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How the next-nearest-neighbor interactions change the phase diagram of a fully frustrated XY model?

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Abstract

We introduce a fully frustrated XY model with nearest-neighbor (nn) and next-nearest-neighbor (nnn) couplings which can be realized in Josephson junction arrays. We study by means of Monte Carlo simulations the phase diagram for $0 \leqslant x \leqslant 1$ (x is the ratio between nnn and nn couplings).

Keywords: Josephson junction arrays; Phase transitions

A variety of two-dimensional systems undergo a phase transition without any spontaneous symmetry breaking, the Kosterlitz–Thouless–Berezinskii (KTB) transition. Josephson junction arrays are experimental realization of the two-dimensional XY model. Until recently, it was thought that Josephson arrays could only model systems with nearest-neighbors (nn) couplings. Theoretical [1,2] and experimental [3] works on infinite range array opened a new field of investigation in these systems. We study the properties of a two-dimensional Josephson junction array with both nn and next-nearest-neighbor (nnn) couplings in an external magnetic field. Proximity-junction arrays may be good candidates to experimentally probe some of the effects discussed in this work. The model system which we consider is defined by the following Hamiltonian:

$$H = -\sum_{\langle\langle i,j\rangle\rangle} J_{ij}\cos(\theta_i - \theta_j - A_{ij}), \tag{1}$$

where $J_{ij} = J > 0$ for nn and $J_{ij} = xJ$ for nnn and the symbol $\langle\langle\cdot\rangle\rangle$ refers to the sum over nn and nnn. The magnetic field enters the model through $A_{ij} = (2\pi/\Phi_0)\int_i^j \mathbf{A}\cdot d\mathbf{l}$ ($\mathbf{A}$ is the vector potential). The magnetic frustration is $f = \sum A_{ij}/(2\pi)$, where the summation runs over the elementary plaquette. The phases θ, the angular variables in the XY model, are the phase of the superconducting order parameter of a given island. We study the case $f = \frac{1}{2}$ on a square lattice.

The system we consider presents characteristics of both the fully frustrated (FF) and unfrustrated XY models. While the elementary square plaquette is FF ($f = \frac{1}{2}$), the smallest square plaquette formed only by nnn couplings is not frustrated. For $x < x_0 \equiv 1/\sqrt{2}$ the ground state (g.s.) of this system turns out to be exactly the same as in the FF model without nnn couplings. On the other hand for $x > x_0$ the g.s. is exactly the same as in the absence of nn coupling and consists of two square sublattices in each of which the nnn interaction is unfrustrated. The relative orientation of these two sublattices can be arbitrary. At the special point $x = x_0 \approx 0.707$ where the energies of both these states coincide they can be transformed into each other by a continuous transformation without increasing the energy. In order to investigate the phase diagram and the critical behavior of this system we have calculated the helicity modulus Γ and the staggered chiral magnetization M by using standard MC simulations for different x. M is the order parameter which

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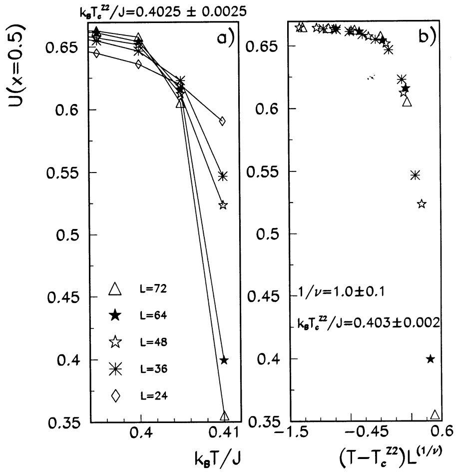

Fig. 1. XY model with nnn interactions with $x = 0.5$. (a) Binder's parameter U versus T for several lattice sizes L. Excluding the data of smallest size ($L = 24$) one can estimate $0.400 < k_B T_c^{Z_2}/J < 0.406$. (b) Collapse of the data in panel (a) (excluding the $L = 24$ data). The scaling parameters are $k_B T_c^{Z_2}/J = 0.403 \pm 0.003$ and $1/\nu = 1.0 \pm 0.1$.

controls the Ising-like transition. In order to obtain a precise determination of the Ising-like critical temperature $T_c^{Z_2}$ and of the critical exponent ν associated to the divergence of the correlation length we have calculated the Binder's cumulant of the staggered chiral magnetization $U = 1 - \langle M^4 \rangle / 3 \langle M^2 \rangle^2$. In Fig. 1 we show the result for $x = 0.5$.

From Fig. 1a we can estimate $k_B T_c^{Z_2}/J = 0.403 \pm 0.003$. The data collapsing in Fig. 1b gives an estimate of $1/\nu = 1.0 \pm 0.1$. We have studied various values of $0 \leqslant x \leqslant 1$. The critical temperature $T_c^{Z_2}$ decreases with increasing x. For $x \geqslant 0.8$, there is no sign of a Ising-like transition which is rather natural since at $x = x_0$ the two-fold degeneracy of the ground state disappears (as shown in Fig. 2).

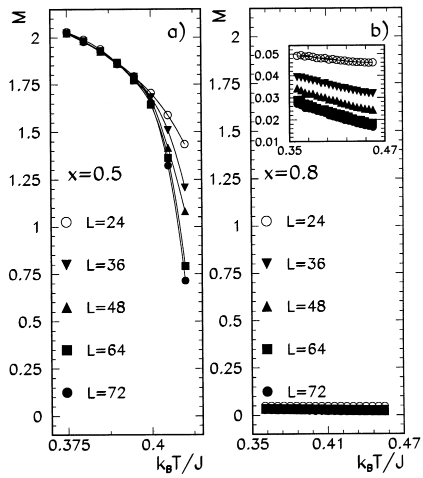

Fig. 2. The staggered chiral magnetization M versus T for the XY model with nnn interactions for $x = 0.5$ (a) and $x = 0.8$ (b) for several lattice sizes L. For large L and low T, M goes to a non-zero value for $x = 0.5$ and vanishes for $x = 0.8$.

Following the procedure proposed by Weber and Minnhagen [4] KBT the critical temperature $T_c^{U(1)}$ is estimated by using the following *ansatz* for the size dependence of the helicity modulus $\pi\Gamma/2T_c = 1 + 1/2(\ln L + \ln l_0)$ where l_0 is a fit parameter. As for $T_c^{Z_2}(x)$ also $T_c^{U(1)}(x)$ decreases with increasing x up to $x = 0.7$.

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