

Andreev and Majorana bound states in quantum dots

Alfredo Levy Yeyati

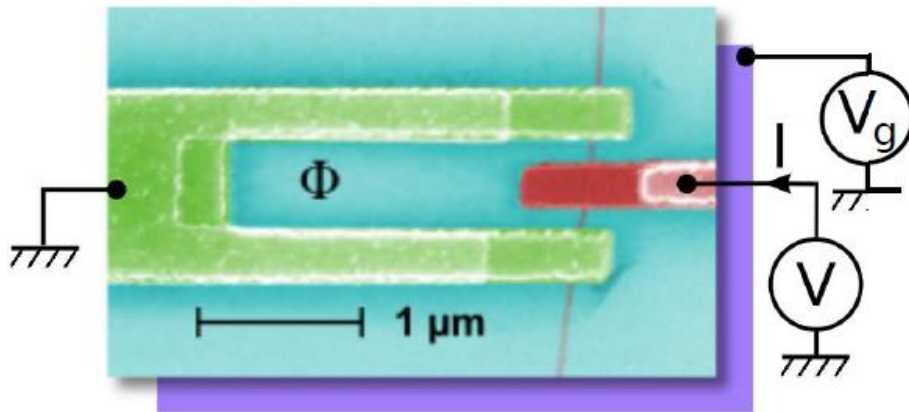
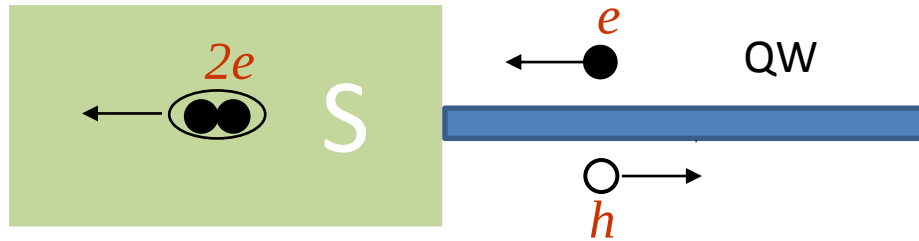


In collaboration with:

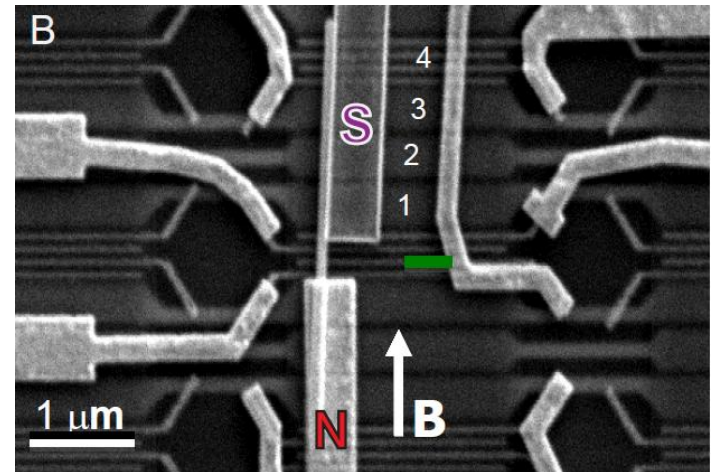
Alvaro Martín-Rodero, Bernd Braunecker (UAM)

Reinhold Egger, Alex Zazunov, Roland Hützen (Dusseldorf)

Exp results: Philippe Joyez (Saclay)



Andreev states spectroscopy in CNTs
J.D. Pillet et al. Nature Phys. (2010)



Search for Majorana states in
semiconducting nanowires
V. Mourik et al. Science (2012)

Effect of e-e interactions $\longrightarrow$ Charging energy in QD regime

Outline

Andreev states in QDs

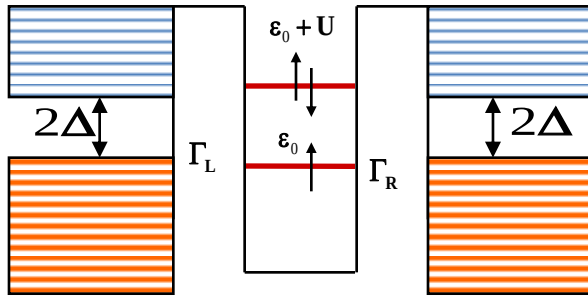
The superconducting Anderson model
Experimental results – Fit by model calculations
NRG vs mean field results

Majorana bound states in QDs

The single charge Majorana transistor
Transport properties: Known limits
Weak blockade regime
General Green functions formalism
Zero band width limit
Equation of motion method
Master equation approach

Conclusions

QD regime: the superconducting Anderson model



$$\Delta \epsilon \gg \Gamma \longrightarrow$$

Single Level

$$H = \sum_{\sigma} \epsilon_0 n_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{k, \sigma} t_k c_k^{\dagger} d_{\sigma} + h.c. + H_L + H_R$$

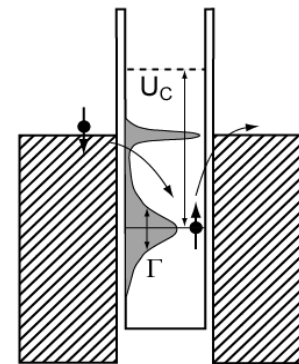
$$H_{L,R} = \sum_{k, \sigma} \epsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + \sum_k \Delta e^{\pm i\phi/2} c_{k\uparrow} c_{-k\downarrow} + h.c.$$

$$\Gamma_L, \Gamma_R, U, \Delta, \epsilon_0, eV = \mu_L - \mu_R$$

Equilibrium (V=0):

Kondo vs Pairing

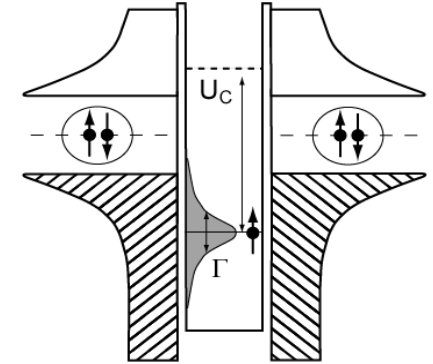
crossover $T_K \approx \Delta$



Source QD Drain



Energy scale : $\sim k_B T_K$



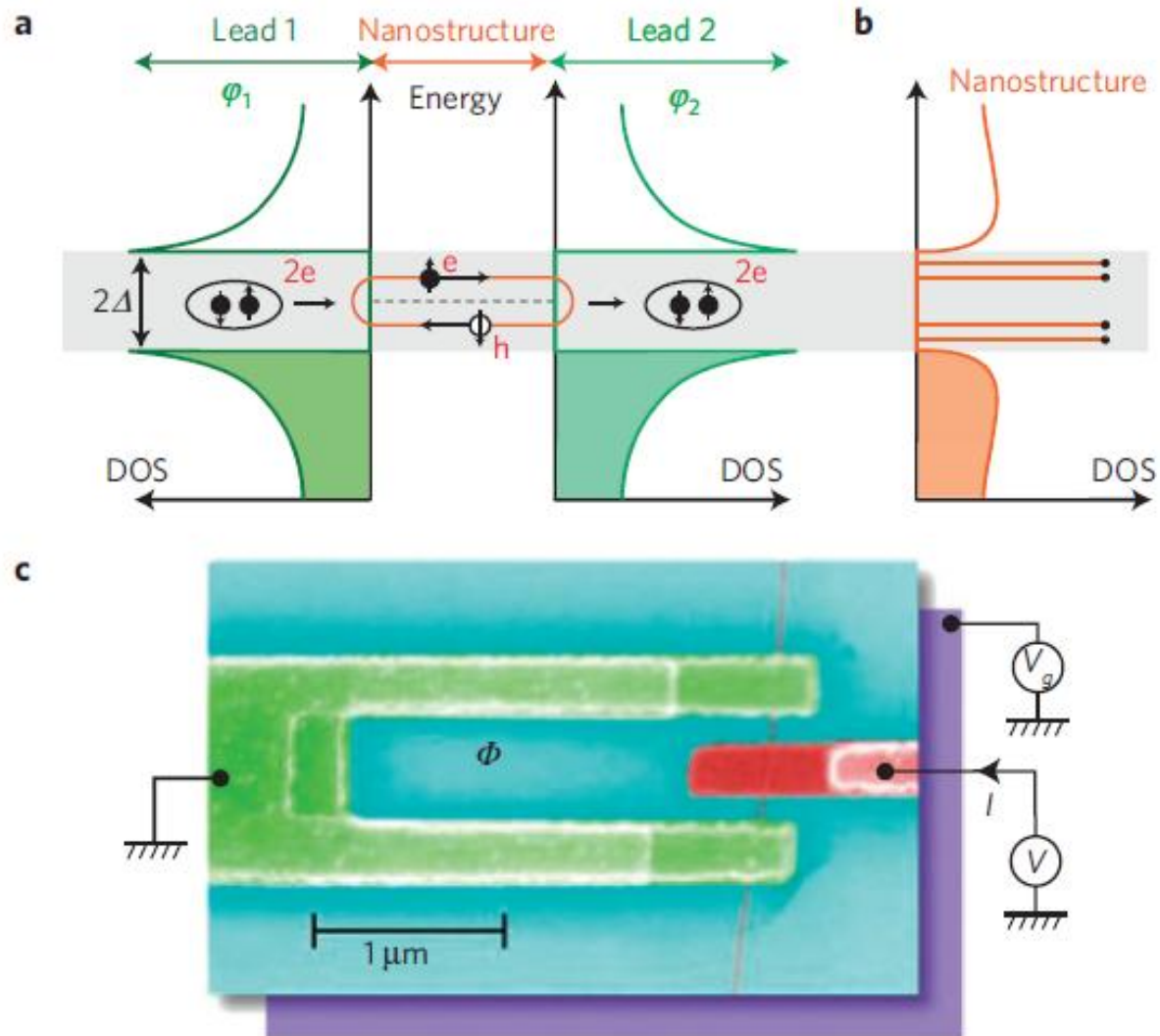
Source QD Drain



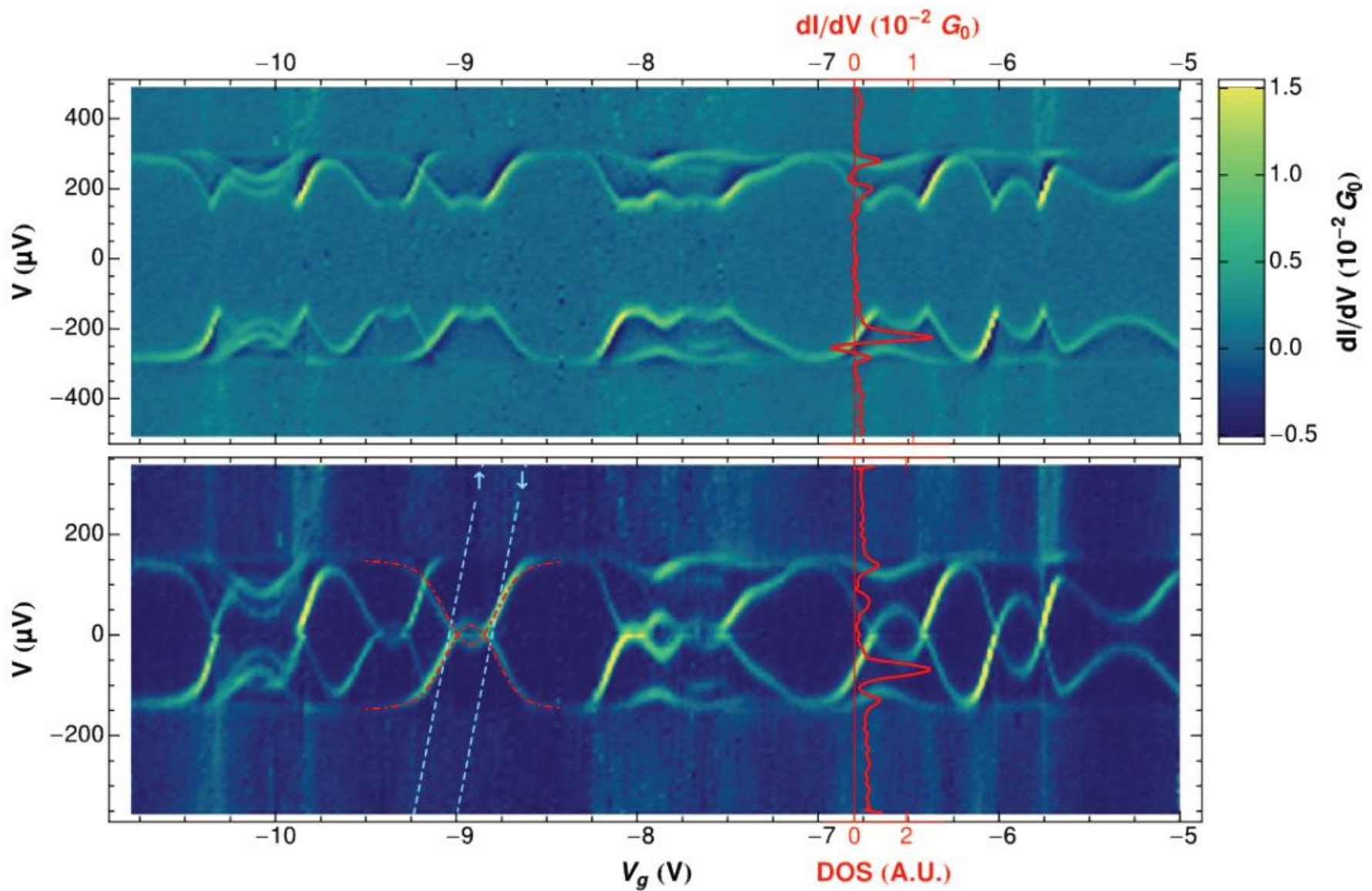
Energy scale : $\sim \Delta$

π-junction behavior!

Spectral properties: Andreev bound states



Experimental results: gate voltage dependence



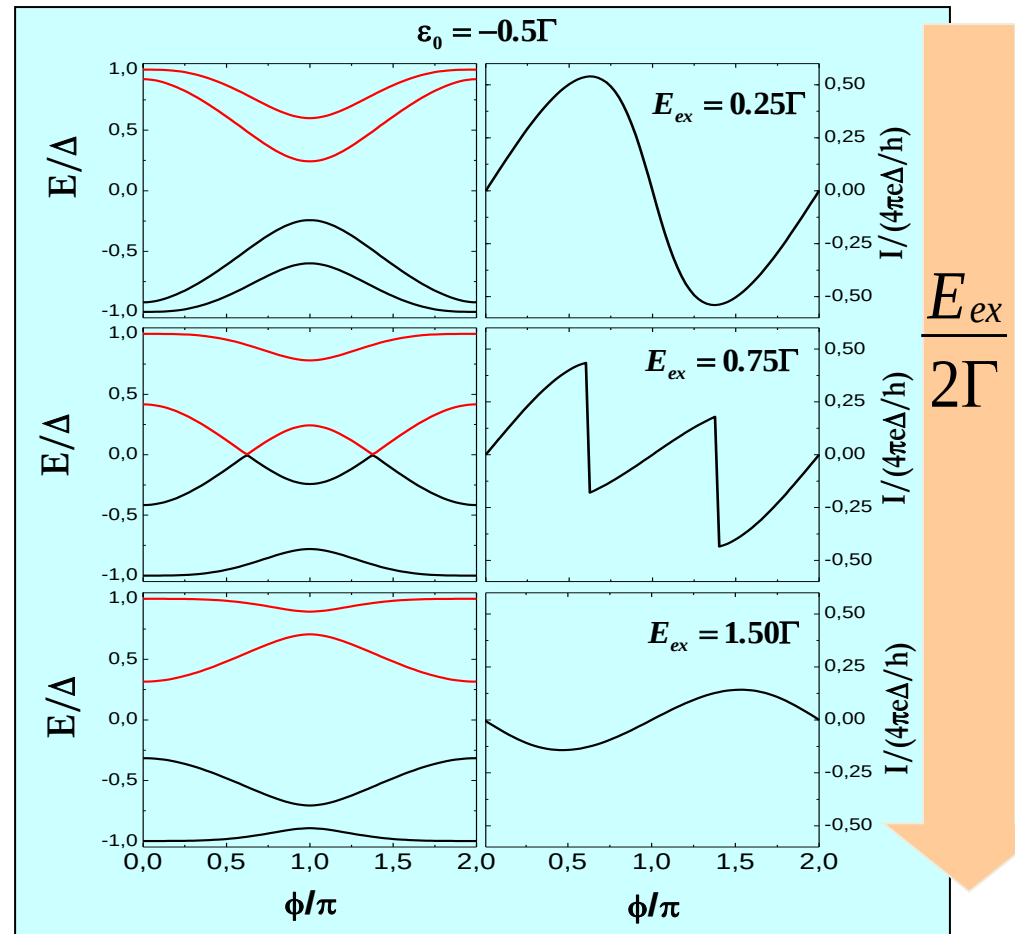
ABS in SC Anderson model: Hartree Fock approximation

$$H = \sum_{\sigma} \varepsilon_0 n_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{k, \sigma} t_k c_k^{\dagger} d_{\sigma} + h.c. + H_L + H_R$$

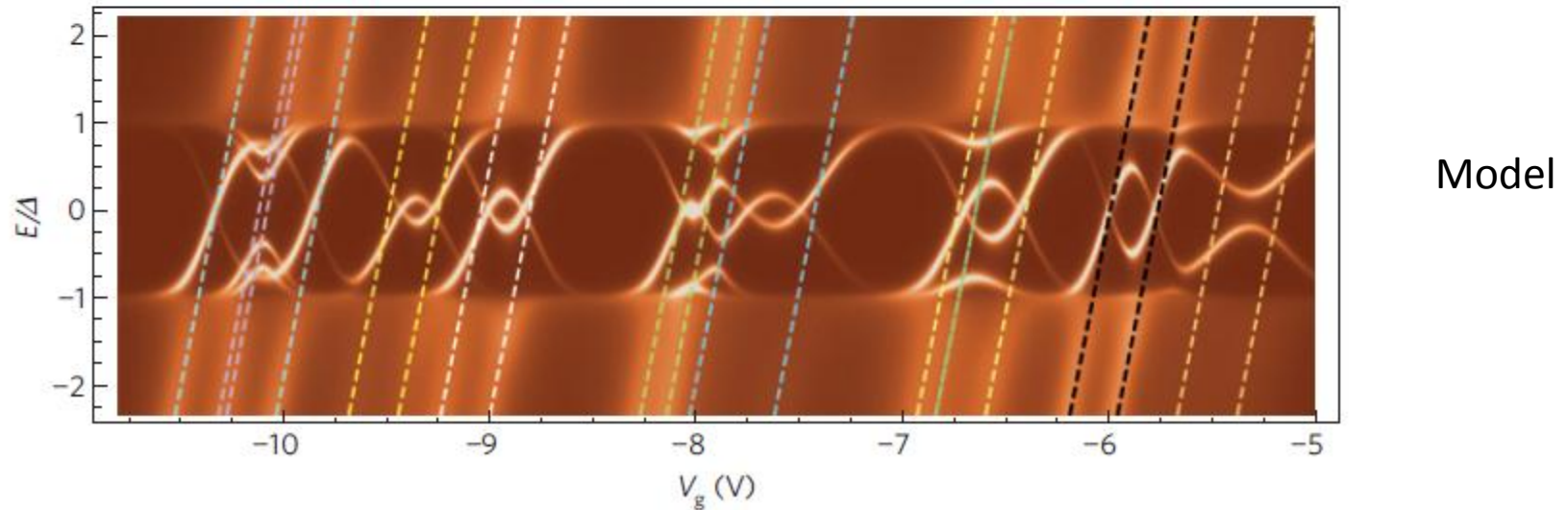
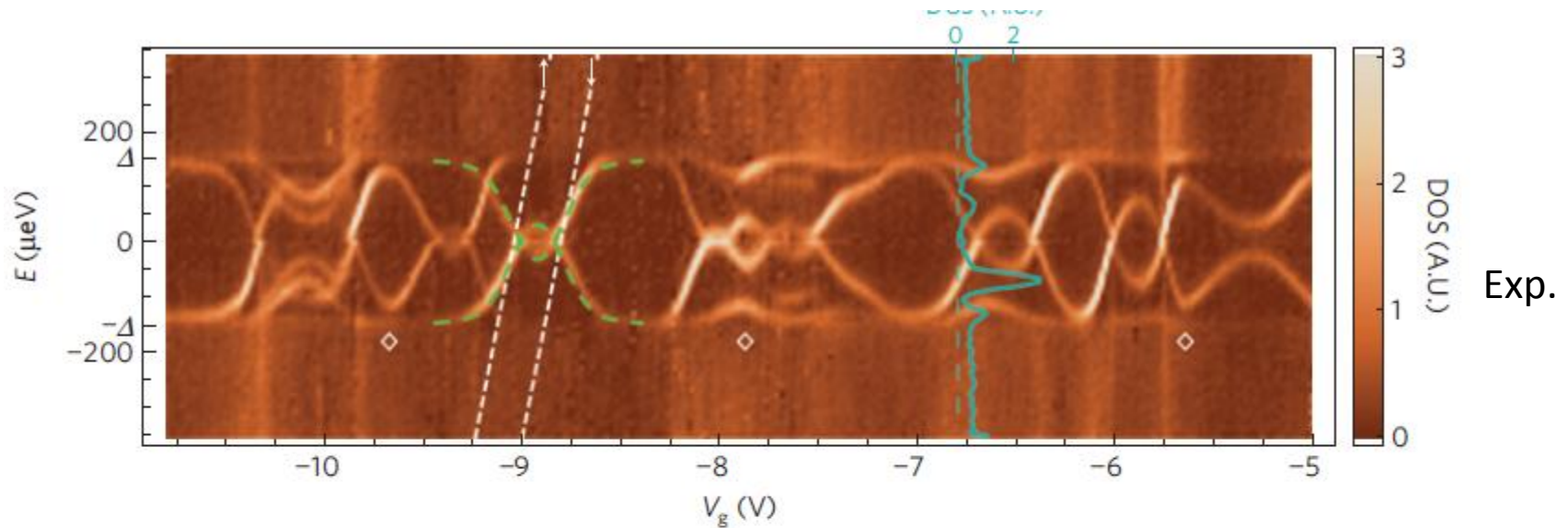
HF approx. $\left\{ \begin{array}{l} \varepsilon_{\sigma} = \varepsilon_0 + U \langle n_{\sigma} \rangle \\ \Delta_{ind} = U \langle d_{\uparrow}^{\dagger} d_{\downarrow}^{\dagger} \rangle \end{array} \right.$

Breaking spin symmetry $n_{\uparrow} \neq n_{\downarrow} \Rightarrow \varepsilon_{\downarrow} \neq \varepsilon_{\uparrow}$

Minimal model $\begin{array}{l} E_{ex} = \varepsilon_{\uparrow} - \varepsilon_{\downarrow} \\ \Delta_{ind} = 0 \end{array}$
(phenomenological parameter)

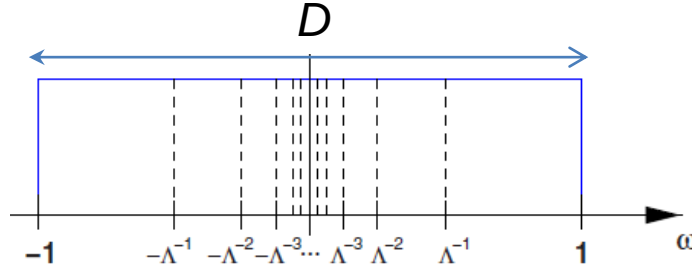


Fitting the experimental data: gate voltage dependence



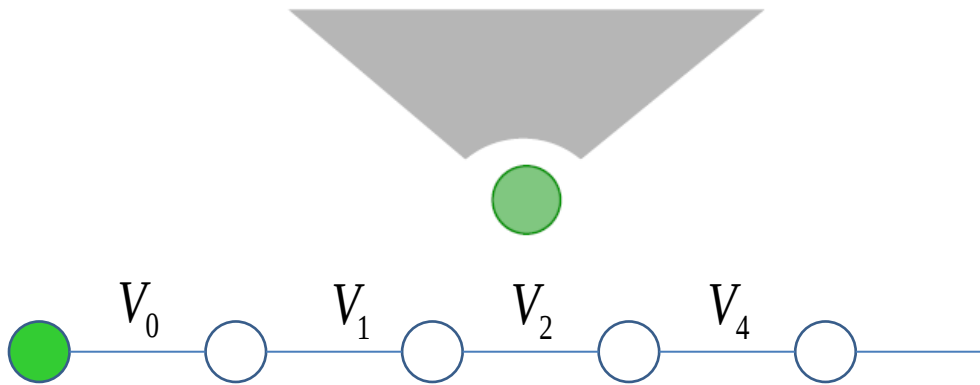
***ABS in SC Anderson model:
Mean field vs “exact” results***

Numerical Renormalization Group: basic ideas



Logarithmic discretization

$$\Lambda > 1$$



Map into semi-infinite chain

$$V_N \propto \Lambda^{-N/2}$$

$$\tilde{H}_{N+1} = \sqrt{\Lambda} \tilde{H}_N + \xi_N \sum_{\mu, \sigma} \left(f_{\mu, N+1, \sigma}^\dagger f_{\mu, N, \sigma} + \text{h. c.} \right)$$

Iterative diagonalization

$$-\Lambda^{N/2} \sum_{\mu} \tilde{\Delta}_{\mu} \left(f_{\mu, N+1, \uparrow}^\dagger f_{\mu, N+1, \downarrow}^\dagger + \text{h. c.} \right)$$

Truncation: #states $< N_c$

$$\xi_N = \frac{1 - \Lambda^{-N-1}}{\sqrt{1 - \Lambda^{-2N-1}} \sqrt{1 - \Lambda^{-2N-3}}} \rightarrow 1 \text{ for } N \gg 1$$

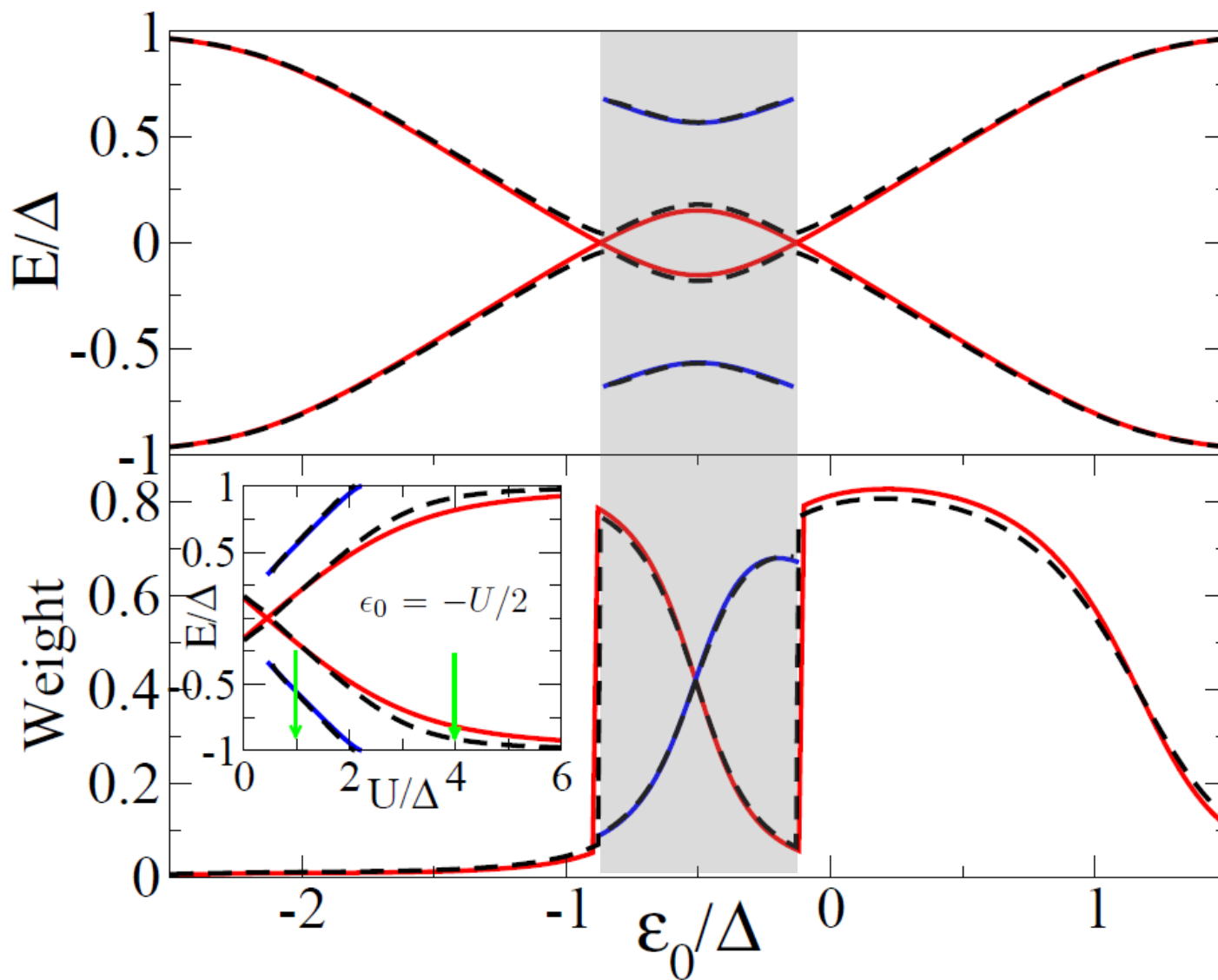
ABS: HF vs NRG results

$\Lambda \approx 4$

$N_c \approx 300$

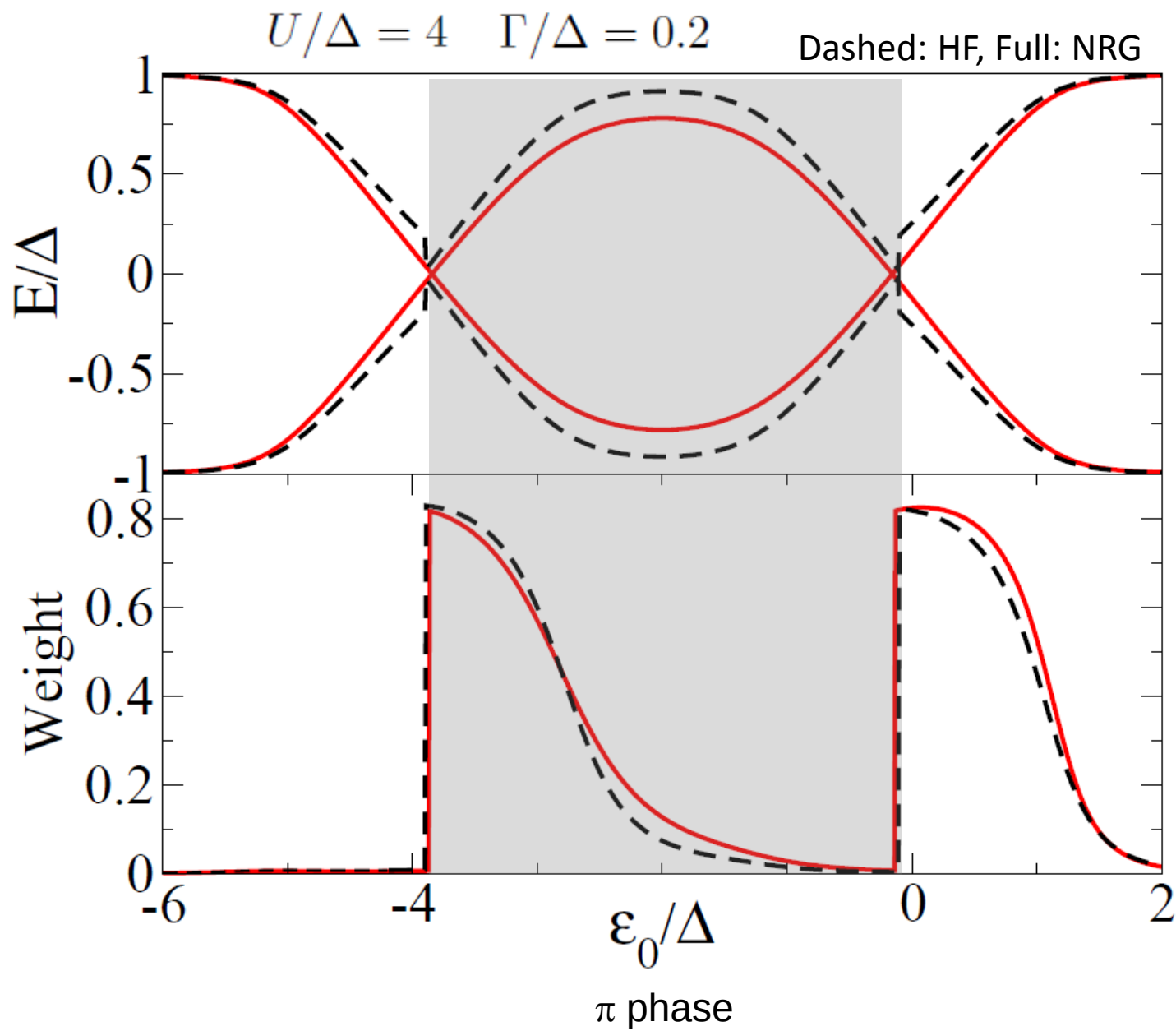
$U/\Delta = 1$ $\Gamma/\Delta = 0.2$

Dashed: HF, Full: NRG

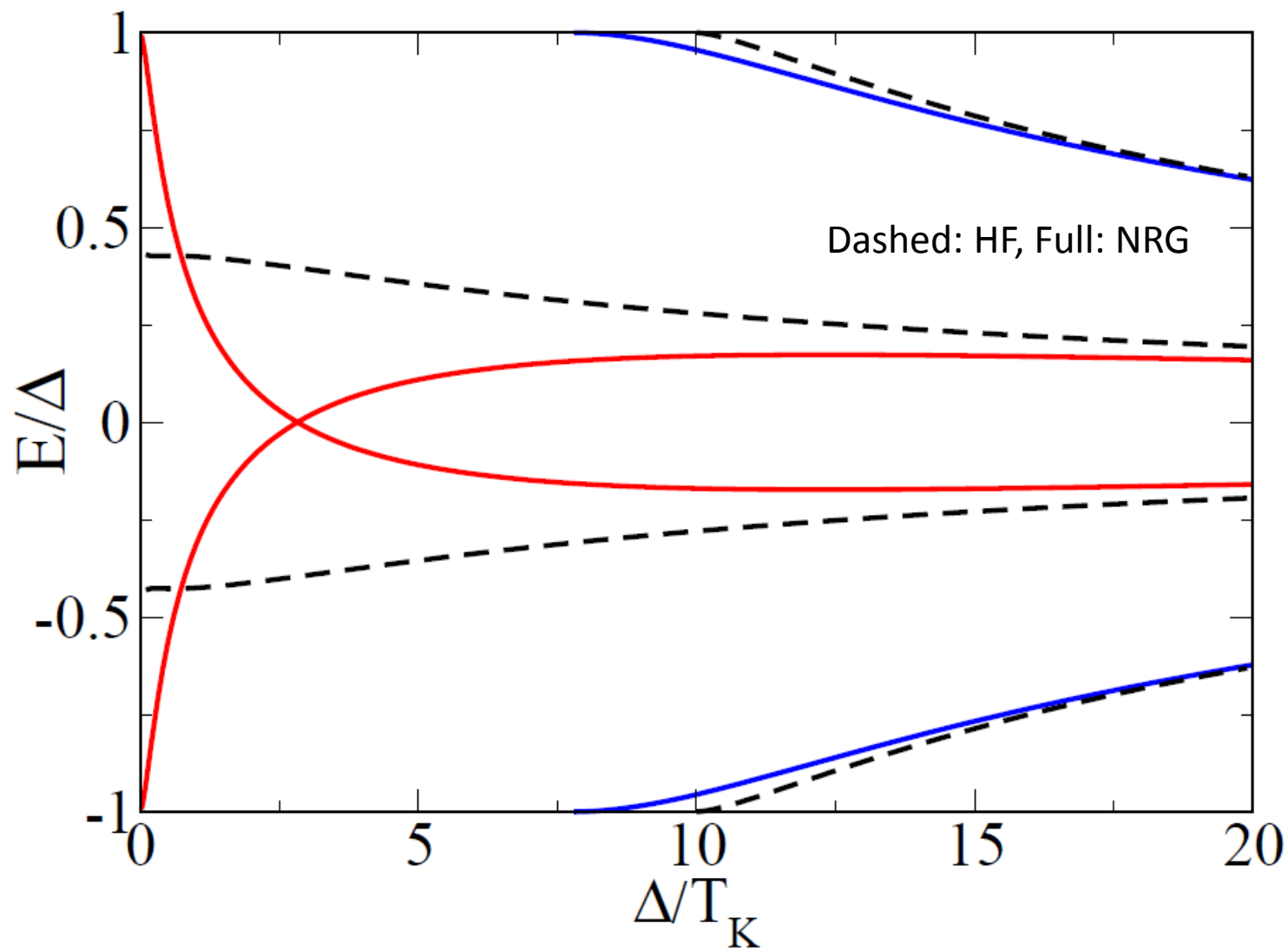


π phase: 4 ABSs

ABS: HF vs NRG results



Symmetric case $\epsilon_0 = -U/2$ $U/\Gamma = 5$



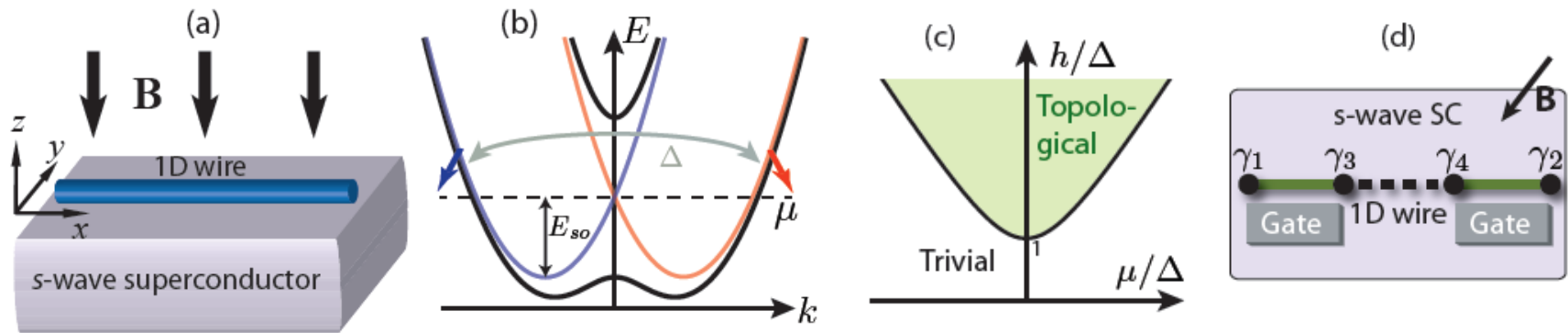
$$T_K = \sqrt{U\Gamma/2} \exp(-\pi U/8\Gamma)$$

Quantum dots with Majorana bound states

A. Zazunov, ALY & R. Egger, PRB (2011)

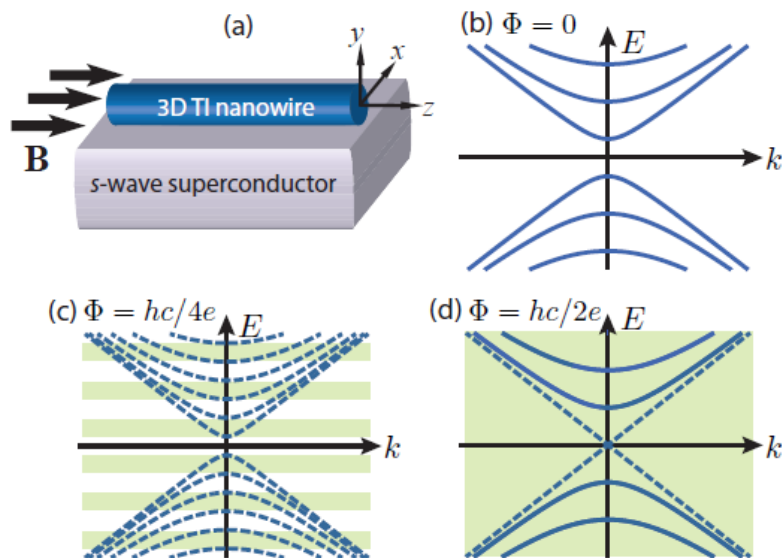
R. Hütten, A. Zazunov, B. Braunecker, ALY & R. Egger, [arXiv:1206.3912](#)

Majorana generation: induced superconductivity in normal or topological semiconducting wires



Review: J. Alicea, [arXiv:1202.1293](https://arxiv.org/abs/1202.1293)

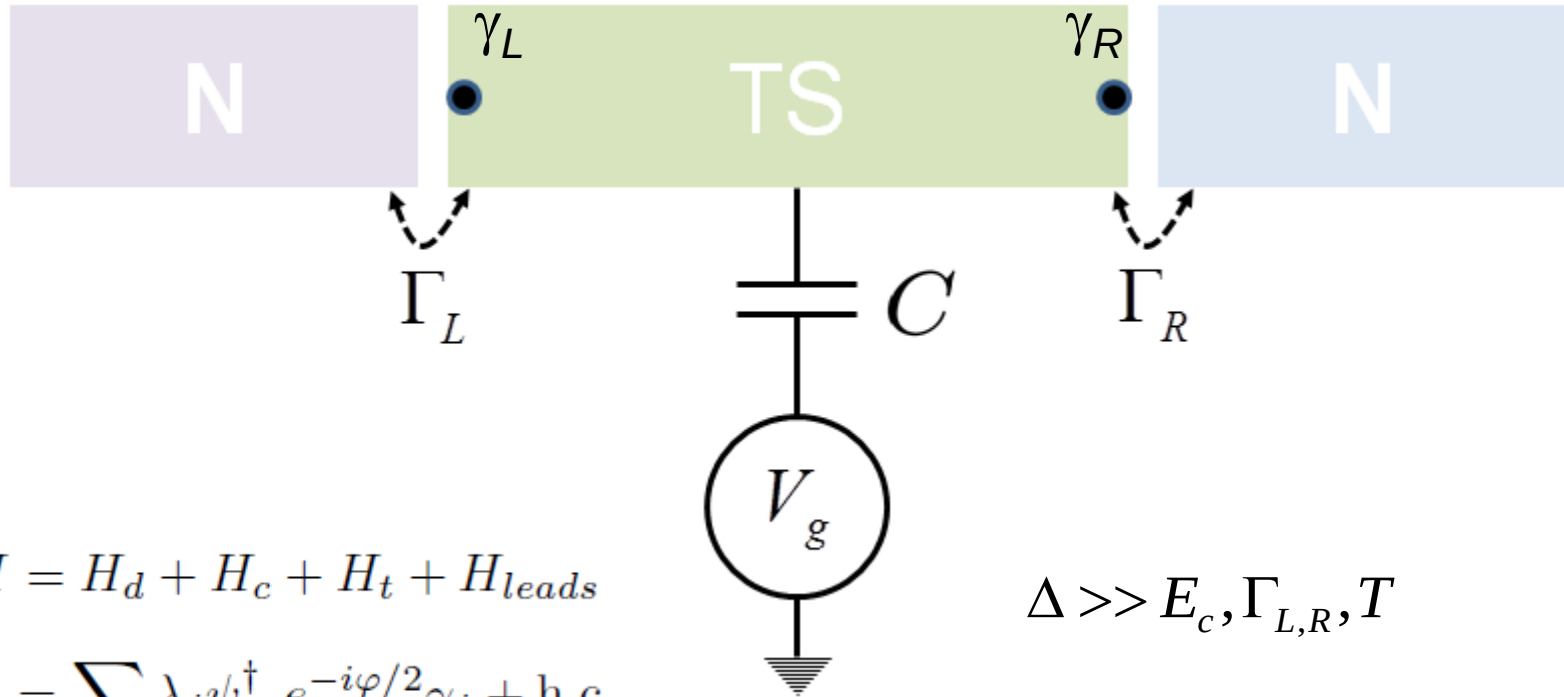
V. Mourik et al. (Delft) Science (2012)



Helical states in TI nanowires

R. Egger, A. Zazunov, and ALY, PRL (2010)

The Majorana Single Charge Transistor



$$H = H_d + H_c + H_t + H_{leads}$$

$$H_t = \sum_{jk} \lambda_j \psi_{jk}^\dagger e^{-i\varphi/2} \gamma_j + \text{h.c.},$$

$$\gamma_j = \gamma_j^\dagger \quad \{\gamma_i, \gamma_j\} = \delta_{ij} \quad j \equiv L, R$$

“Non-local” fermion $d = \frac{1}{\sqrt{2}} (\gamma_L + i\gamma_R)$

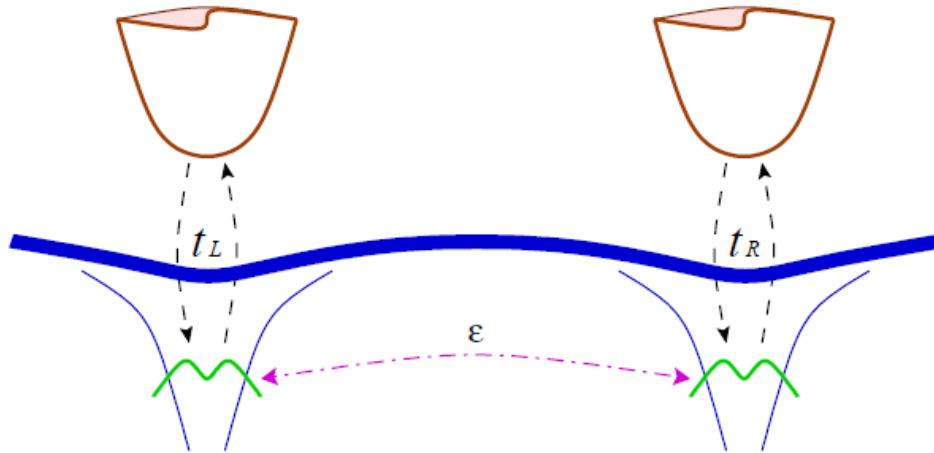
$$\Delta \gg E_c, \Gamma_{L,R}, T$$

$$H_c = E_c (\hat{n} - n_g)^2$$

$$\hat{n} = 2N + d^\dagger d$$

Cooper pairs number

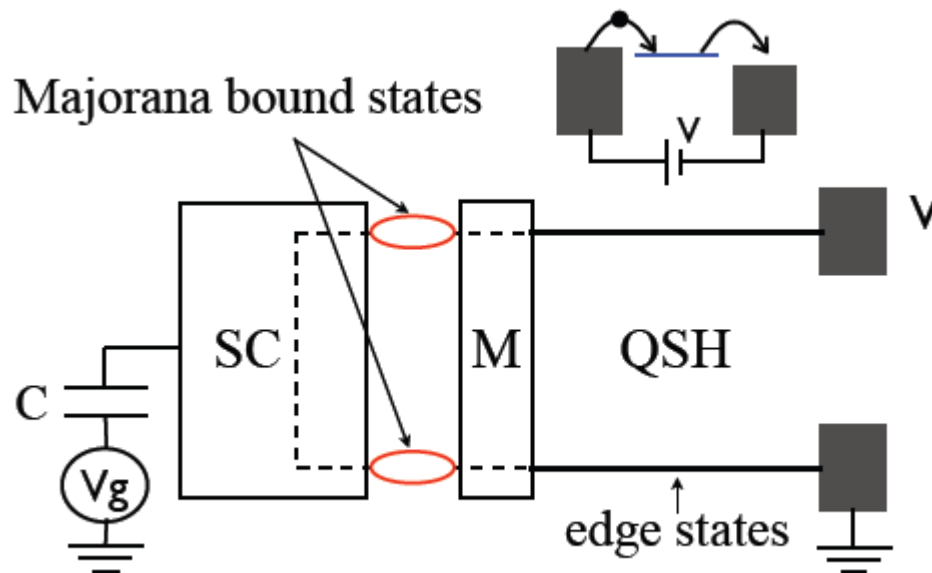
Known limits



$$E_c = 0$$

Bolech & Demler, PRL (2007)
“resonant Andreev reflection”

$$G \rightarrow \frac{2e^2}{h}$$



$$E_c/\Gamma \rightarrow \infty$$

L. Fu, PRL (2010)
“electron teleportation”

$$G \rightarrow \frac{e^2}{h}$$

$$H = H_d + H_c + H_t + H_{leads}$$

$$H_c = E_c (\hat{n} - n_g)^2 \quad \gamma_L = \frac{1}{\sqrt{2}} (d + d^\dagger) \quad , \quad \gamma_R = \frac{-i}{\sqrt{2}} (d - d^\dagger)$$

$$H_t = \sum_{jk} \lambda_j \psi_{jk}^\dagger e^{-i\varphi/2} \gamma_j + \text{h.c.}, \quad [\varphi/2, \hat{n}] = i$$

Equivalent representation: Cooper pairs + d fermion

$$[\chi, \hat{N}] = i \quad H_c = E_c (2\hat{N} + \hat{n}_d - n_g)^2$$

$$H_t = \frac{1}{\sqrt{2}} \sum_k \left[\lambda_L \psi_{Lk}^\dagger (d + e^{-i\chi} d^\dagger) + \lambda_R \psi_{Rk}^\dagger (-i) (d - e^{-i\chi} d^\dagger) + \text{h.c.} \right].$$

$(N,1) \rightarrow (N,0)$
“normal” e tunneling

$(N,0) \rightarrow (N-1,1)$
“anomalous” tunneling (Cooper pair splitting)

Weak blockade regime

A. Zazunov, A.L.Y. & R. Egger, PRB (2011)

Relevant dregree of freedom $\phi = \chi/2$

Keldysh path integral formulation $\phi_{\pm} \rightarrow \phi_c = \frac{1}{2} (\phi_+ + \phi_-)$ $\phi_q = \phi_+ - \phi_-$

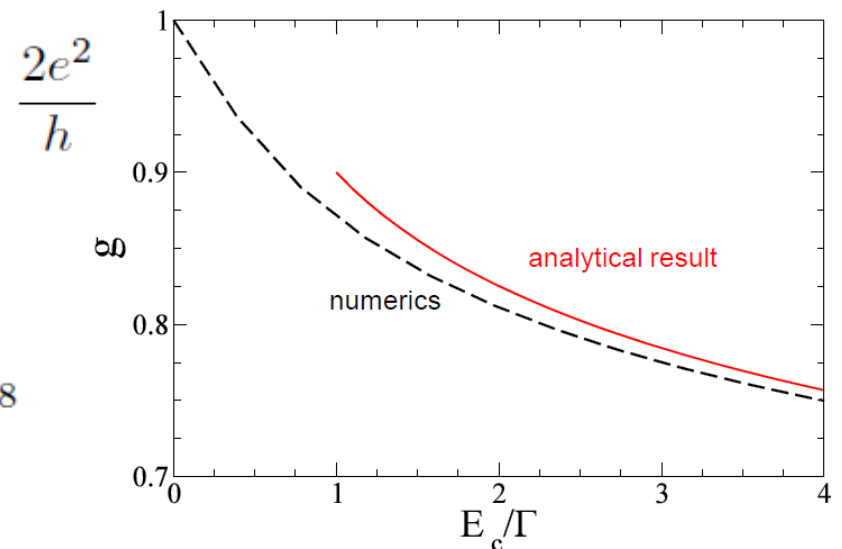
Second order expansion equivalent to semi-classical Langevin equation

$$\frac{1}{E_c} \ddot{\phi}_c(t) + \int^t dt' \eta(t-t') \dot{\phi}_c(t') = \xi(t) \quad J(t-t') = \frac{1}{2} \langle [\phi_c(t) - \phi_c(t')]^2 \rangle_{\xi}$$

$$I_j = \Gamma_j \int dt' G_j^R(t-t') F(t-t') \sin(\mu_j(t-t')) e^{-J(t-t')} \quad \text{Current in "P(E)" form}$$

$$G_j^R(\omega) = \frac{1}{\omega + i\Gamma_j} \quad F(\omega) = \tanh\left(\frac{\omega}{2T}\right)$$

$$\Gamma \lesssim E_c \quad G(V \rightarrow 0) \propto (E_c/\Gamma)^{-1/8}$$



Keldysh GFs formulation: general current formula

Define Nambu spinors

$$\Psi_j = \begin{pmatrix} \psi_{jk} \\ \psi_{jk}^\dagger \end{pmatrix} \quad \Psi_d = \begin{pmatrix} d \\ d^\dagger \end{pmatrix}$$

$$H_t = \sum_j \lambda_j \Psi_j^\dagger V_j \Psi_d \quad \text{and} \quad I_j = \frac{ie}{\hbar} \lambda_j \Psi_j^\dagger \sigma_z V_j \Psi_d$$

$$V_j = \begin{pmatrix} v_j^* & v_j e^{-i\chi} \\ -v_j^* e^{i\chi} & -v_j \end{pmatrix} \quad \begin{aligned} v_L &= 1/\sqrt{2} \\ v_R &= i/\sqrt{2} \end{aligned}$$

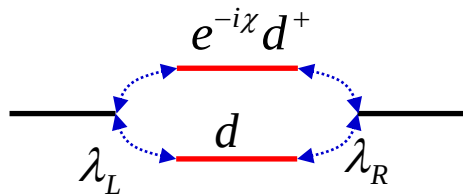
$$\langle I_j \rangle = \frac{2e}{h} \Gamma_j \int d\omega \text{Tr} \left[\sigma_z \begin{pmatrix} f_{je} & 0 \\ 0 & f_{jh} \end{pmatrix} \text{Im} G_{\eta_j}^A(\omega) \right]$$

Exact current formula!

$$f_{je,h} = 1/(1 + e^{\beta(\omega \pm \mu_j)})$$

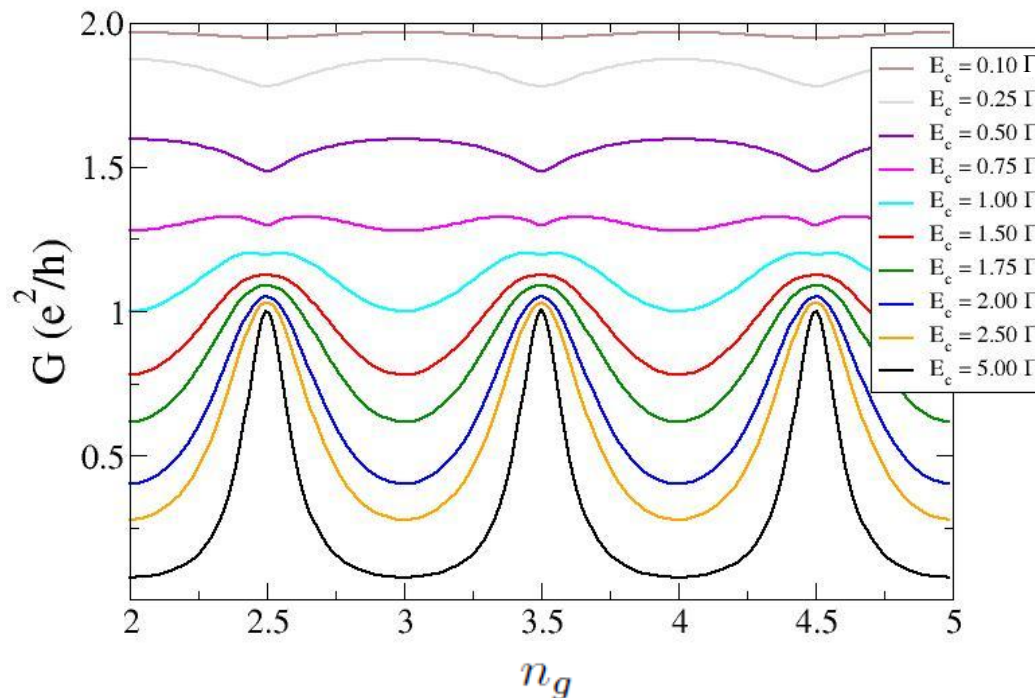
$$G_{\eta_j}^A(t, t') = i\theta(t' - t) \langle \left\{ \eta_j(t); \eta_j^\dagger(t') \right\} \rangle \quad \eta_j = V_j \Psi_d$$

Linear conductance: evaluation within ZBWM



$$G_j = \frac{\partial \langle I_j \rangle}{\partial \mu_j} \Big|_{\mu_j=0} = \frac{4e^2 \Gamma_j}{h} \left(\text{Im} G_{\eta_j}^A(0) \right)_{11}$$

$$\left(G_{\eta_j}^A(\omega) \right)_{11} = \sum_{n,m} \frac{e^{-\beta E_n}}{\mathcal{Z}} \left[\frac{|\langle E_n | (v_j^* d + v_j e^{i\chi} d^\dagger) | E_m \rangle|^2}{(E_n - E_m) + i\Gamma} + \frac{|\langle E_n | (v_j d^\dagger + v_j^* e^{i\chi} d) | E_m \rangle|^2}{(E_n - E_m) - i\Gamma} \right]$$



Evaluation using EOM (Equation of Motion method)

$$G_{dd}^R(t) = -i\Theta(t)\langle\{\Psi_d(t), \Psi_d^\dagger(0)\}\rangle \quad \Psi_d = \begin{pmatrix} d \\ e^{-i\chi}d^\dagger \end{pmatrix}$$

$$(\epsilon - E_0 + V_g + i\hat{\Gamma})G_{dd}^R = 1 + 4E_c\Gamma_{Ndd}^R \quad V_g = 2E_cn_g$$

$$\Gamma_{N^m dd}^R(t) = -i\Theta(t)\langle\{\hat{N}^m(t)\Psi_d(t), \Psi_d^\dagger(0)\}\rangle$$

$$(\epsilon - E_0 + V_g + i\hat{\Gamma})\Gamma_{Ndd}^R = A - i\tilde{\Gamma}G_{dd}^R + 4E_c\Gamma_{N^2 dd}^R \quad A = \langle\{\hat{N}\Psi_d, \Psi_d^\dagger\}\rangle$$

Truncation $\Gamma_{N^2 dd}^R = B\Gamma_{Ndd}^R$

Self-consistency

$$-\frac{1}{\pi} \int d\epsilon f_d(\epsilon) \text{Im}[G_{dd}^R(\epsilon)]_{11} = \langle \hat{n}_d \rangle,$$

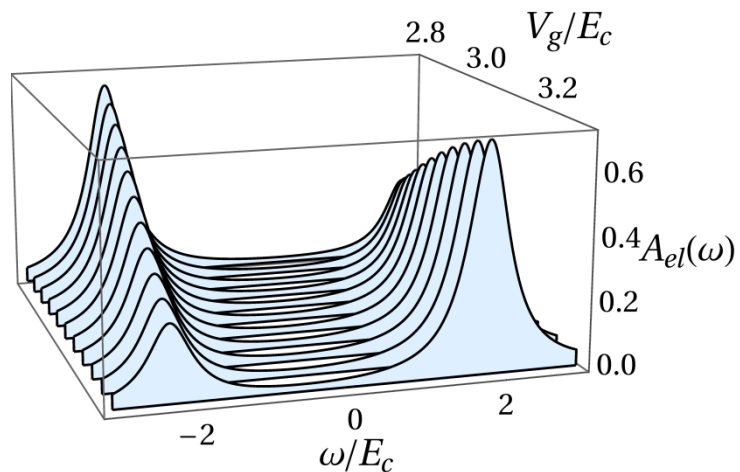
$$-\frac{1}{\pi} \int d\epsilon f_d(\epsilon) \text{Im}[G_{dd}^R(\epsilon)]_{22} = 1 - \langle \hat{n}_d \rangle,$$

$$-\frac{1}{\pi} \int d\epsilon f_d(\epsilon) \text{Tr}\{\text{Im}\Gamma_{Ndd}^R(\epsilon)\} = \langle \hat{N} - (1 - \hat{n}_d) \rangle,$$

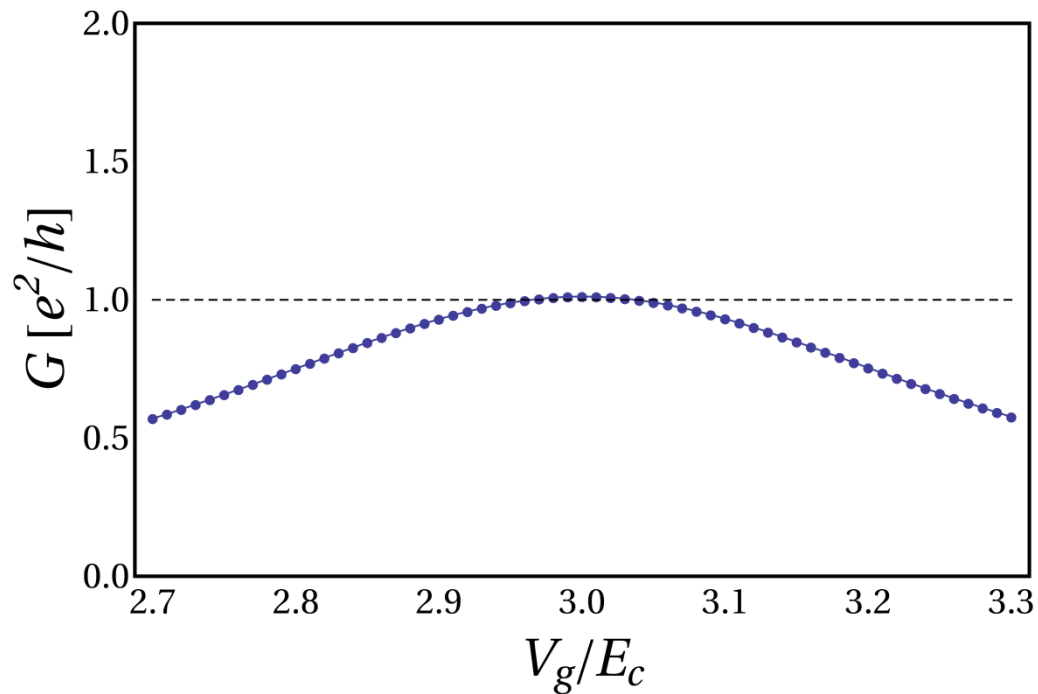
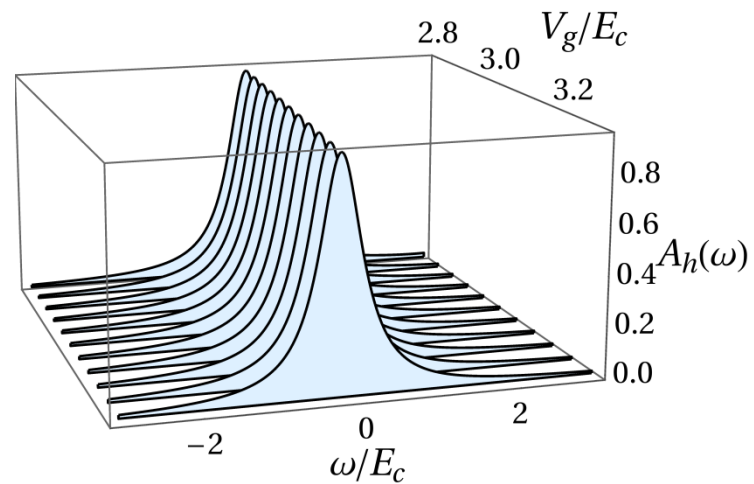
EOM results

$$E_c/\Gamma = 3$$

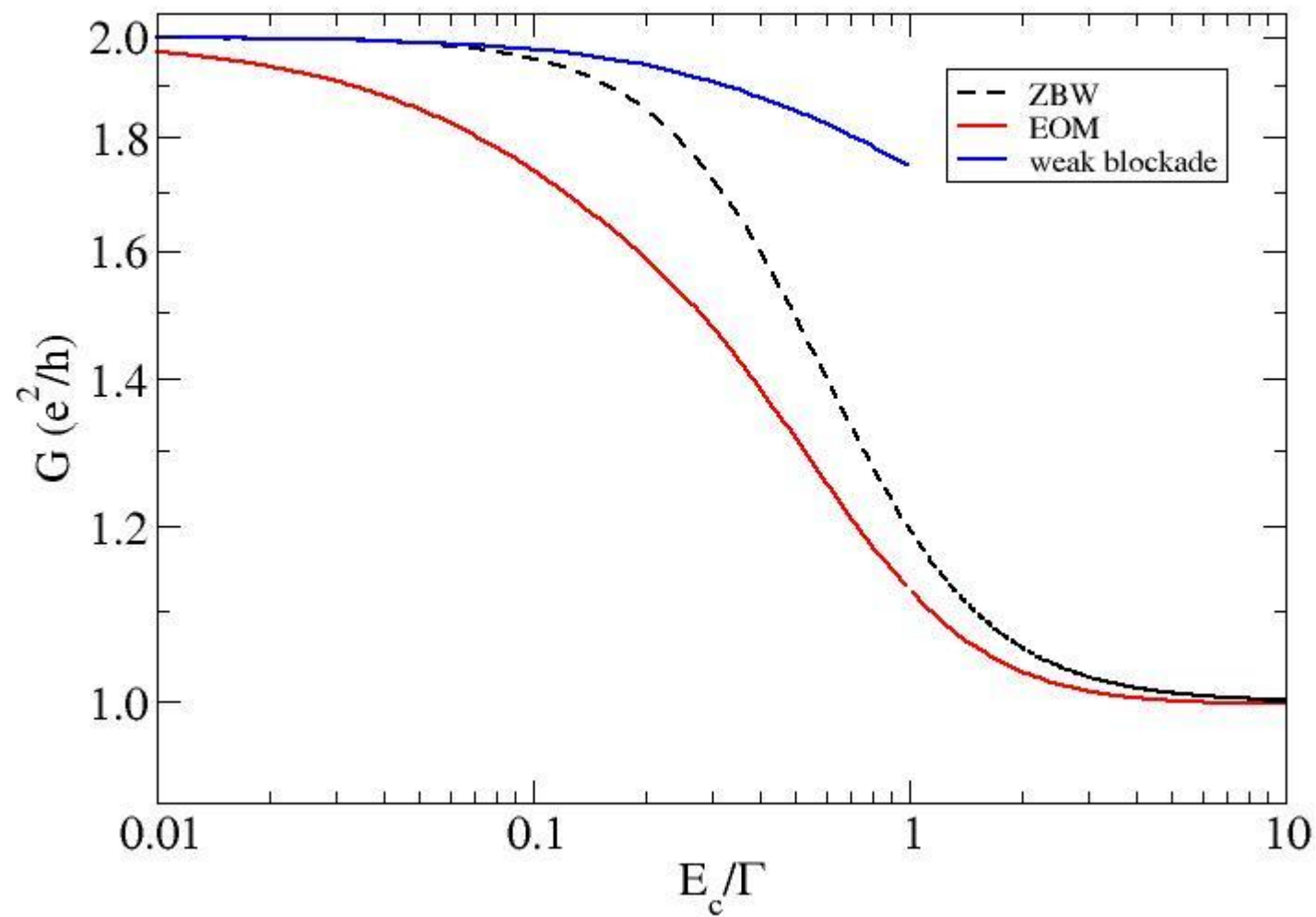
e-spectral density



"h"-spectral density



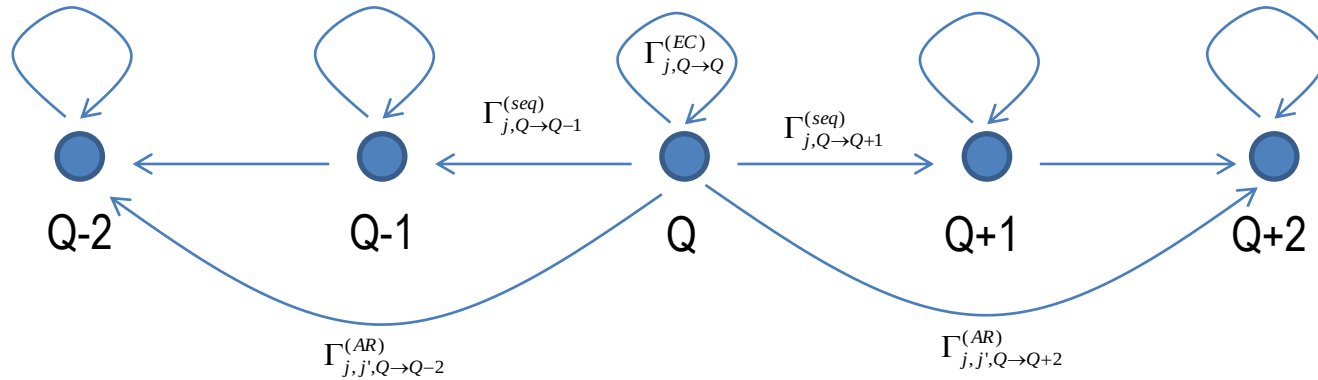
Crossover of peak conductance



Finite temperature: Master equation approach

$$Q = 2N + n_d$$

$$E_Q = E_c(Q - n_g)^2$$



$$\Gamma_{j,Q \rightarrow Q \pm 1}^{(\text{seq})} = (\Gamma_j/2) f(E_{Q \pm 1} - E_Q \mp \mu_j)$$

$$\Gamma_{j,Q}^{(\text{EC})} = \frac{\Gamma_L \Gamma_R}{8\pi} \int d\epsilon f(\epsilon - \mu_j) [1 - f(\epsilon - \mu_{-j})] \\ \times \left| \frac{1}{\epsilon - (E_{Q+1} - E_Q) + i0} - \frac{1}{\epsilon - (E_Q - E_{Q-1}) - i0} \right|^2$$

$$\Gamma_{j,j',Q \rightarrow Q \pm 2}^{(\text{AR})} = \frac{1 + \delta_{j,-j'}}{2} \frac{\Gamma_j \Gamma_{j'}}{8\pi} \int d\epsilon \int d\epsilon'$$

$$\times f(\pm(\epsilon - \mu_j)) f(\pm(\epsilon' - \mu_{j'})) \delta(\epsilon + \epsilon' \mp (E_{Q \pm 2} - E_Q))$$

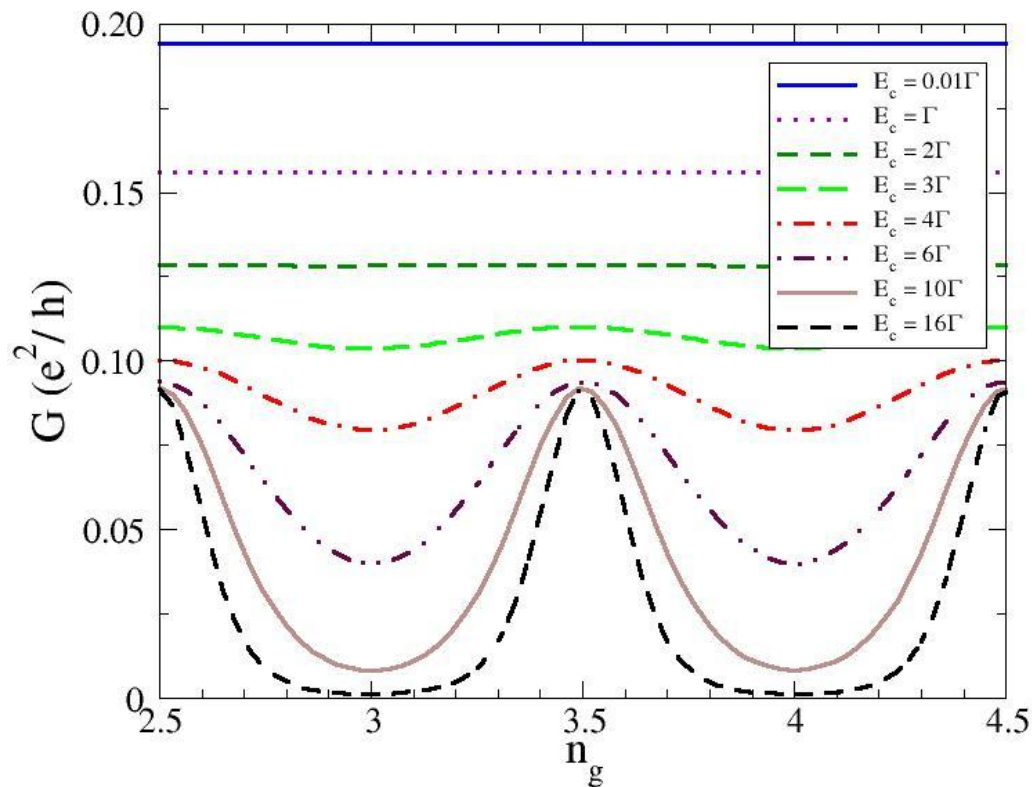
$$\times \left| \frac{1}{\epsilon \mp (E_{Q \pm 1} - E_Q) + i0} - \frac{s_j s_{j'}}{\epsilon' \mp (E_{Q \pm 1} - E_Q) + i0} \right|^2$$

$$s_L = +1$$

$$s_R = -1$$

Results from Master equation approach

CB oscillations $T = 2\Gamma$

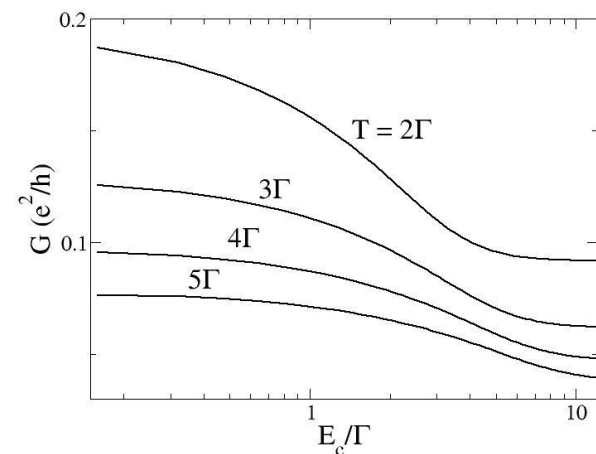


$$G_{\text{valley}}(\delta) = \frac{e^2}{h} \frac{\Gamma_L \Gamma_R}{E_c^2} \frac{1}{(1 - 4\delta^2)^2}$$

$$\delta = n_g - [n_g]$$

$$(\Gamma, T) \ll E_c$$

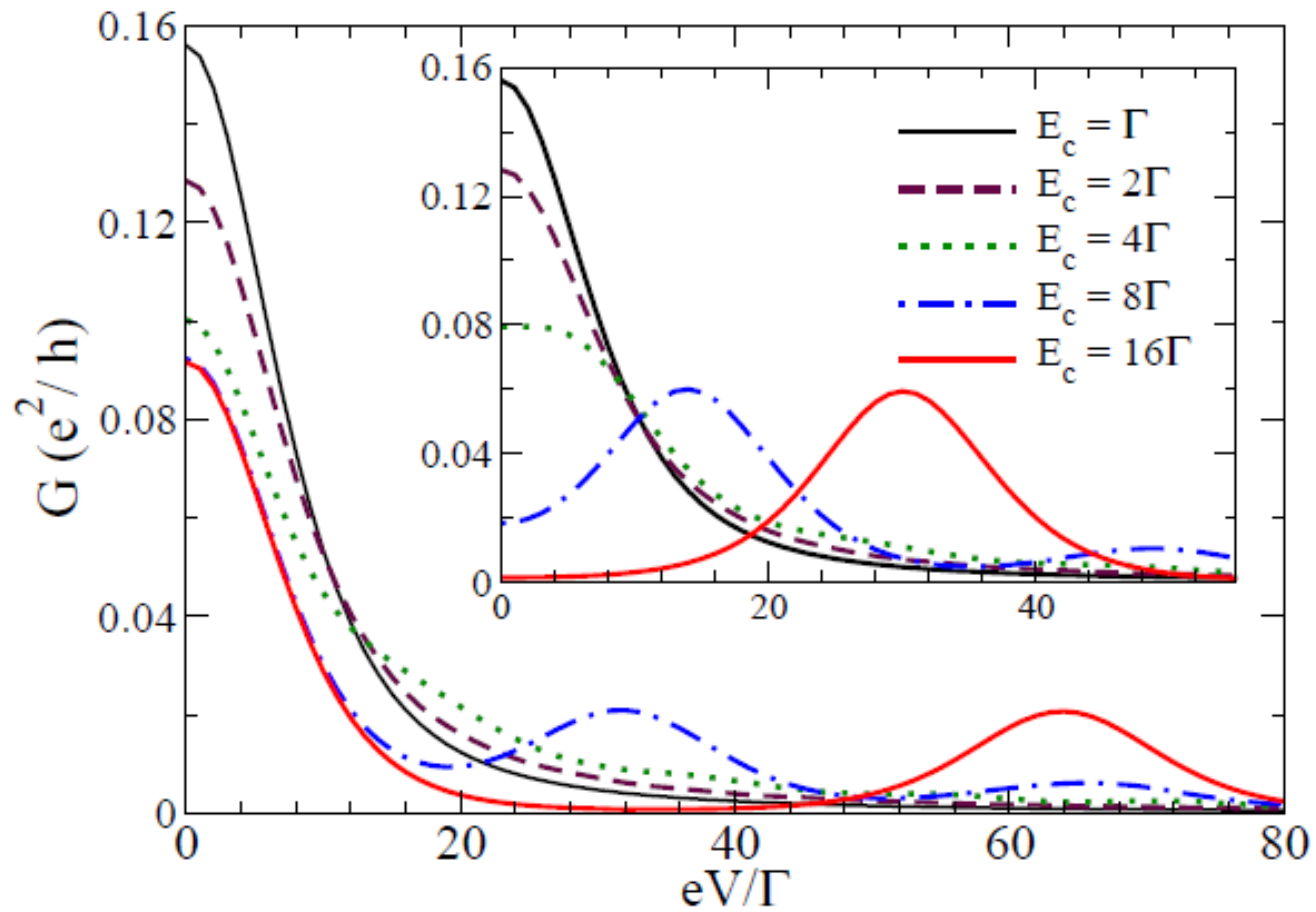
Peak conductance



$$G_{\text{peak}}(\delta) = \frac{e^2}{h} \frac{\pi \Gamma}{16T} \frac{1}{\cosh^2(\delta E_c/T)}$$

$$\delta = n_g - [n_g] - 1/2$$

Finite voltage sideband peaks



Conclusions

Andreev bound states in QDs

- * Qualitative description of CNTs results using phenomenological models
- * Validity of mean field (HFA): good agreement with NRG for $\Delta \gg T_K$

Majorana Single Charge Transistor

- * Insight from several different methods (WB, ZBWM, EOM, ME)
- * Crossover of peak conductance from $2e^2/h$ to e^2/h as a function of E_c/Γ
- * Coulomb blockade oscillations and side band peaks in non-linear conductance
- * Work in progress: consequences for non-local transport (crossed Andreev)