

Quantum Impurity Physics with Microwave Photons

Moshe Goldstein (Simons Fellow @ Yale),

Michel Devoret (Yale),

Manuel Houzet (CEA, Grenoble),

Leonid Glazman (Yale)

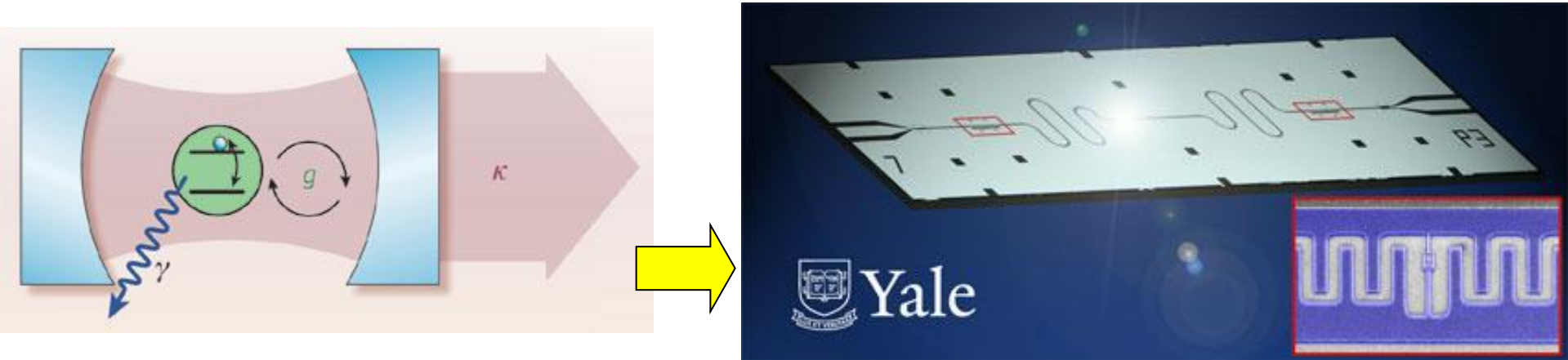


Outline

- Introduction
 - Quantum impurities
 - Microwave photons
- System and relation to anisotropic **Kondo**
- **AC conductance**: Photon **elastic** scattering
- Photon **inelastic** scattering

Circuit QED

- **Quantum optics** with **microwave** circuits:



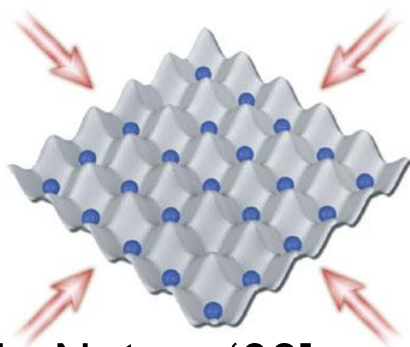
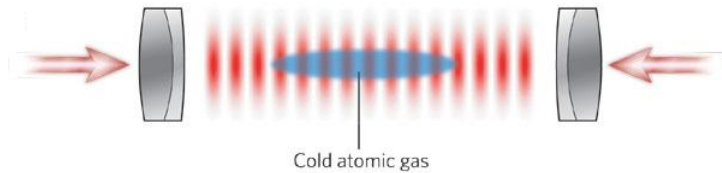
[Scholkoepf and Girvin, Nature '08]

- optical cavity ► microwave resonator
- atom ► qubit
- **Small** “cavity”, **large** “atom” ► **strong** light-matter interaction

Many Body Physics

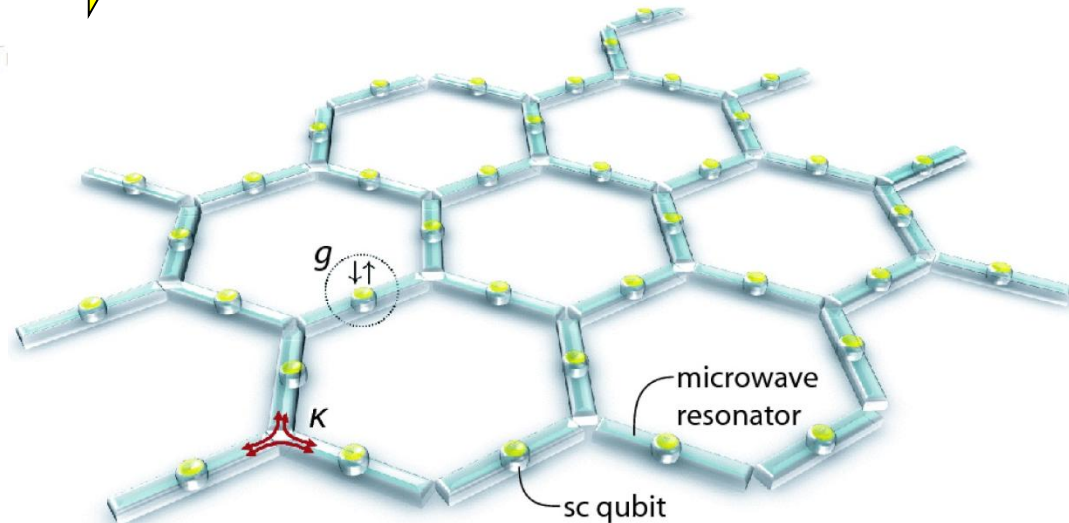
- Controllable **simulators** of **many-body** physics

Ultracold atoms in an optical lattice



[Bloch, Nature '08]

Microwave photons in a circuit:



[Koch et al., PRA '10]

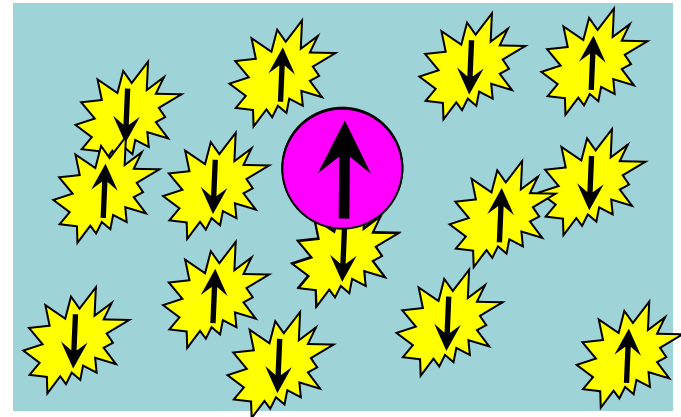
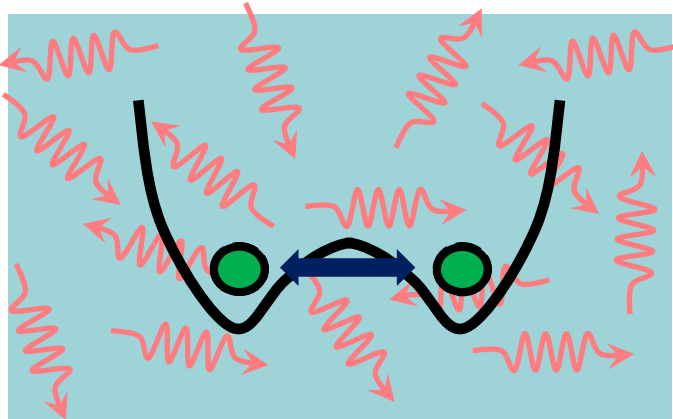
- Could we start with something simpler?**

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Quantum Impurities

- **Small** system coupled to quantum **environment**
 - **2 level system** in a **bosonic bath** (**spin-boson**)
 - **Magnetic impurity** in a **Fermi sea** (**Kondo**)

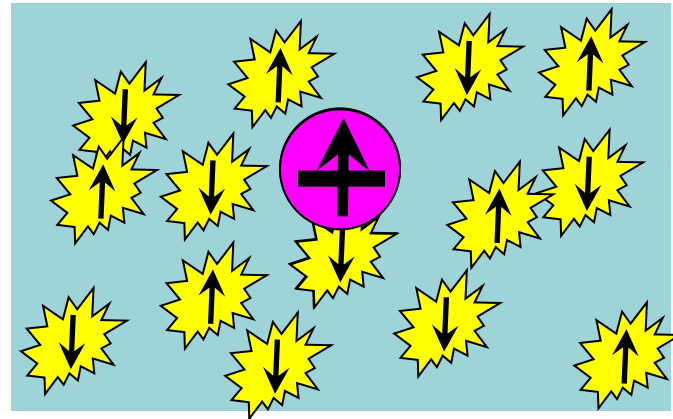


- Easy to study:
 - **Experimentally**: **nanophysics** (QDs, nanograins, ...)
 - **Theoretically** (RG, bosonization, CG, CFT, Bethe, NRG, DMRG, ...)
- Teach us about:
 - **Strong correlations** (asymptotic freedom, quantum phase transitions, non Fermi liquid, ...)
 - **Nanophysics** and **quantum computation** (qubits)

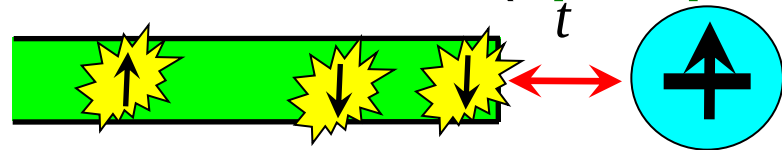
Example: Kondo

- Realizations

- Magnetic impurity



- Quantum dot with **odd** electron number (**spin qubit**)



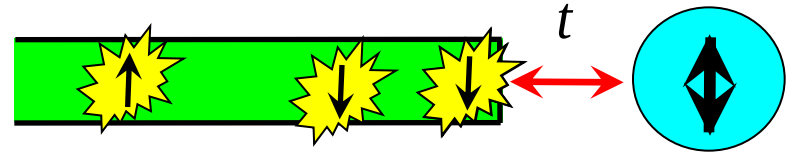
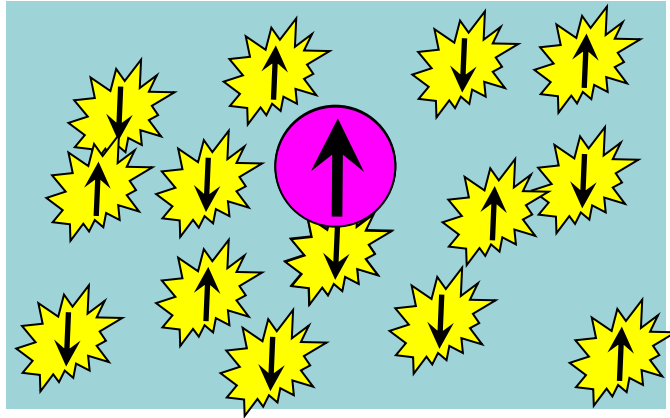
- Anderson impurity model:

$$H = H_{\text{env}}^0 + \left(\varepsilon + \frac{B_z}{2} \right) n_{\uparrow} + \left(\varepsilon - \frac{B_z}{2} \right) n_{\downarrow} + U n_{\downarrow} n_{\uparrow} + t \left[c_{\uparrow}^{\dagger} \psi_{\uparrow}(x=0) + c_{\downarrow}^{\dagger} \psi_{\downarrow}(x=0) + \text{H.c.} \right]$$

$$H_{\text{env}}^0 = \sum_{k,s} \varepsilon_k c_{k,s}^{\dagger} c_{k,s} \quad \psi_s(0) = \sum_k c_{k,s}$$

$\Gamma = \pi v |t|^2$: level width

The Kondo Problem



- **Local moment regime** [$\Gamma \ll \varepsilon + U, |\varepsilon|$] ► **Kondo model:**

$$H_K = H_{\text{env}}^0 + \frac{I_{xy}}{2} [S_+ s_-(0) + S_- s_+(0)] + I_z S_z s_z(0) + B_z S_z$$

Impurity spin: $\vec{S}$

Environment spin: $\vec{s}(0) = \frac{1}{2} \sum_{k,k',s,s'} c_{k,s}^+ \vec{\sigma}_{s,s'} c_{k',s'}$

$I_{xy}=I_z=I \sim t^2/U > 0$: exchange
 B_z : local magnetic field

- Problem: **divergences** [Kondo '64]
 - Example: **susceptibility**

$$\chi \sim \frac{1}{T} \left[1 + (vI) \ln \left(\frac{\omega_0}{T} \right) + (vI)^2 \ln^2 \left(\frac{\omega_0}{T} \right) + \dots \right]$$

ω_0 : bandwidth



Kondo Physics

- RG equations** ($\mathbf{B}_z=0$)

[Anderson]:

$$\frac{d(vI_z)}{d \ln \omega_0} = -v^2 I_{xy}^2$$

$$\frac{d(vI_{xy})}{d \ln \omega_0} = -v^2 I_z I_{xy}$$

ω_0 : bandwidth
 v : local DoS

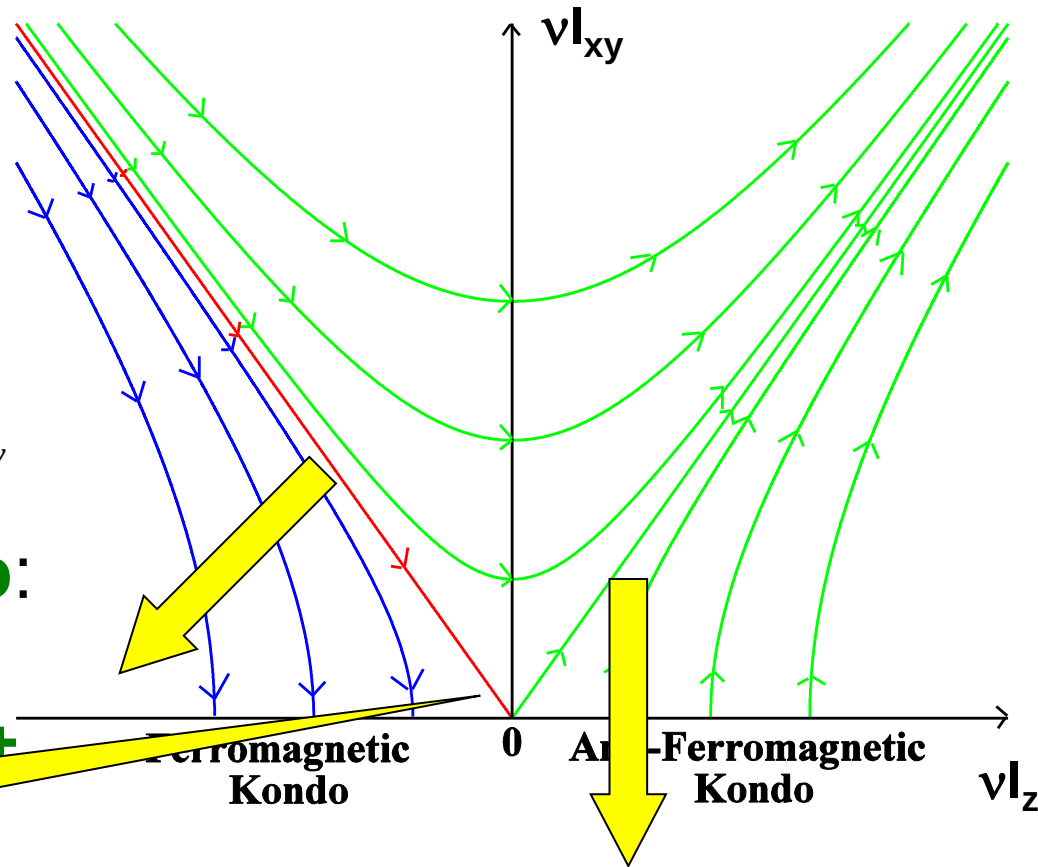
- Ferromagnetic Kondo:**

- impurity **decoupled**
- susceptibility: $\chi_{zz} \sim c(I)/T +$

Kosterlitz-Thouless transition

Antiferromagnetic Kondo:

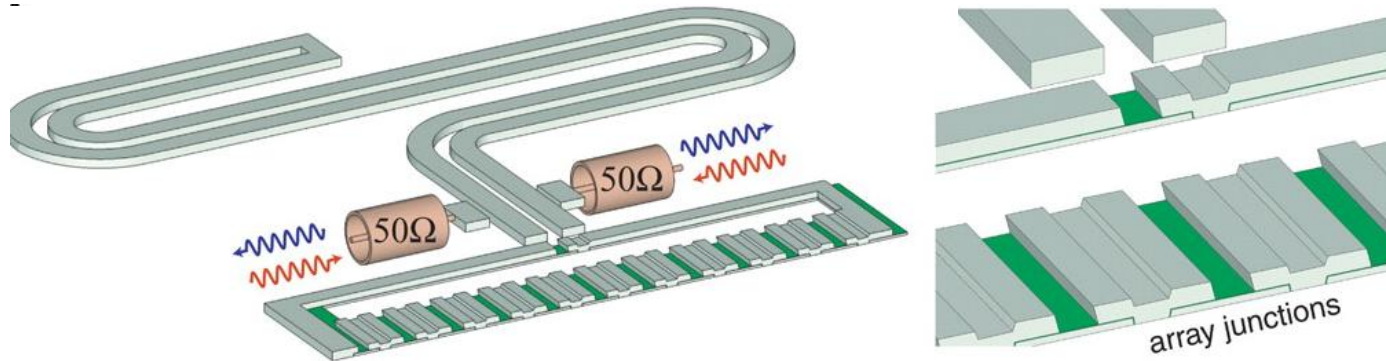
- impurity **strongly-coupled** (asymptotic freedom)
 - susceptibility: $\chi_{zz} \sim 1/T_K + \dots$ (**Fermi liquid**)
- T_K : Kondo temperature $T_K = \omega_0 \exp[-1/(vI)]$



Outline

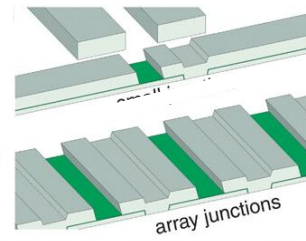
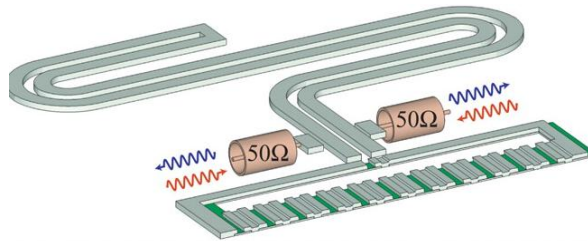
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Superconducting Grain Array



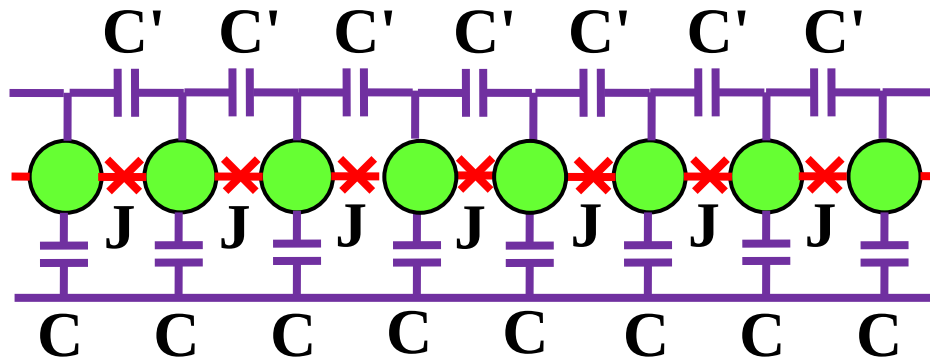
[Manucharyan et al.,
Science '09]

Superconducting Grain Array

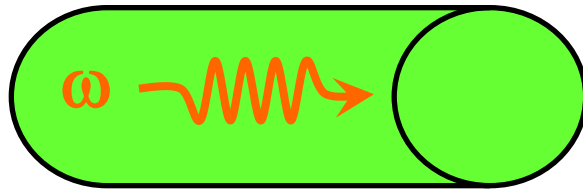


[Manucharyan et al.,
Science '09]

Assuming $C \gg C'$



$$H_{\text{lead}} = \sum_i \left\{ J \cos(\phi_i - \phi_{i+1}) + \frac{Q_i^2}{2C} \right\}$$



$$H_{\text{lead}} = \frac{v}{2\pi} \int \left\{ g [\partial_x \phi(x)]^2 + g^{-1} [\pi \rho(x)]^2 \right\} dx$$

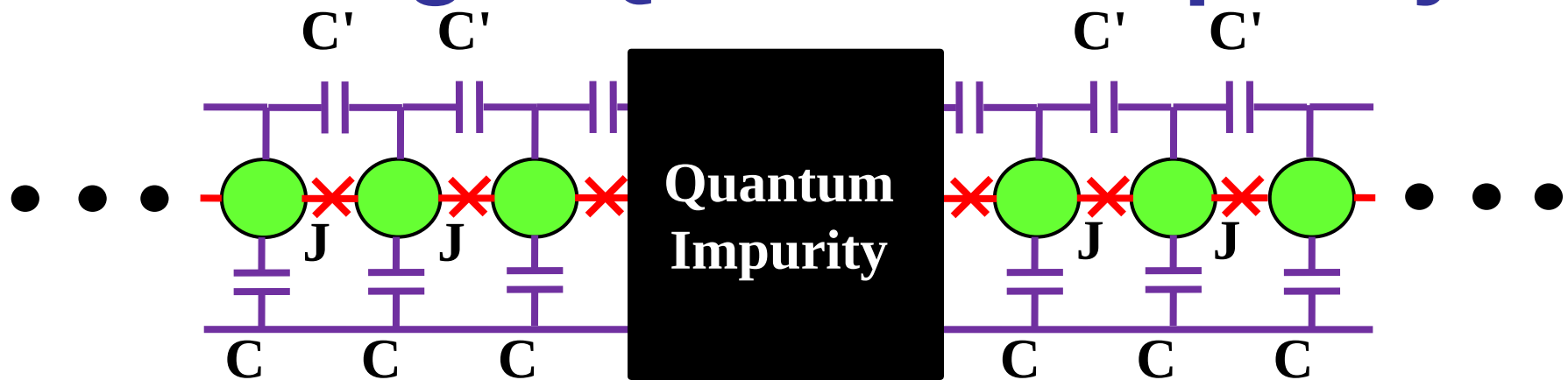
$$i(x) \propto \partial_x \phi(x)$$

Velocity: $v/a = \omega_0 = \sqrt{C/J}$

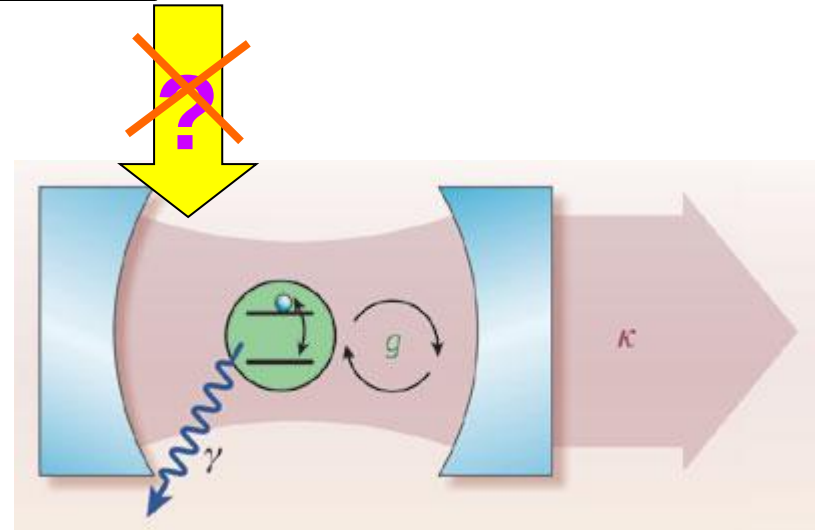
Admittance: $g = \frac{\pi \sqrt{JC}}{e^*} = \frac{R_Q}{2Z}$

- Waveguide for microwave photons
- Usually $g \gg 1$, but $g \sim 1$ possible
- [$g < 1$: Glazman & Larkin, PRL '97]

Adding a Quantum Impurity

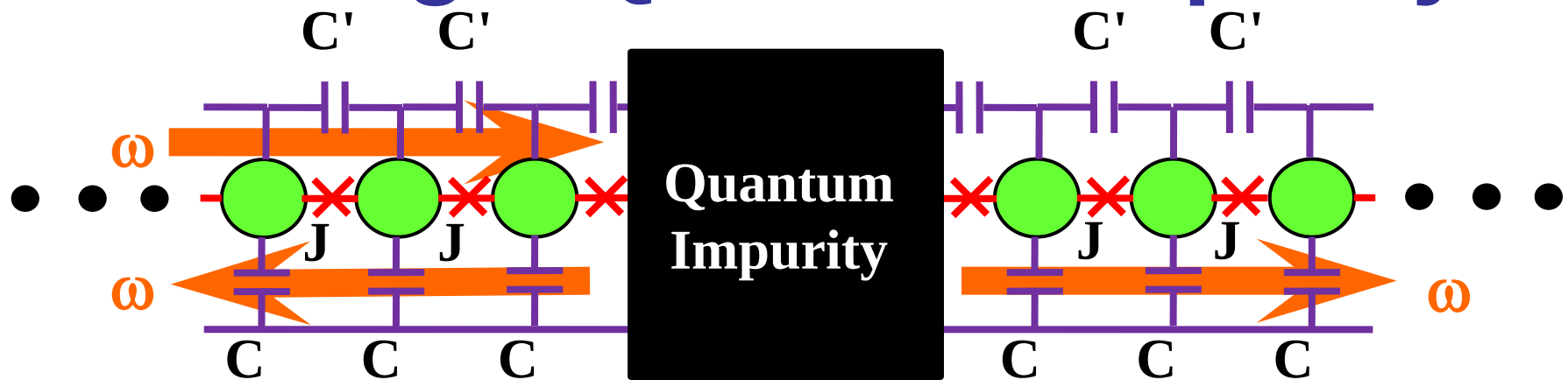


- **Artificial atom** in microwave waveguide



- Motivations:
 - **Quantum optics** ► **many-body** effects
 - **Condensed matter** ► **bosons**

Adding a Quantum Impurity



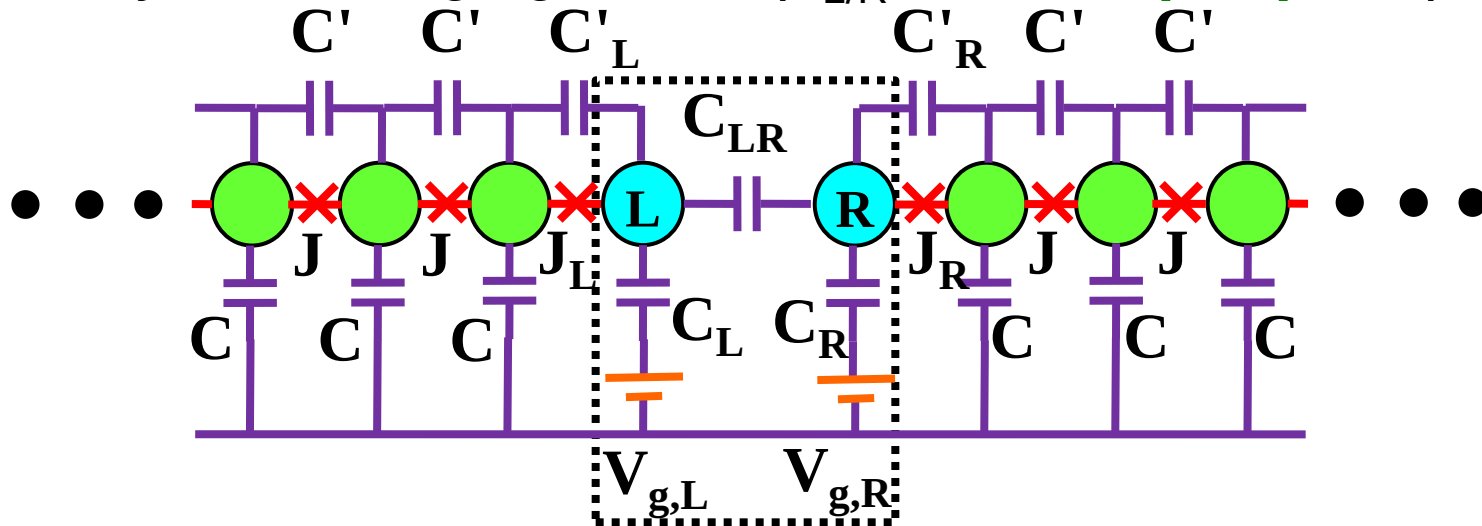
- **Transport Measurement:**
 - **Charge** ► **conserved**
 - **Energy** ► **not conserved** (dissipation)
- **Where does energy go?**
 - **Photons** at **different frequencies!**

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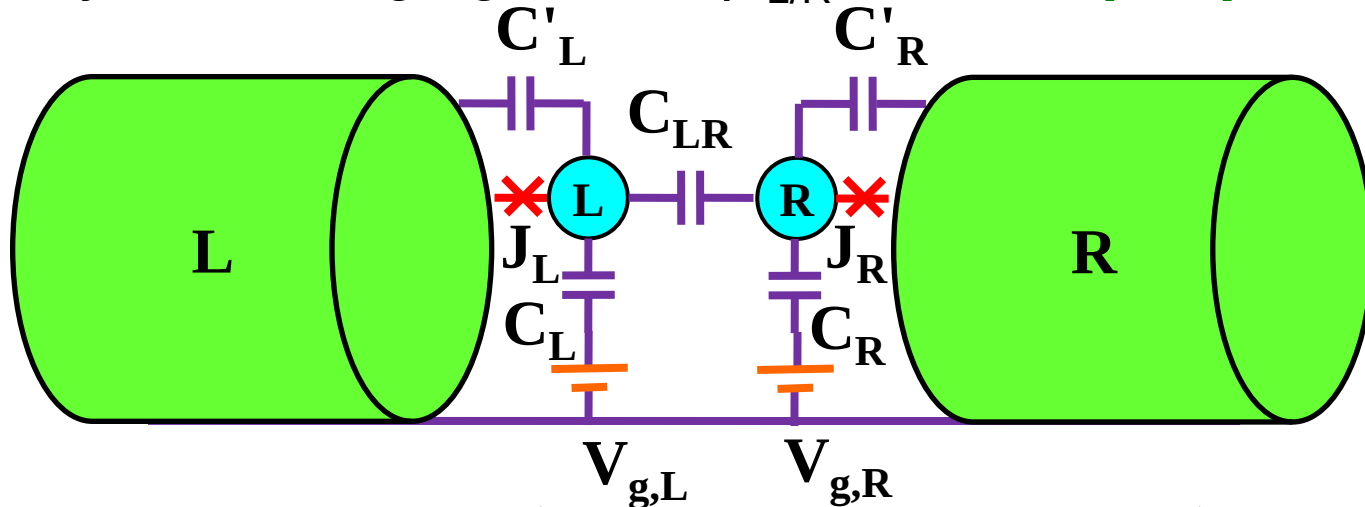
System

- Quantum impurity: two **capacitively** coupled grains, **weakly** coupled ($J_{L/R} \ll J$) to the leads
 - Only **two** charging states ($n_{L/R} = 0, 1$ **Cooper pairs**)



System

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 - Only **two** charging states ($n_{L/R} = 0, 1$ **Cooper pairs**)

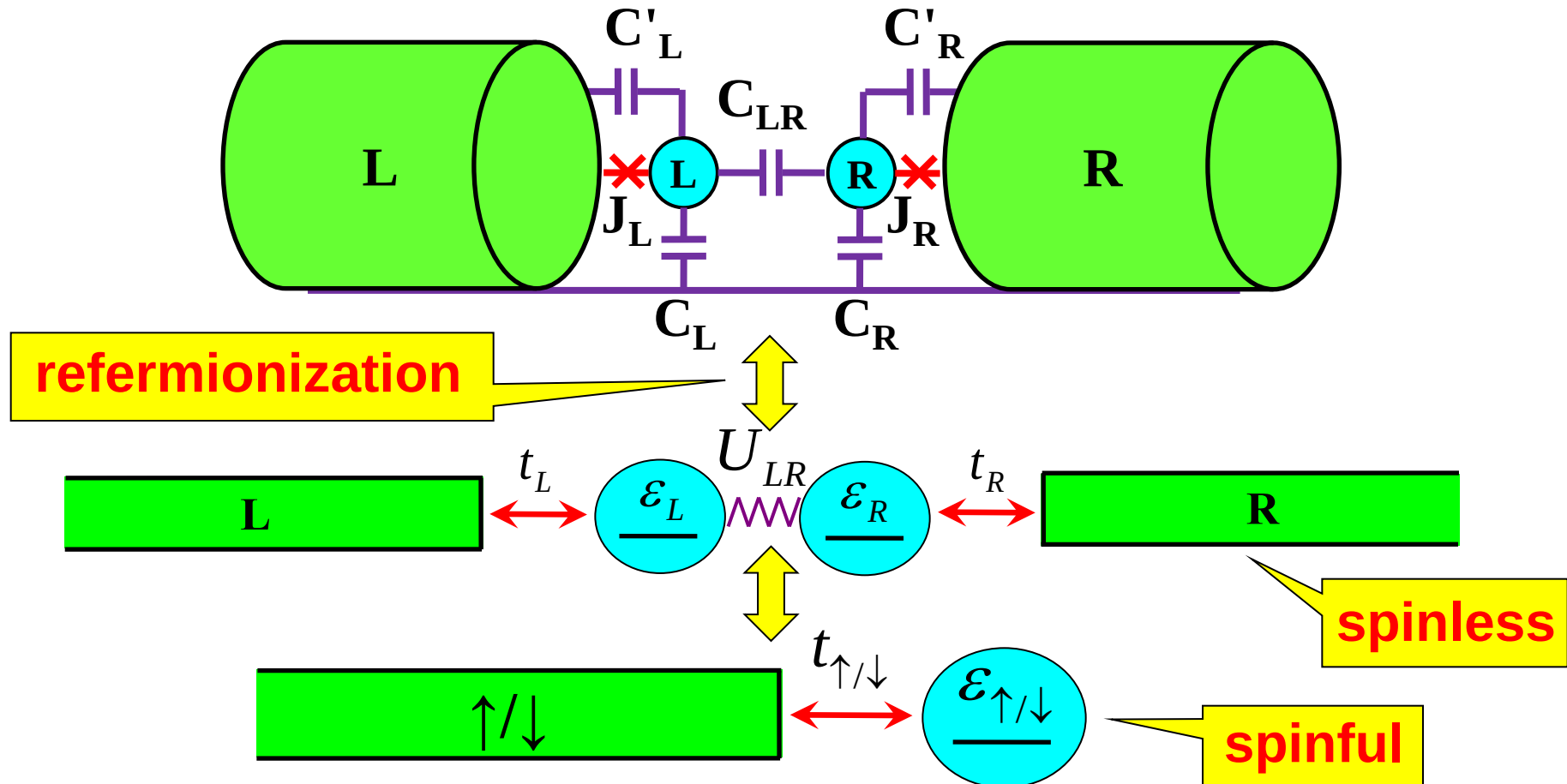


$$H_{\text{lead}} = \frac{v}{2\pi} \sum_{\ell=L,R} \int \left\{ g [\partial_x \phi_\ell(x)]^2 + g^{-1} [\pi \rho_\ell(x)]^2 \right\} dx$$

$$H_{\text{imp}} = \sum_{\ell=L,R} \varepsilon_\ell n_\ell + U_{LR} n_L n_R$$

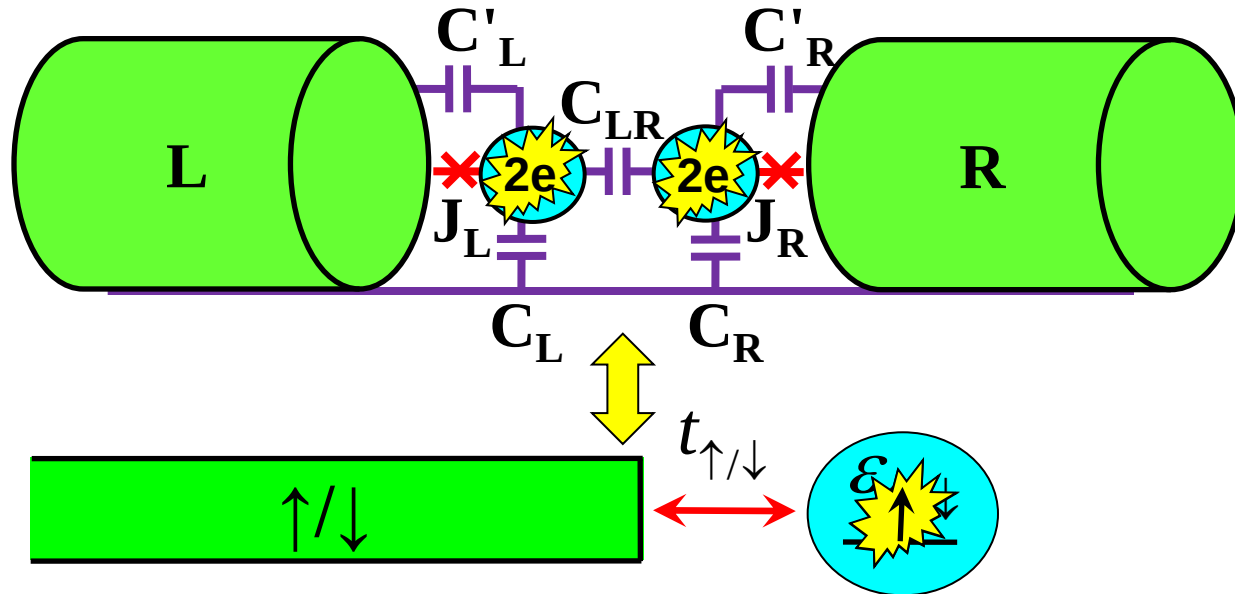
$$H_{\text{lead-imp}} = \sum_{\ell=L,R} \left\{ U_\ell n_\ell \rho_\ell(0) + J_\ell |1_\ell\rangle \langle 0_\ell| e^{-i\phi_\ell(0)} + \text{H.c.} \right\}$$

Relation with Anderson Impurity



- **Generalized Anderson impurity model:**
 - Spin **anisotropy**
 - **Luttinger liquid** ($g \neq 1$)
 - Level-lead **interaction**

Kondo Description



- In **Coulomb blockade** valley – “**local moment**” regime $\{n_L + n_R \approx 1\}$:
 - **Singly occupied** states – “**spin**” $\{S_z = (n_L - n_R)/2\}$
 - Equivalent to **Kondo** with **noninteracting lead**:

$$H_K = H_{\text{env}}^0 + \frac{I_{xy}}{2} [S_+ s_-(0) + S_- s_+(0)] + I_z S_z s_z(0) + B_z S_z$$

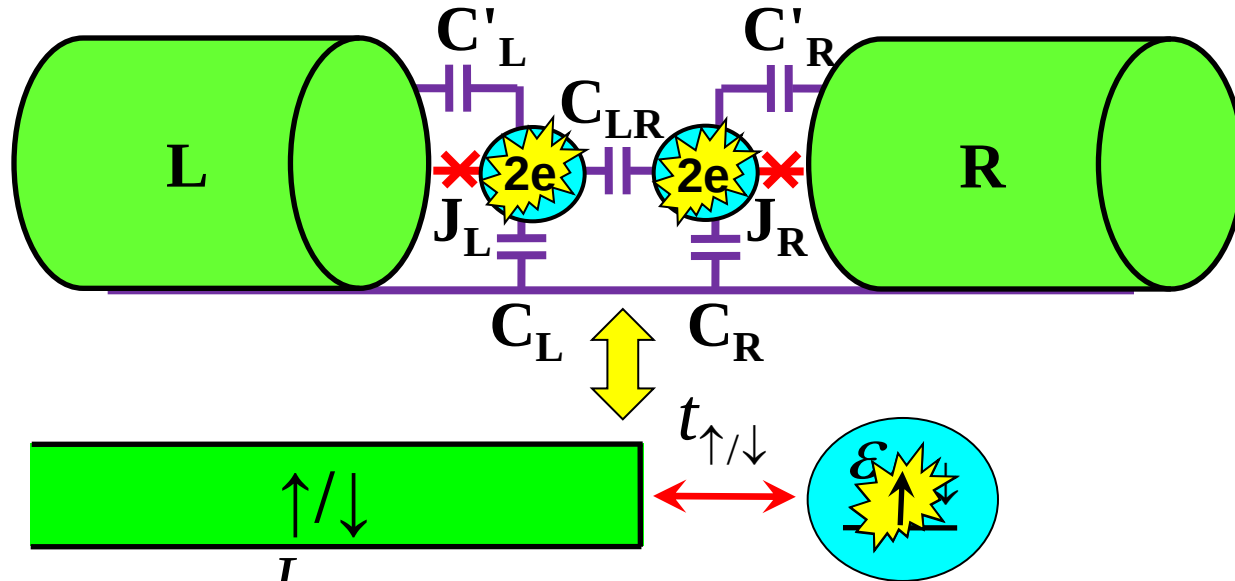
$$H_{\text{env}}^0 = \sum_{k,s} \epsilon_k c_{k,s}^+ c_{k,s}$$

Impurity spin: $\vec{S}$

Environment spin:

$$\vec{s}(0) = \frac{1}{2} \sum_{k,k',s,s'} c_{k,s}^+ \vec{\sigma}_{s,s'} c_{k',s'}$$

Kondo Parameters

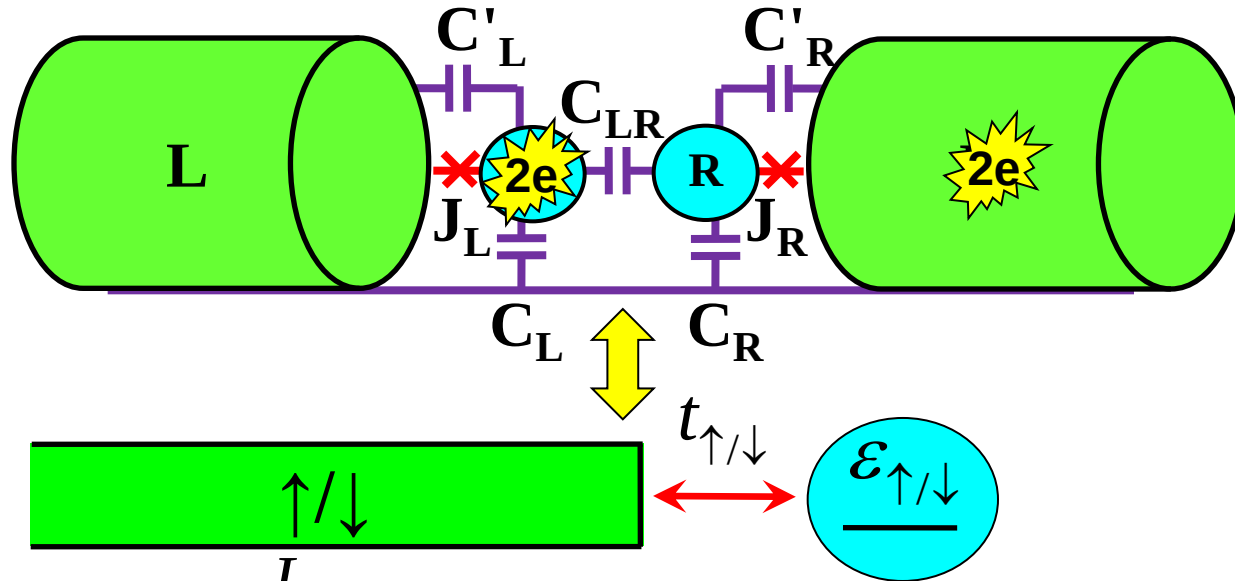


$$H_K = H_{\text{env}}^0 + \frac{I_{xy}}{2} [S_+ s_-(0) + S_- s_+(0)] + I_z S_z s_z(0) + B_z S_z$$

- Schrieffer-Wolf** (simplified expressions):

$$B_z \approx \epsilon_L - \epsilon_R \rightarrow V_{g,L} - V_{g,R}$$

Kondo Parameters



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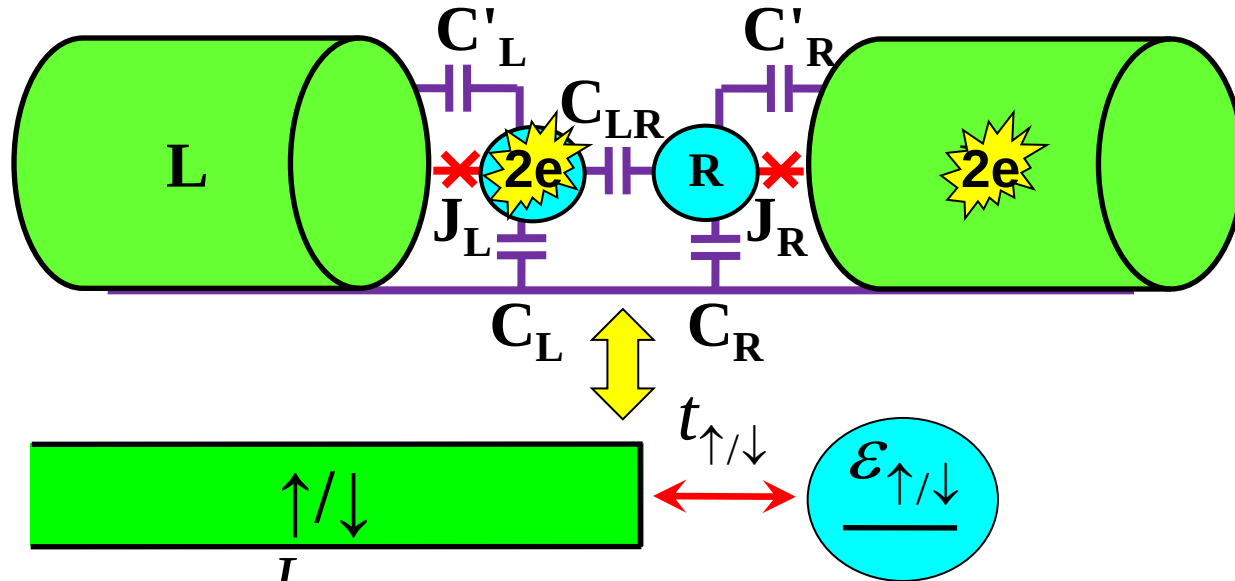
- Schrieffer-Wolf** (simplified expressions):

$$B_z \approx \varepsilon_L - \varepsilon_R \rightarrow V_{g,L} - V_{g,R}$$

$$\nu I_{xy} \approx 2 \frac{J_L J_R}{\omega_0} \left(\frac{1}{|\varepsilon_0|} + \frac{1}{\varepsilon_0 + U_{LR}} \right)$$

$\varepsilon_0 \equiv (\varepsilon_L + \varepsilon_R)/2$
 ω_0 – bandwidth
 ν – local DoS

Kondo Parameters



$$H_K = H_{\text{env}}^0 + \frac{I_{xy}}{2} [S_+ s_-(0) + S_- s_+(0)] + I_z S_z s_z(0) + B_z S_z$$

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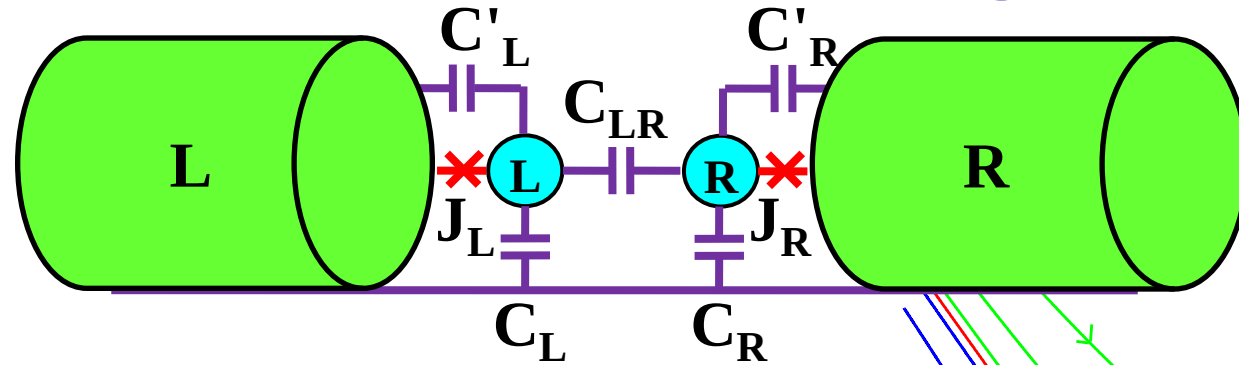
$$\nu_{I_z} \approx 1 - \frac{\alpha_L^2 + \alpha_R^2}{2} + \sum_{\ell=L,R} \frac{J_\ell^2}{\omega_0} \left(\frac{1}{|\varepsilon_\ell|} + \frac{1}{\varepsilon_\ell + U_{LR}} \right)$$

$$\varepsilon_0 \equiv (\varepsilon_L + \varepsilon_R)/2$$

ω_0 – bandwidth
 v – local DoS

$$\alpha_\ell = \frac{1}{\sqrt{g}} \left(1 - \frac{g U_\ell}{\pi v} \right)$$

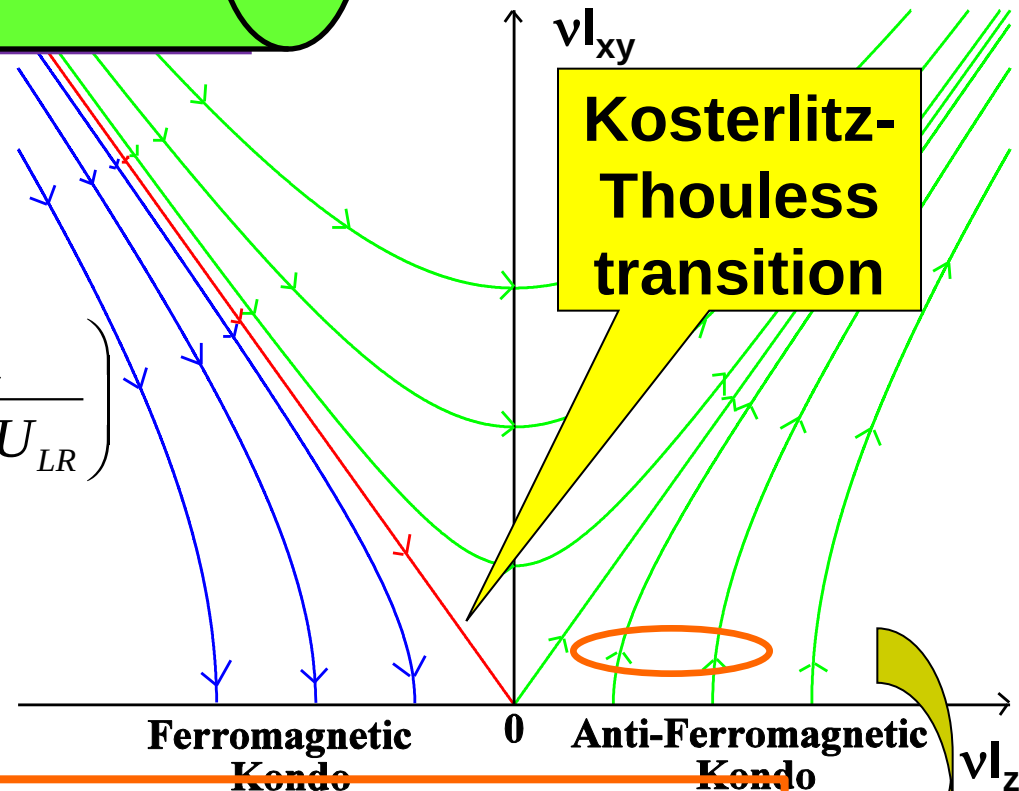
Relevant Regime for SC



$$vI_{xy} \approx 2 \frac{J_L J_R}{\omega_0} \left(\frac{1}{|\varepsilon_0|} + \frac{1}{\varepsilon_0 + U_{LR}} \right)$$

$$vI_z \approx 1 - \frac{\alpha_L^2 + \alpha_R^2}{2} + \sum_{\ell=L,R} \frac{J_\ell^2}{\omega_0} \left(\frac{1}{|\varepsilon_\ell|} + \frac{1}{\varepsilon_\ell + U_{LR}} \right)$$

$$\alpha_\ell = \frac{1}{\sqrt{g}} \left(1 - \frac{gU_\ell}{\pi v} \right)$$



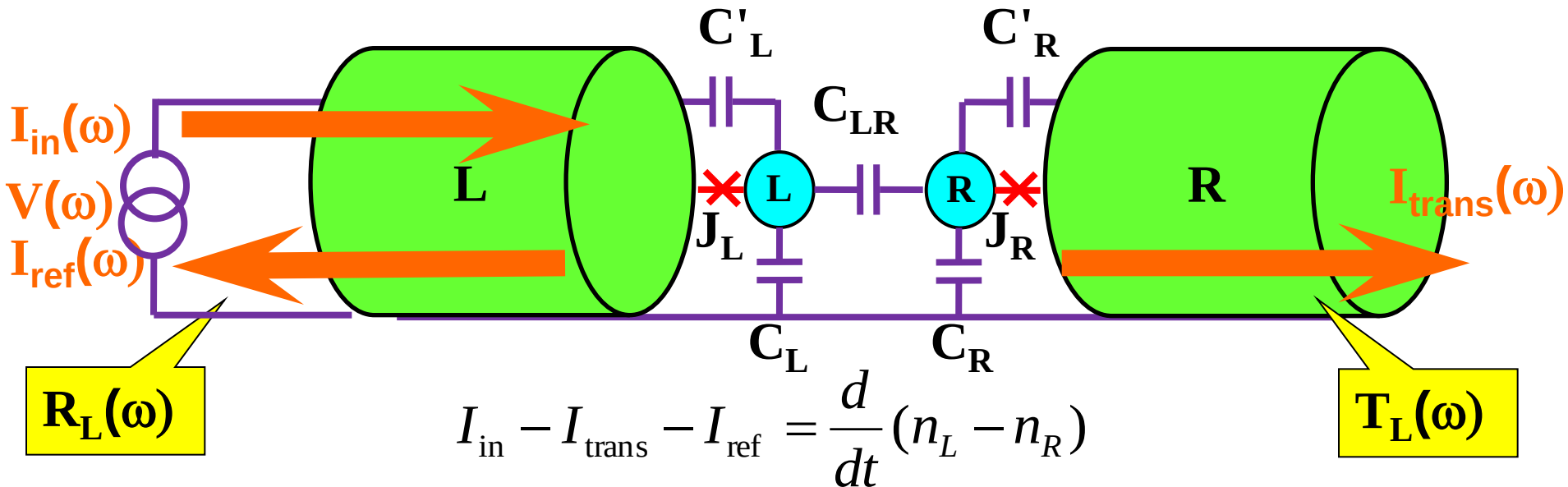
• Typically: $g \gg 1$, $U_{L/R} > 0$ $\blacktriangleright$ $I_z \gg I_{xy}$, **AFM Kondo**

• $g < 1$ possible [Glazman & Larkin, '97] $\blacktriangleright$ $I_z < -I_{xy}$, **FM Kondo**

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AC conductance



- Current **transmission coefficient**:

$$T_L(\omega) \equiv \frac{I_{\text{trans}}(\omega)}{I_{\text{in}}(\omega)} = \frac{G(\omega)}{G_0(\omega)} = \frac{\langle\langle i(x_{\text{in}}); i(x_{\text{out}}) \rangle\rangle_{\omega}}{\langle\langle i(x_{\text{in}}); i(x_{\text{out}}) \rangle\rangle_{\omega}^{(0)}}$$

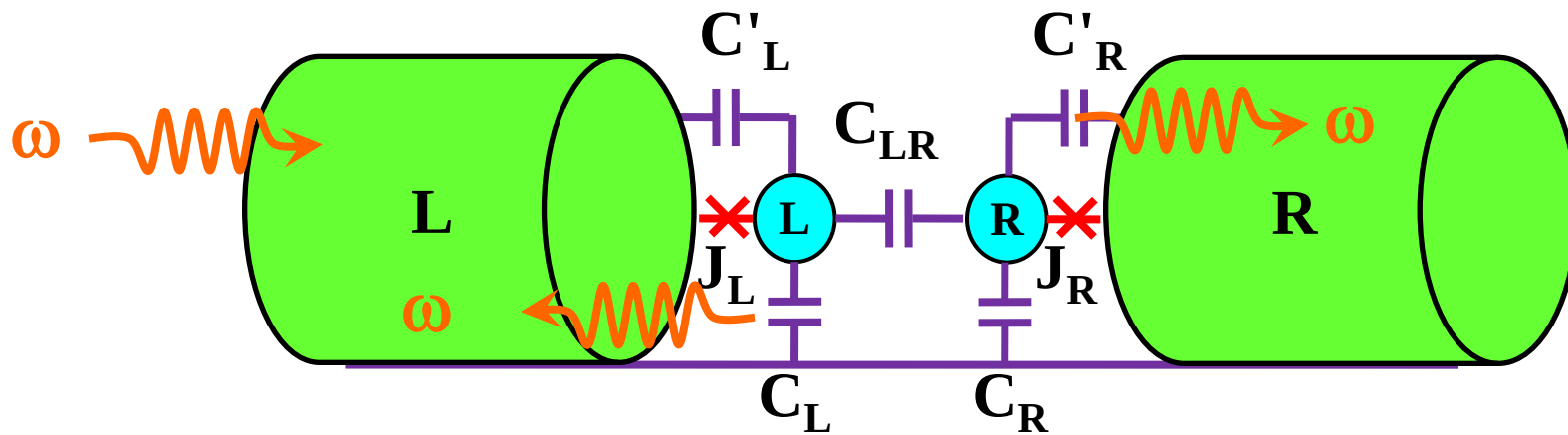
– Similarly for **reflection**

linear response

$$i(x) \propto \partial_x \phi(x)$$

$$(x_{\text{in}} < 0, x_{\text{out}} > 0)$$

Elastic Scattering



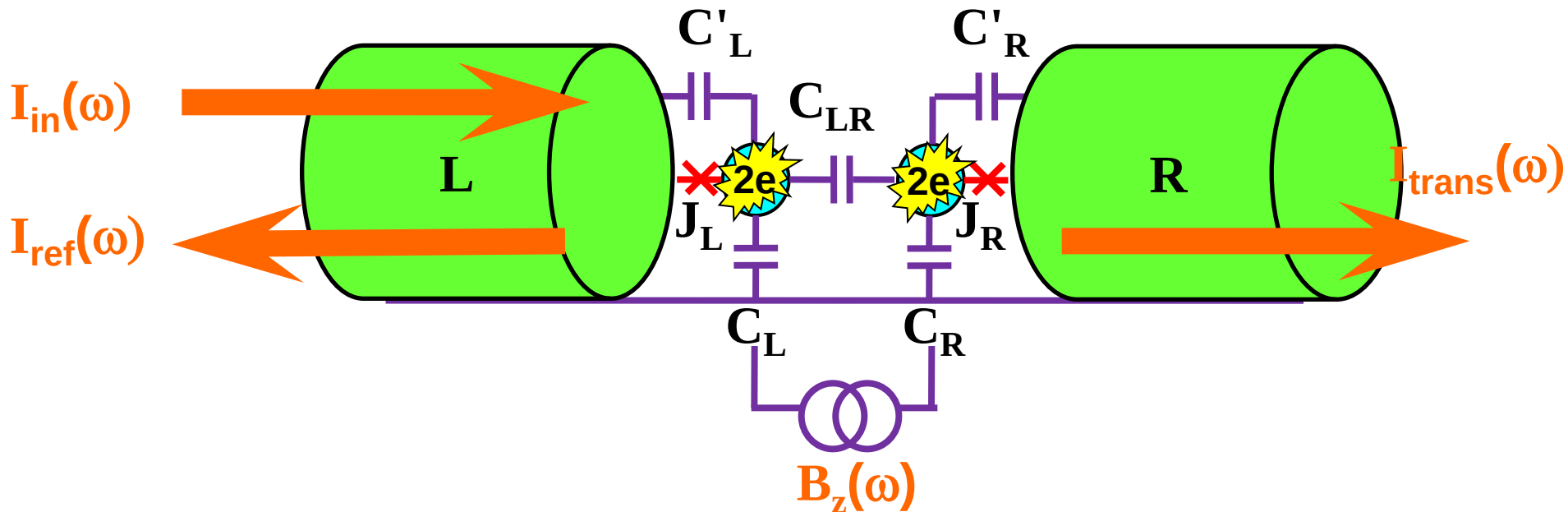
- Elastic T matrix:**

$$2\pi i \hat{T}_{\ell'\ell}^{\text{el}}(\omega) = \begin{bmatrix} R_L(\omega) - 1 & T_R(\omega) \\ T_L(\omega) & R_R(\omega) - 1 \end{bmatrix} \quad \ell, \ell' = L, R$$

– Generalization (**finite temperature**) of **photon elastic scattering amplitudes**:

$$\hat{\mathcal{G}}_{\text{ph}}(x, x'; \omega) = \hat{\mathcal{G}}_{\text{ph}}^{(0)}(x, x'; \omega) + \hat{\mathcal{G}}_{\text{ph}}^{(0)}(x, 0; \omega) \hat{T}^{\text{el}}(\omega) \hat{\mathcal{G}}_{\text{ph}}^{(0)}(0, x'; \omega)$$

Scattering and Susceptibility



- **T matrix** Related to **local spin susceptibility** (by equations of motion):

$$\hat{T}_{\ell'\ell}^{\text{el}}(\omega) = (-1)^{\delta_{\ell\ell'}} \alpha_{\ell} \alpha_{\ell'} \omega \chi_{zz}(\omega)$$

$$\alpha_{\ell} = \frac{1}{\sqrt{g}} \left(1 - \frac{g U_{\ell}}{\pi v} \right)$$

- **Experimental probe** for **dynamic susceptibility**!

Kondo Susceptibility

- New low energy scale:**

$$T_K \sim \omega_0 (\nu I_{xy})^{1/\nu I_z}$$

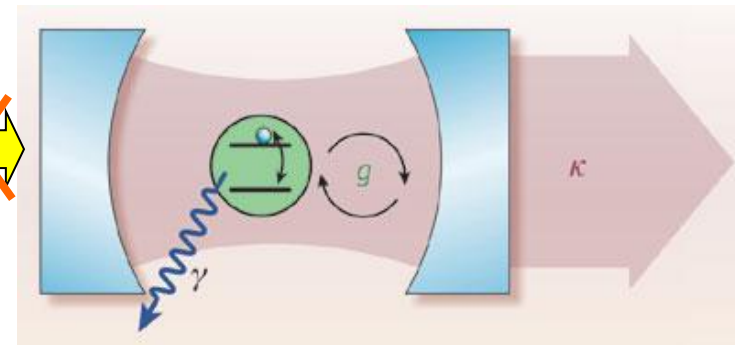
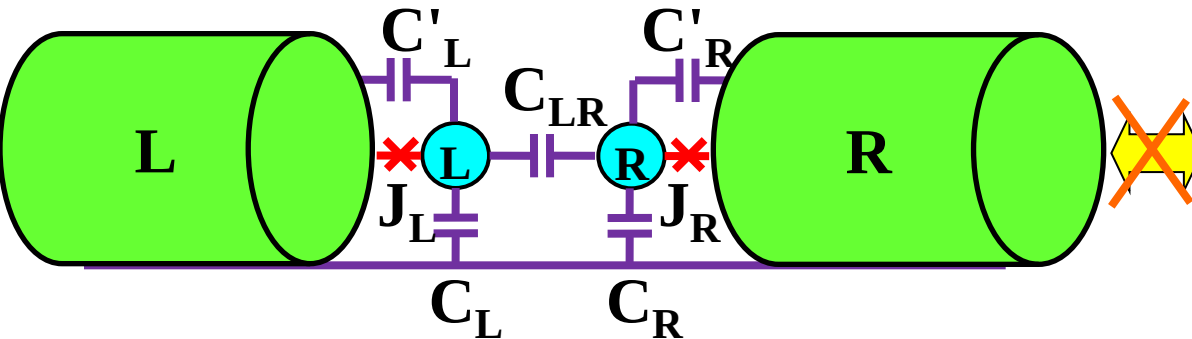
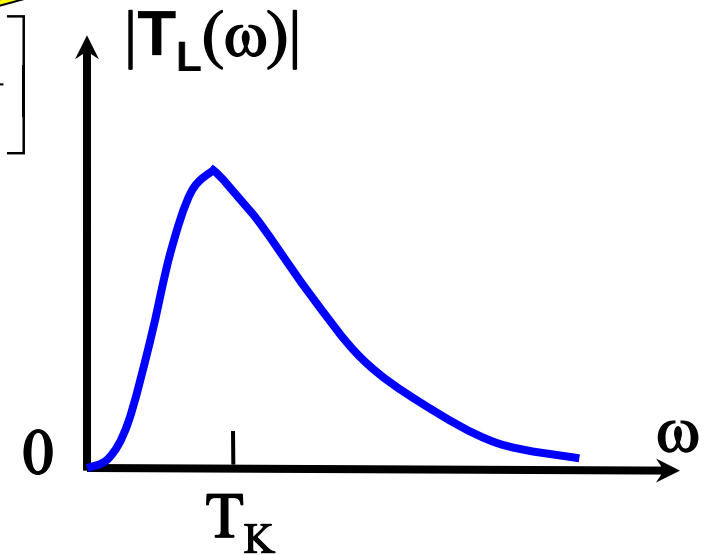
$$\alpha^2 \equiv \alpha_L^2 + \alpha_R^2 \approx 2(1 - \nu I_z)$$

- For $T, B_z \ll T_K$:

- **Low ω** [Shiba]: $\chi_{zz}(\omega) \sim \frac{1}{T_K} \left[1 + i\pi\alpha^2 \frac{\omega}{T_K} \right]$

- **High ω :**

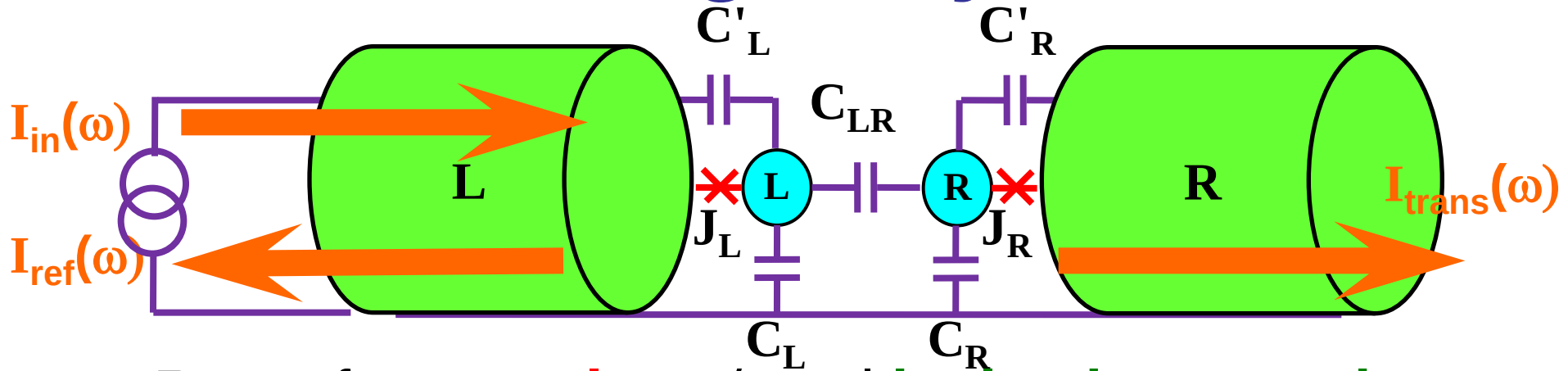
$$\chi_{zz}(\omega) \sim i \frac{[\nu I_{xy}(\omega)/2]^2}{\omega} = \frac{i}{\omega} \left(\frac{T_K}{\omega} \right)^{\nu I_z}$$



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Is Scattering Fully Elastic?



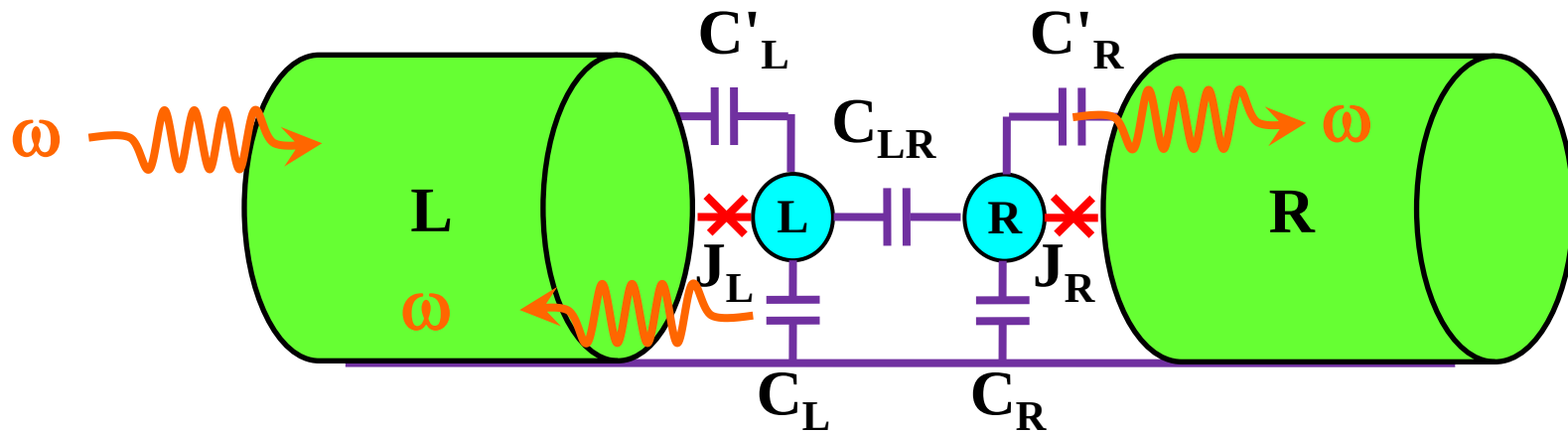
- Rate of **energy loss** / total **inelastic scattering probability** for a **photon** at frequency ω

$$\gamma_{\ell}(\omega) = 1 - |T_{\ell}|^2 - |R_{\ell}|^2 =$$

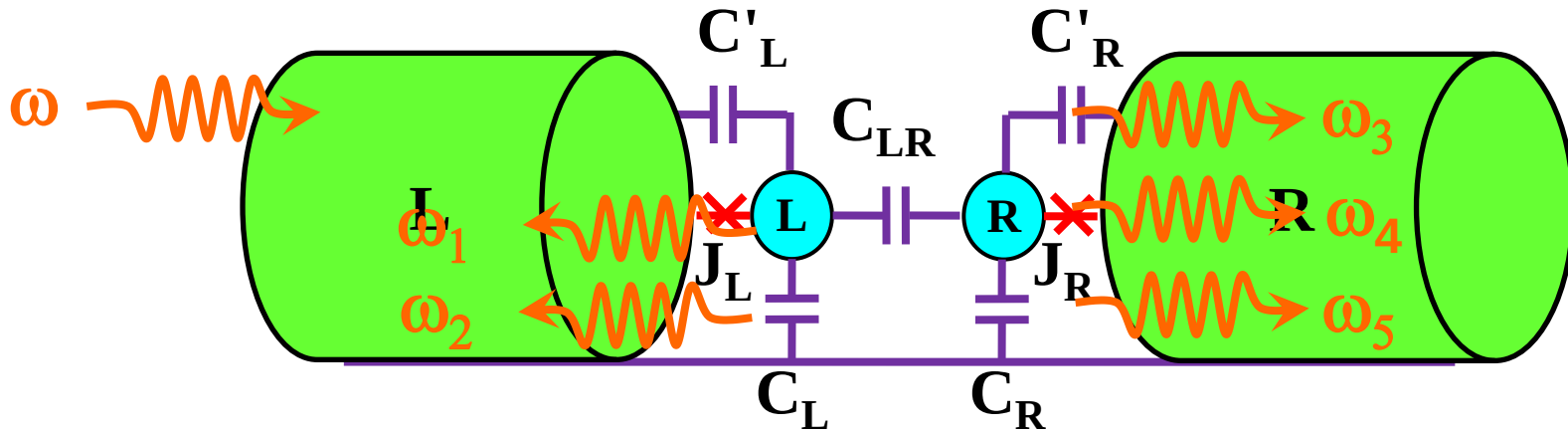
$$4\pi\alpha_{\ell}^2\omega\text{Im}[\chi_{zz}(\omega)] - 4\pi^2\alpha_{\ell}^2(\alpha_L^2 + \alpha_R^2)\omega^2|\chi_{zz}(\omega)|^2$$

- Zero** for **harmonic** systems
 - Kondo** at $\omega \ll T_K$: vanishes to **$O(\omega^2)$** [Nozieres, Shiba]
- Nonzero in general !**
 - Kondo** at $\omega \gg T_K$: **$O([\nu I_{xy}/2]^2)$** – **dominating** over elastic transmission, **$O([\nu I_{xy}/2]^4)$**

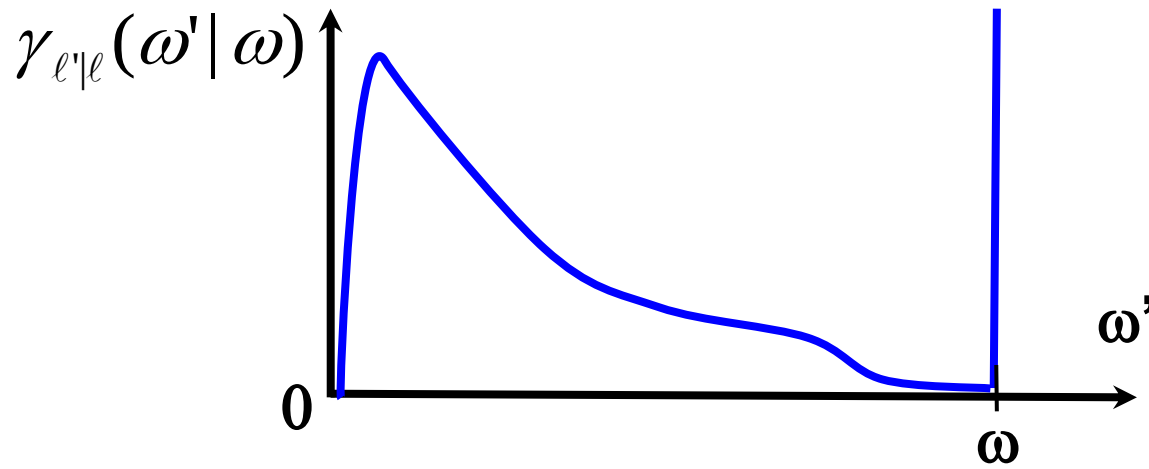
Inelastic Scattering



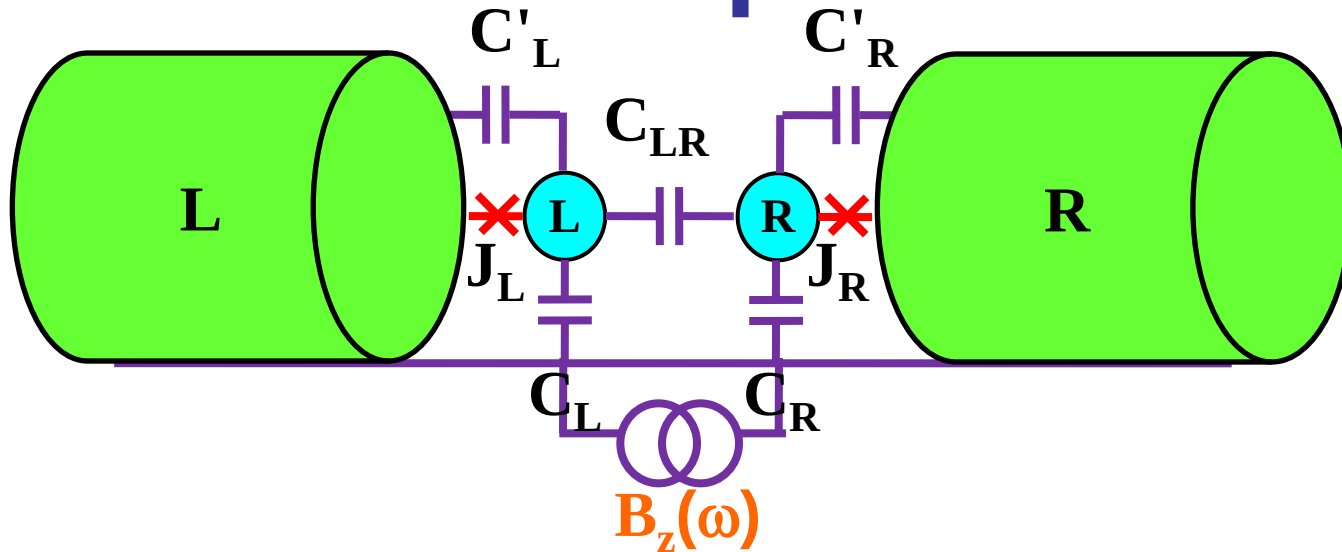
Inelastic Scattering



- **Spectrum** of **outgoing** photons (“**Raman**”)
 # of **outgoing** photons in lead ℓ' at frequency interval $[\omega', \omega' + d\omega']$ per each **incoming** photon in ℓ and ω



Inelastic Spectrum



- linear** response in **energy flux** –
2nd order in **charge current**

$$\gamma_{\ell'|\ell}(\omega'|\omega) = \frac{\dot{G}_{n_{\omega'};i;i}^r(\omega, -\omega)}{\dot{G}_{n_{\omega};i;i}^{r;(0)}(\omega, -\omega)}$$
- Reducible to **local** correlators (**Keldysh formalism**):

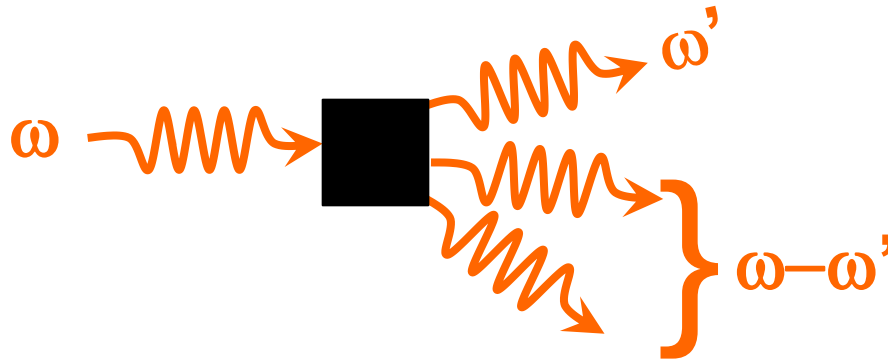
$$\gamma_{\ell'|\ell}(\omega'|\omega) = \pi \alpha_{\ell'}^2 \alpha_{\ell}^2 \omega \omega' \left[\underbrace{\langle S_z^c S_z^c S_z^q S_z^q \rangle_{\omega', -\omega'}^{\omega, -\omega}}_{\frac{\delta^2 \langle S_z S_z \rangle_{\omega'}}{\delta B_z(\omega) \delta B_z(-\omega)}} + \cot \frac{\omega'}{2T} \left(\underbrace{\langle S_z^c S_z^q S_z^q S_z^q \rangle_{\omega', -\omega'}^{\omega, -\omega}}_{\frac{\delta^3 \langle S_z(\omega') \rangle}{\delta B_z(\omega') \delta B_z(\omega) \delta B_z(-\omega)}} - \langle S_z^q S_z^c S_z^q S_z^q \rangle_{\omega', -\omega'}^{\omega, -\omega} \right) \right]$$

Perturbative Regime (I)

- To **lowest** (**2nd**) order in $I_{xy} \propto \mathbf{J}_L \mathbf{J}_R$ ($\omega \gg T_K$):

$$\gamma_{\ell|\ell}(\omega'|\omega) = 4\pi\alpha_{\ell}^2\alpha_{\ell'}^2 \frac{(I_{xy}/4\pi a)^2}{\omega\omega'} \times$$

$$\left\{ \theta(\omega - \omega') \left[1 + n_B(\omega') \right] \left[1 + n_B(\omega - \omega') \right] - n_B(\omega') n_B(\omega - \omega') \right\} \text{Im} \left\langle \left\langle e^{i\alpha\phi(0)}; e^{-i\alpha\phi(0)} \right\rangle \right\rangle_{\omega - \omega'} + \dots \right\}$$



- Equivalent to **Boltzmann equation**:

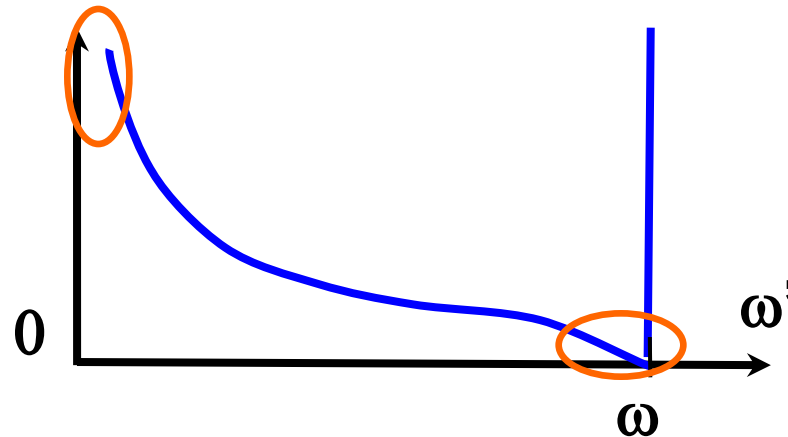
$$\frac{dn_{\omega}}{dt} = \left(\frac{I_{xy}}{2\pi a} \right)^2 \sum_{N, N'} \frac{|\alpha|^{N+N'}}{N! N'!} \int \frac{d\omega_1}{\omega_1} \dots \int \frac{d\omega_N}{\omega_N} \int \frac{d\omega'_1}{\omega'_1} \dots \int \frac{d\omega'_{N'}}{\omega'_{N'}} n_{\omega_1} \dots n_{\omega_N} (1 + n_{\omega'_1}) \dots (1 + n_{\omega'_{N'}}) \times$$

$$\left[(1 + n_{\omega}) \delta(\omega_1 + \dots + \omega_N - \omega'_1 - \omega'_{N'} - \omega) - n_{\omega} \delta(\omega + \omega_1 + \dots + \omega_N - \omega'_1 - \omega'_{N'}) \right]$$

Perturbative Regime (II)

- At **T=0**:

$$\gamma_{\ell'|\ell}(\omega'|\omega) = \pi \alpha_{\ell}^2 \alpha_{\ell'}^2 (\nu I_{xy})^2 \frac{\omega_0}{\omega \omega'} \left(\frac{\omega - \omega'}{\omega_0} \right)^{1-2\nu I_z} = \pi \alpha_{\ell}^2 \alpha_{\ell'}^2 \frac{\omega - \omega'}{\omega \omega'} \left(\frac{T_K}{\omega - \omega'} \right)^{2\nu I_z}$$

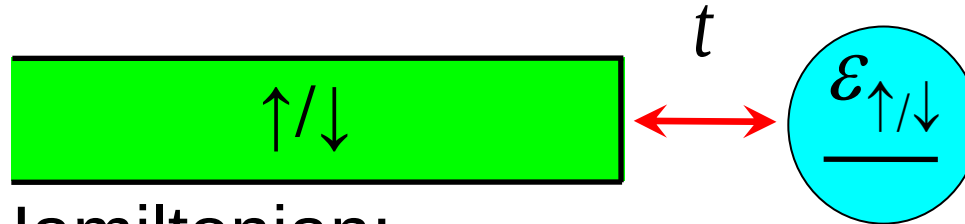


- Obeys **energy conservation**:

$$\sum_{\ell'=L,R} \int \gamma_{\ell'|\ell}(\omega'|\omega) \omega' d\omega' = \gamma_{\ell}(\omega) \omega$$

- What happens for $\omega - \omega'$ or ω' **small** w.r.t. **T_K**?

Bosonic Kondo at Low Energy (I)



- **Kondo** Hamiltonian:

$$H_K = H_{\text{env}}^0 + I_z S_z \frac{\partial_x \phi_s(0)}{\pi\sqrt{2}} + \frac{I_{xy}}{4\pi a} \left[S_+ e^{-i\sqrt{2}\phi_s(0)} + \text{H.c.} \right]$$

$$H_{\text{env}}^0 = \frac{v}{4\pi} \int [\partial_x \phi_s(x)]^2 dx$$

- Equivalent to **spin-boson**:

$$H_{SB} = U^\dagger H_K U = H_{\text{env}}^0 + \cancel{\Delta I_z S_z \frac{\partial_x \phi_s(0)}{\pi\sqrt{2}}} + \frac{I_{xy}}{2\pi a} S_x$$

$$U = e^{i\sqrt{2}S_z\phi_s(0)}$$

$$\Delta I_z = I_z - 2\pi v$$

$I_z/(4v)$ is a
phase shift

- At “**strong coupling Toulouse point**”, $I_z/(4v) = \pi/2$:

$$\Delta I_z = 0$$

- Spin in $\mathbf{S}_x$ eigenstate

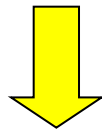
Bosonic Kondo at Low Energy (II)

- **Deviations** from “**Toulouse point**”:

$$H_{SB} = H_{\text{env}}^0 + \frac{I_{xy}}{2\pi a} S_x + \Delta I_z S_z \frac{\partial_x \phi_s(0)}{\pi\sqrt{2}}$$

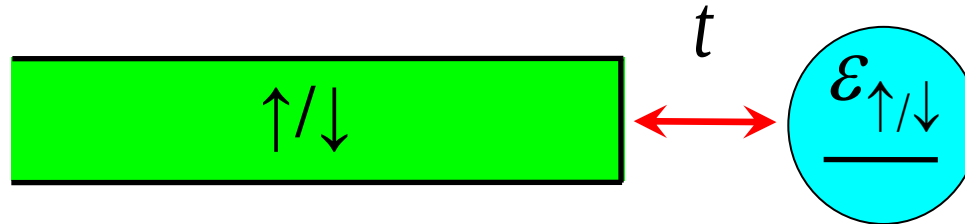
- At low energy, flipping **S_x** is **not** allowed. Thus:

$$H \rightarrow H_{\text{env}}^0 + \frac{a(\Delta I_z)^2}{\pi I_{xy}} [\partial_x \phi_s(0)]^2$$



$$H_{\text{eff}} = H_{\text{env}}^0 + \frac{v^2}{T_K} [\partial_x \phi_s(0)]^2$$

Kondo at Low Energy



- Low energy description for Fermions [**Nozières**]:

$$H_{\text{eff}} = \sum_{k,s} \varepsilon_k c_{k,s}^+ c_{k,s} - \sum_{k,k',s} \frac{\varepsilon_k + \varepsilon_{k'}}{2\pi v T_K} c_{k,s}^+ c_{k',s} + \frac{1}{\pi v^2 T_K} \rho_{\uparrow} \rho_{\downarrow}$$

- Bosonization:

$$H_{\text{eff}} = H_{\text{env}}^0 + \frac{v^2}{T_K} [\partial_x \phi_s(0)]^2$$

$$H_{\text{env}}^0 = \frac{v}{4\pi} \int [\partial_x \phi_s(x)]^2 dx$$

- **Lowest** term allowed by **symmetries** (**time reversal**)
- **No** inelastic scattering of **bosons** to **$O(\omega^2)$**

Low Energy Inelastic Scattering

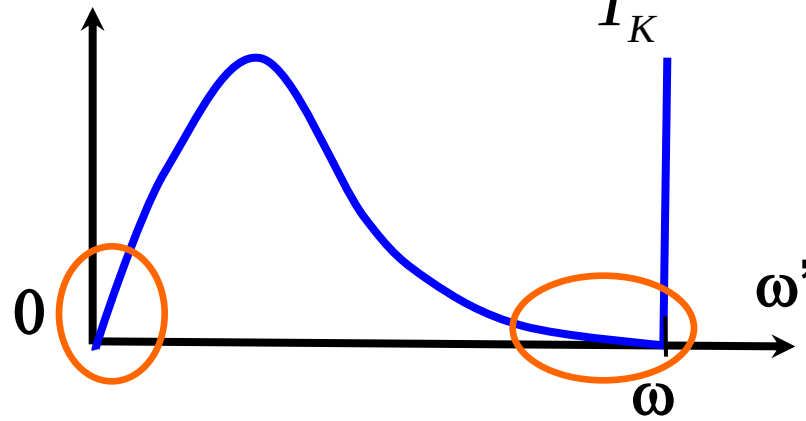
- Least-irrelevant **many boson** term:

- no magnetic field** $\frac{v^4}{T_K^3} [\partial_x \phi_s(0)]^4 \propto \sqrt{k_1 k_2 k_3 k_4} a_{k_1}^+ a_{k_2} a_{k_3} a_{k_4} + \dots$

- with magnetic field** $\frac{B_z v^3}{T_K^3} [\partial_x \phi_s(0)]^3 \propto \sqrt{k_1 k_2 k_3} a_{k_1}^+ a_{k_2} a_{k_3} + \dots$

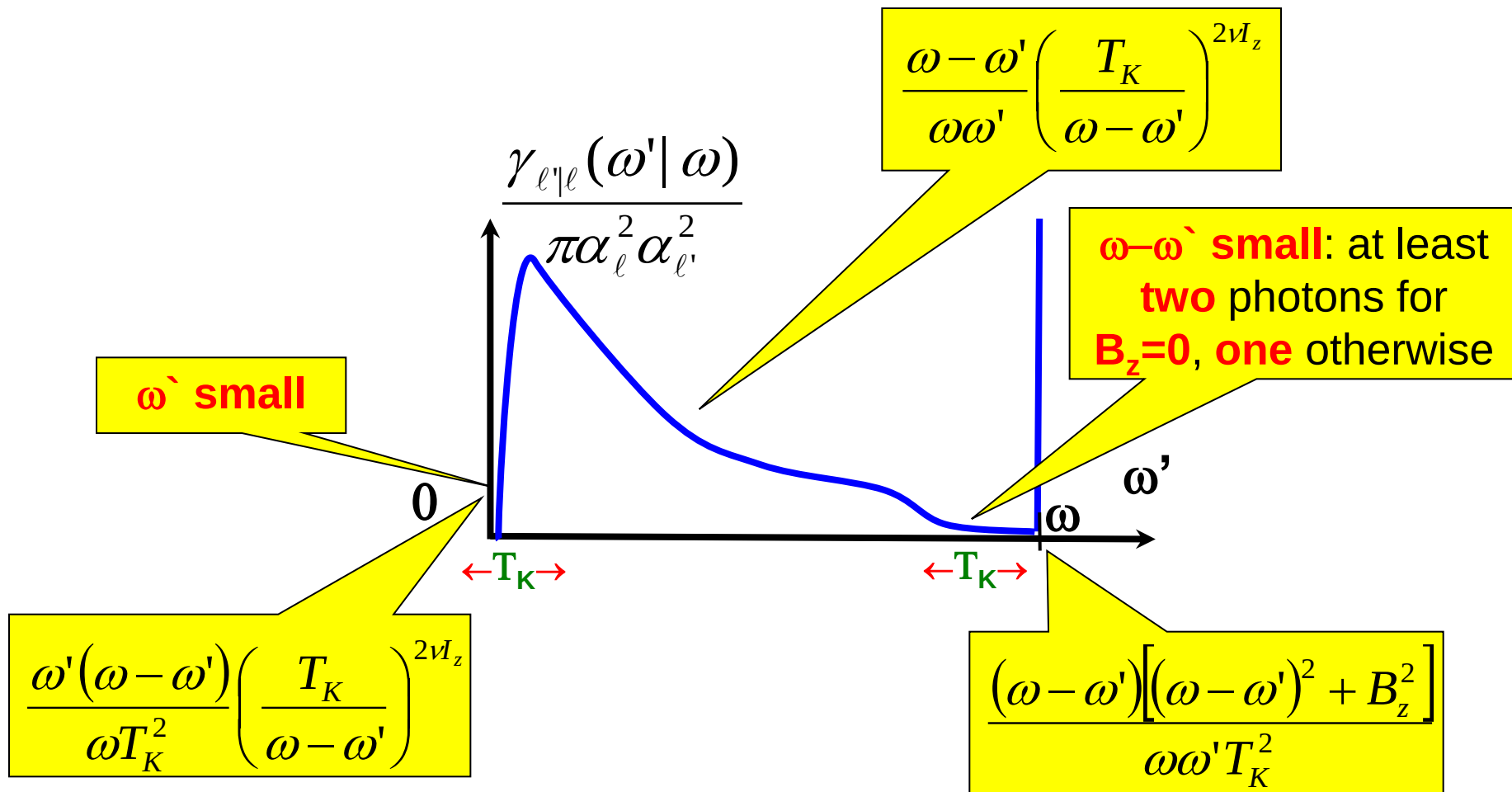
- Inelastic spectrum** for $\omega \ll T_K$:

$$\gamma_{\ell'\ell}(\omega' | \omega) \sim \alpha_\ell^2 \alpha_{\ell'}^2 \frac{\omega \omega' (\omega - \omega') [(\omega - \omega')^2 + B_z^2]}{T_K^6}$$



Back to High Energies

- For $\omega \gg T_K$ (following considerations similar to $\omega \ll T_K$):



Conclusions

- **Quantum impurity** in **circuit QED**: **Many body** physics with **photons**
 - **No dissipative** elements, yet **dissipative** linear response of **charge current**
 - **Missing energy?** ► **inelastic** photon scattering
 - **Scattering** amplitudes ► **local response** functions
- **Example**: **anisotropic Kondo** for **microwaves**
 - **N** leads ► **SU(N)** [Carmi et al., PRL'11]; **2 channel?**