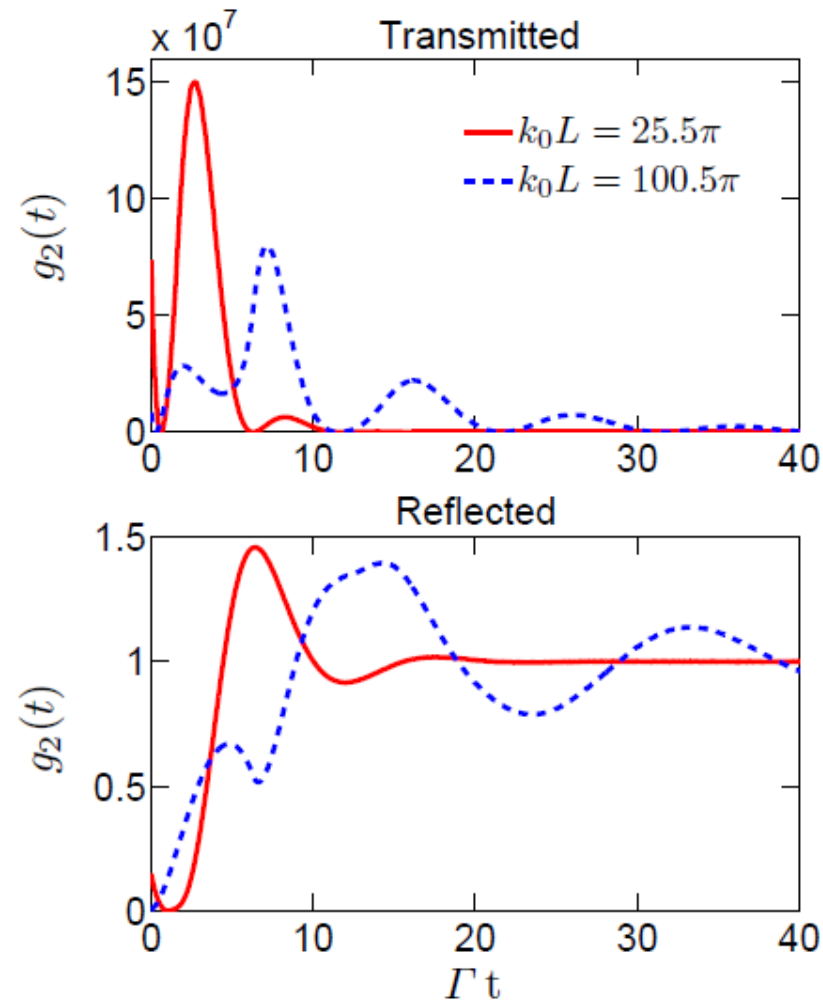
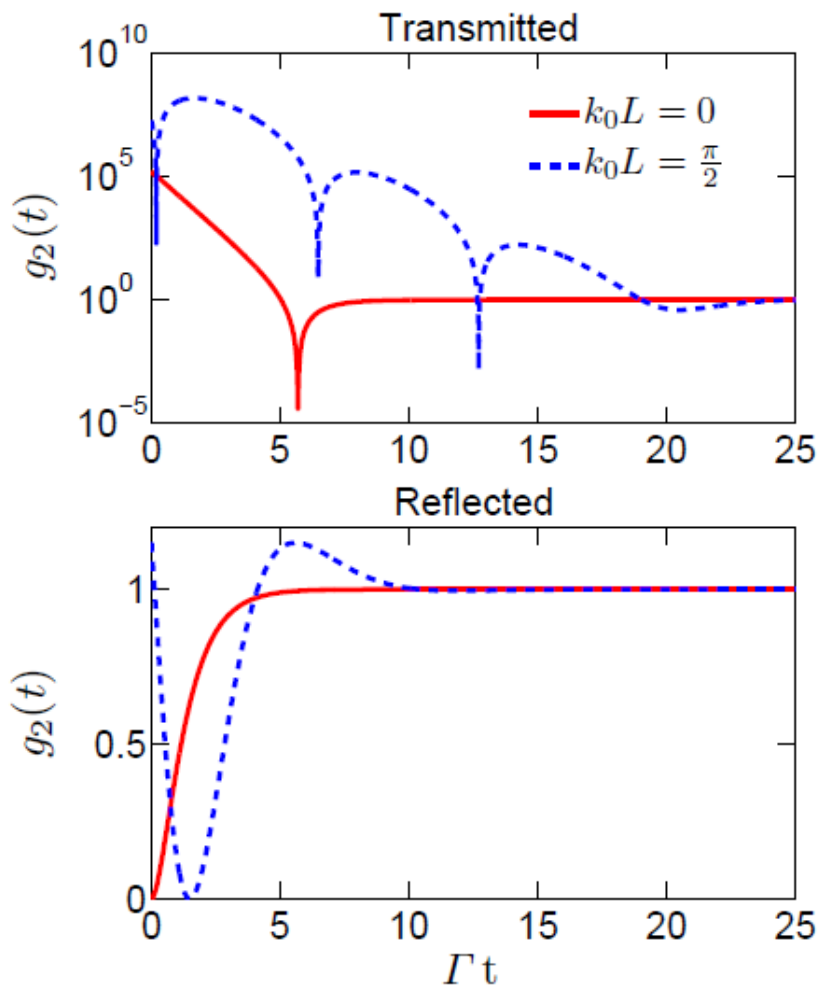
- $k_0L = \pi/2$

Quantum beats!

$$g_2(t) = \frac{|\# + \$|^2}{|\#|^2}$$

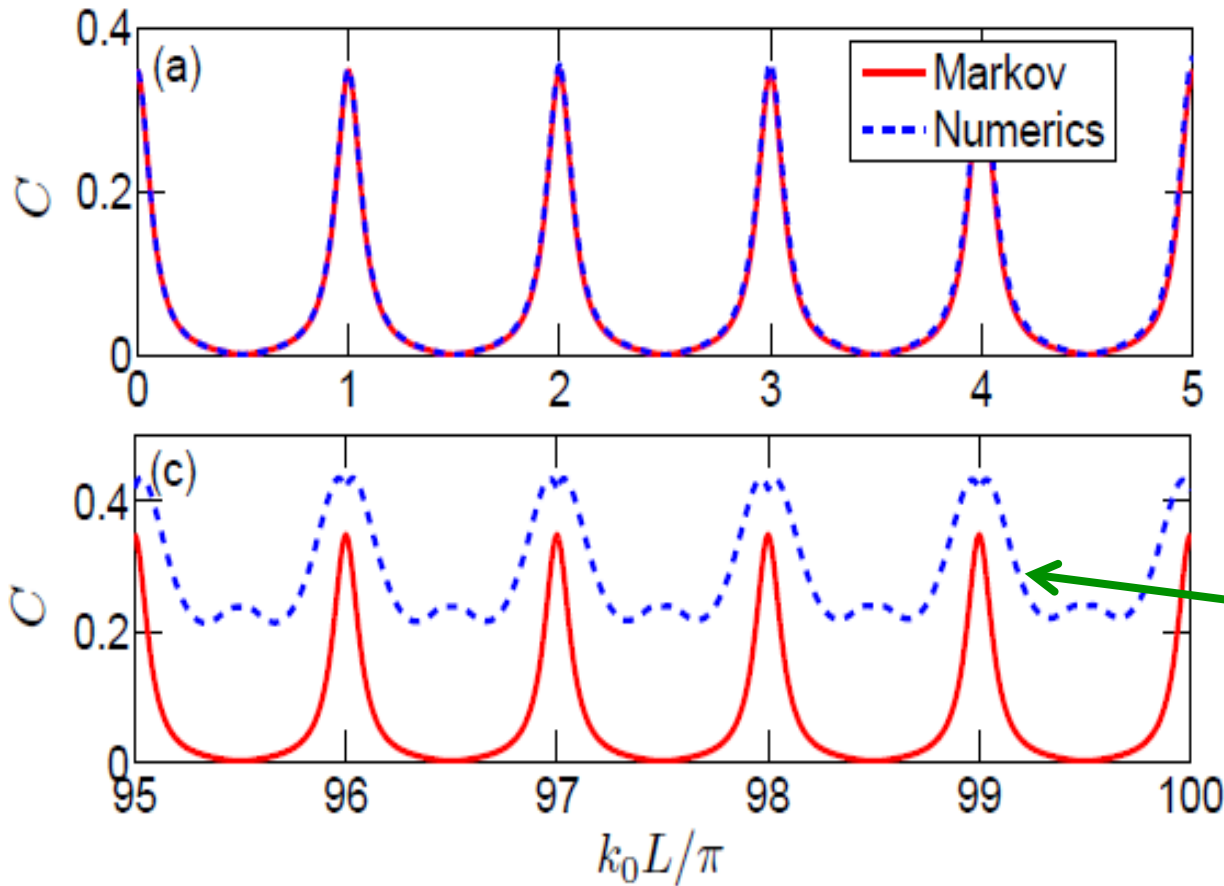


Markovian vs. Non-Markovian Regime



Dynamics on a very long time scale!

Qubit-Qubit Entanglement: Concurrence



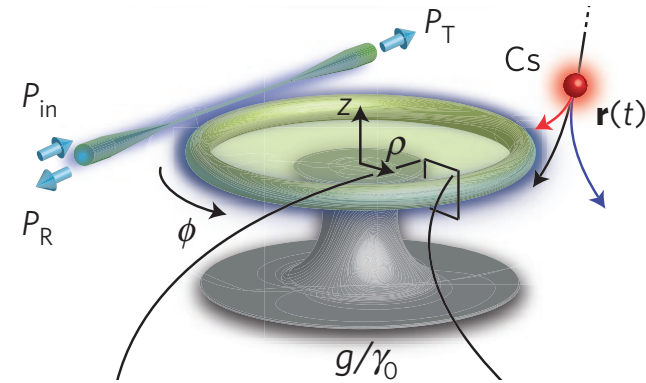
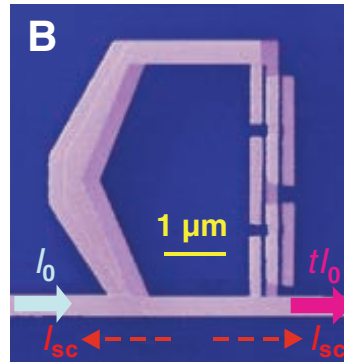
$C=0$: product state
 $C=1$: maximally entangled state

$$\Omega_1 = 0.1\Gamma; \Omega_2 = 0$$

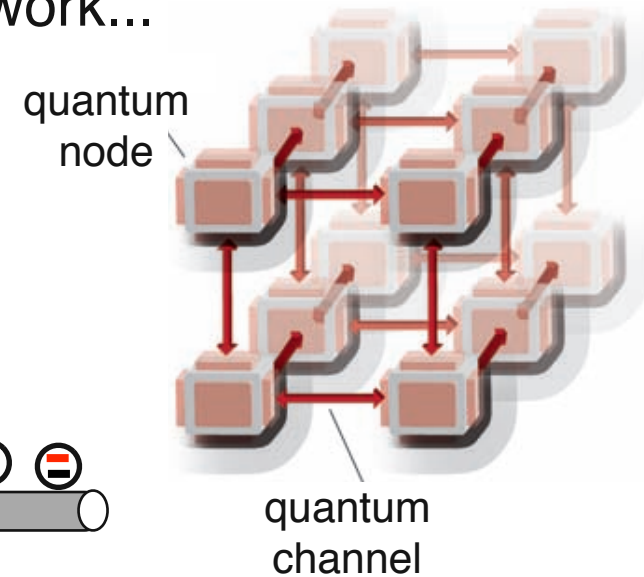
Long-distance entanglement!

Future Directions

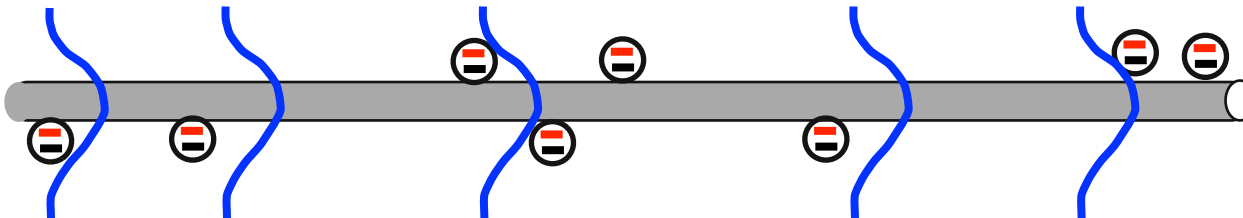
1. best way to observe correlated photon effects?



2. quantum gates: transfer q. information from qubit to photon
-- key building block of a quantum network...
-- photon-qubit pi-phase gate possible



3. effect of many 2LS?
-- correlations of many photons, QPT?



CONCLUSIONS

**Waveguide QED → interactions between photons
→ interest for q.optics, q.information, and QPT**

➔ **1D waveguide + strongly coupled 2LS or 4LS**

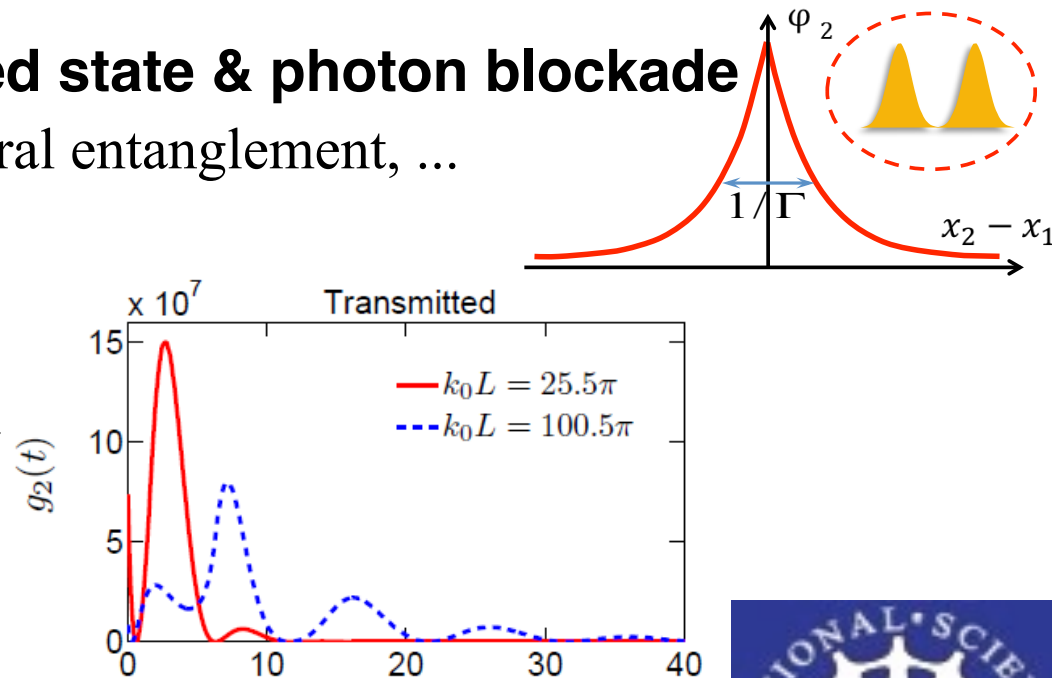
- effective photon-photon interaction

➔ **photon-photon correlated state & photon blockade**

- g_2 , number statistics, spectral entanglement, ...

➔ **two qubit system**

- persistent quantum beats,
long-distance entanglement



Huaixiu Zheng, Dan Gauthier, and HUB

PRA 2010; PRL 2011; PRA 2012; arXiv 2012

Fitzpatrick Institute
for Photonics

