Effect of dissipation on the low-temperature properties of a tunnel-junction chain

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A regular chain of tunnel junctions with linear ohmic dissipation is investigated. Account is taken in the electrostatic energy both of the capacitance of the superconducting grains making up the chain and of the mutual capacitance of the neighboring grains. The instanton-gas concept is used to draw the zero-temperature phase diagram. It is shown that the line separating the regions of existence of coherent (superconducting) and incoherent states of the chain as a whole is subdivided by singular points into four segments. The zero-temperature superconductivity is destroyed at arbitrarily low temperature and manifests itself only in a power-law decrease of the chain resistance R with temperature ($R \propto T^{\lambda}$ as $T \to 0$). In the semiclassical approximation this decrease can be attributed to a quantum-fluctuation phase slip at the tunnel junctions. The values of the exponent λ are obtained in various regions of the phase diagram, including the universal values assumed by the exponent on various segments of the phase boundary. Two auxiliary phase-transition lines, on which the exponent becomes discontinuous, are present in the region of existence of the coherent state.

1. INTRODUCTION

The simplest model that can be used to describe a regular chain of tunnel junctions can be specified by the Hamiltonian

$$H=\sum_{j=-\infty}^{\infty}\left[\frac{E_C}{2}\hat{n}_j^2-E_J\cos(\varphi_j-\varphi_{j-1})\right], \qquad (1)$$

where the first term is the energy of the electrostatic interaction between the grains making up the chain and the conducting substrate ($E_c = 4e^2C^{-1}$, C^{-1} is the diagonal element of the reciprocal-capacitances matrix), while the second is the Josephson-interaction energy of the neighboring grains ($E_j = \hbar I_c/2e$, I_c is the nonrenormalized critical current of a single junction). The variable n_j, which has the meaning of the charge of the jth grain in $2e$ units, is conjugate to the order-parameter phase φ_j at the same grain: $\hat{n}_j = -i\partial/\partial\varphi_j$.

The model (1) can be described by a single dimensionless parameter[1)] $\varkappa = \pi(E_J/E_c)^{1/2}$. When $\varkappa$ is varied at zero temperature, a phase transition takes place in the model (1) between the coherent (superconducting) and incoherent states of the chain as a whole.[1,2] This phase transition belongs to the same universality class as the Berezinskiĭ–Kosterlitz–Thouless phase transition[3–5] in the two-dimensional classical XY model since, as shown in Ref. 1, the partition functions of these models are isomorphic.

It is possible to retain in (1) only the diagonal element of the reciprocal–capacitance matrix solely in the presence of a conducting substrate.[1] In the opposite case the Coulomb interaction of the remote grains destroys the superconductivity at all ratios of the model parameters.[6] In the presence of a conducting substrate the appearance of induced charges causes a more rapid decrease of the electrostatic-interaction energy of the remote grains with increase of distance, and leads now only to small quantitative (but not qualitative) differences from the model (1), so that it can be discarded. Allowance for the nonadiabaticity of the alignment of the induced charges would correspond to consideration of frequency-dependent corrections to the first term of (1), which are likewise immaterial.

At finite temperatures, the zero-temperature superconductivity of the chain as a whole is disturbed and is manifested only via a power-law decrease of the linear resistance R of the chain with temperature[2]: $R \propto T^{\lambda}$ ($T \to 0$). In an analysis within the context of the semiclassical approximation (to which we propose to adhere here for the most part) the appearance of finite resistance in the chain can be due to the finite probability of slippage of the phase P in the absence of an external current I. In these case, for small external currents, a phase slip by $\pm 2\pi$, accompanied by a chain-energy change $\varepsilon = \mp 2\pi I\hbar/2e$, will have a probability $(1-\varepsilon/T)P$, leading to an average rate of phase loss:

$$\left\langle\frac{\partial}{\partial t}\Delta\varphi\right\rangle=\frac{(2\pi)^2\hbar I}{2eT}P,$$

corresponding to a linear resistance of the chain (per junction) equal to

$$R=\frac{\hbar}{2e}\left\langle\frac{\partial}{\partial t}\Delta\varphi\right\rangle\Big/ I=4\pi^2R_Q\frac{\hbar P}{T},\qquad R_Q=\frac{\hbar}{4e^2}. \qquad (2)$$

Here and henceforth the Boltzmann constant is included in the definition of the temperature. It is shown in Ref. 2 that $P(T)$ has a power-law dependence as $T \to 0$, and the exponent λ of the temperature dependence of the resistance takes on for $\varkappa \gg 2$ the value $\lambda = 2\varkappa - 3$. As the transition point is approached, λ tend to unity.

At zero temperature the phase-slip probability in the coherent state turns out to have a nonlinear dependence on the current, meaning a nonlinear current-voltage (IV) characteristic: $U \propto I^{\lambda+1}$.

The purpose of the present paper is an investigation of the influence exerted on the position of the phase-transition point and on the temperature dependence of the chain resistance by the dissipative properties of the chain junctions.

It was shown in the papers of Ambegaokar *et al.*,[7] devoted to a macroscopic derivation of an effective Euclidean action that determines the partition function of a single tunnel junction

$$Z=\int D\theta(\tau)\exp\{-S[\theta(\tau)]\}$$

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(where θ is the phase difference at the junction), that exclusion of the electron variables leads to the appearance in S, besides the terms corresponding to the Coulomb and Josephson energies, also of a nonlocal term of the form

$$S_D=\iint d\tau\, d\tau' K(\tau-\tau')\left[\sin\frac{\theta(\tau)-\theta(\tau')}{4}\right]^2, \tag{3}$$

whose periodicity corresponds to single-electron tunneling.

Electron tunneling between the superconducting banks of the junction leads to a short-range kernel $K(\tau)$ that permits, with good accuracy, to replace (3) by a local term

$$S_D=\int d\tau \frac{M}{2}\left(\frac{\partial\theta}{\partial\tau}\right)^2, \quad M=\frac{3\pi}{32}\frac{R_Q}{R_N}\frac{\hbar}{\Delta},$$

whose form is equivalent to renormalization of the intrinsic capacitance of the junction[7] (here R_N is the tunnel resistance of the junction in the normal state).

In Sec. 2 we consider the influence exerted on the properties of a superconducting-junction chain by the mutual capacitance of neighboring grains; this capacitance, as we have noted, can be strongly renormalized by quasiparticle tunneling.

The bulk of the paper, however, will be devoted to an investigation of a chain of junctions with linear ohmic dissipation, introduced by Caldeira and Leggett[8] from semiphenomenological considerations and described by the quadratic nonlocal term in the Euclidean action:

$$S_D=\iint d\tau\, d\tau' \frac{\eta}{4\pi}\left[\frac{\theta(\tau)-\theta(\tau')}{\tau-\tau'}\right]^2, \tag{4}$$

the form of which corresponds to presence, in the junction, of a shunting normal resistance $R_{sh}=R_Q/\eta$ not connected with the direct tunneling of the quasiparticle between the junction banks.

Zero-temperature phase diagrams of regular d-dimensional ($d=1,2,3$) lattices of superconducting junctions with different dissipation mechanisms have been the subject of many earlier studies.[9–20] However, the methods used there, e.g. variational approximation[9,10,18] and mean-field theory[11–15,19–20] provide answers that depend only insignificantly on the dimensionality. Understandably, for these methods to be applicable to a description of the low-dimensionality ($d=1$) systems of interest to us an additional verification with adequate allowance for quantum fluctuations is necessary.

We analyze in the present paper the question of the zero-temperature phase diagram of a chain of superconducting junctions with linear ohmic dissipation, starting from the properties of topological excitations of the effective action of the system. We investigate also the temperature dependence of the chain in the low-temperature limit. We shall deal throughout only with the case when a conducting substrate is present and leads to the existence of a zero-temperature phase transition and in the absence of dissipation. The main results are contained in the concluding section.

2. ALLOWANCE FOR THE MUTUAL CAPACITANCE OF NEIGHBORING GRAINS

We consider in the present section a chain of superconducting junctions described by a dimensionless Euclidean action

$$S_0\{\varphi\}=\sum_{j=-\infty}^{\infty}\int_{-\infty}^{\infty} d\tau\left\{\frac{m}{2}\left(\frac{\partial\varphi_j}{\partial\tau}\right)^2+\frac{M}{2}\left(\frac{\partial\varphi_j}{\partial\tau}-\frac{\partial\varphi_{j-1}}{\partial\tau}\right)^2-V\cos(\varphi_j-\varphi_{j-1})\right\}, \tag{5}$$

where $m=\hbar/E_c=(\hbar/4e^2)C_0$, $V=E_j/\hbar=I_c/2e$, and $M=(\hbar/4e^2)C_1$ is connected with the effective mutual capacitance C_1 of neighboring grains and can be due, in particular, to quasiparticle tunneling.

If at least one of the following conditions

$$mV\gg 1, \quad MV\gg 1$$

is met, the quantum fluctuations of the variables $\theta_j=\varphi_j-\varphi_{j-1}$ turn out to be small compared with period of the cosinusoidal potential, so that a semiclassical approximation can be used. The decisive role in the zero-temperature partition function corresponding to the action (5) will be played in this case by instantons—extremal trajectories on which one of the variables, θ_j, passes through the maximum of the periodic potential, and the action has a local minimum. In addition to trajectories corresponding to single instantons and constituting exact extrema of the action, it is natural to consider also trajectories corresponding to a superposition of several instantons located at different points of space-time.

The case $M=0$ [corresponding to the Hamiltonian (1)] was considered in Ref. 1 and it was shown that the space-time distribution of the field φ on the extremal trajectory turns out to be the same as in a vortex in the classical two-dimensional XY model. Accordingly, the interaction of the instantons is found to be logarithmic, and when the factor $2\pi(mV)^{1/2}$ preceding the logarithm changes, a Berezinskiĭ–Kosterlitz–Thouless phase transition takes place—instanton depairing that leads to loss of phase coherence.

According to Kosterlitz's renormalization-group analysis,[5] the phase transition occurs when

$$2\pi(mV)^{1/2}=4+O(y^2), \tag{6}$$

where y has the meaning of the chemical activity of the instanton and is a pre-exponential factor whose calculation requires allowance for the fluctuations in the vicinty of the trivial and instanton extremal trajectories.

The finite M leads even in the limit $M\gg m$ to no qualitative changes whatever. Instanton interaction at large distance in space-time remains logarithmic as before, with the factor preceding the logarithm remaining the same (and

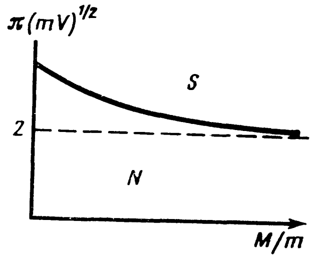

FIG. 1. Phase diagram of a chain of tunnel junctions with account taken of the mutual capacitance of neighboring grains: S and N are respectively the existence regions of the coherent and incoherent states.

hence independent of M). As the same time, the chemical activity y decreases exponentially with increase of M so that, according to (6), the critical value of $(mV)^{1/2}$ tends to $2/\pi$ as $M/m \to \infty$. The corresponding phase diagram is shown in Fig. 1.

To illustrate the statements of the preceding paragraph, we can transform to a single-particle effective action.[2] Only one of the variables θ_j (say, θ_l) deviates substantially on the instanton trajectory from its equilibrium value and passes to the neighboring minimum of the periodic potential. The remaining variables θ_j with $j\neq l$ do not deviate very significantly from their equilibrium values and remain near the bottoms of the corresponding potential wells. This makes it possible to replace the cosinusoidal potential by a harmonic one in the calculation of the action on the extremal trajectory for all the variables θ_j except θ_l, and then eliminate them from the action by using Gaussian integration, going over thus to a single-particle effective action:

$$S_{eff}\{\theta(\tau)\}=\frac{1}{2}\iint d\tau\, d\tau' G_{eff}(\tau-\tau')\theta(\tau)\theta(\tau') - \int d\tau V\cos\theta(\tau), \quad (7)$$

where the Fourier transform of the propagator $G_{\rm eff}(\tau)$ takes in the case of the model (5) the form

$$G_{eff}^{-1}(\omega)=\left\{\frac{m}{4}\left[V+\left(M+\frac{m}{4}\right)\omega^2\right]\right\}^{1/2}|\omega|+\left(M+\frac{m}{4}\right)\omega^2, \quad (8)$$

which describes a particle with an effective mass

$$m_{eff}=\lim_{\omega\to\infty}{}^{-1}\,\omega^2 G_{eff}(\omega)=\left[\frac{m}{4}\left(M+\frac{m}{4}\right)\right]^{1/2}+M+\frac{m}{4}, \quad (9)$$

moving in a medium with linear ohmic dissipation corresponding to an effective-viscosity coefficient

$$\eta_{eff}=\lim_{\omega\to 0}{}^{-1}\,|\omega|G_{eff}(\omega)={}^1/_2(mV)^{1/2}.$$

Such an approximation works fairly well also if $M=0$, and becomes practically exact for $M\gg m$, since the smaller the ratio m/M the smaller the deviations of the variables θ_j with $j\neq l$ from equilibrium. For $M\gg m$ we can assume that $m_{\rm eff}\approx M$.

After these transformations we can use the properties, investigated relatively in detail, of systems with linear ohmic dissipation and with a periodic potential. According to Refs. 21 and 22, in the limiting case $m_{\rm eff}V\gg\eta_{\rm eff}^2$, corresponding in our problem to satisfaction of the condition $M\gg m$, the preexponential factor can be calculated without allowance for the effective viscosity and is equal to half the hopping frequency of a particle of mass $m_{\rm eff}$ in a potential $-V\cos\theta$:

$$y=\Delta_0/2=4\left(\frac{V^3}{\pi^2 m_{eff}}\right)^{1/4}\exp[-8(m_{eff}V)^{1/2}].$$

At the same time, long-range interaction of instantons in imaginary time is determined completely by the effective viscosity and takes the form

$$S_{int}=2\pi\eta_{eff}\ln\frac{V\tau^2}{m_{eff}}, \qquad V\tau^2\gg m_{eff}. \quad (10)$$

For widely spaced instantons it is necessary to replace $V\tau^2$ in the logarithm of (10) by $V\tau^2+mR^2$.

In Refs. 11 and 15 the phase diagram of the model (5) was investigated in the mean-field approximation (or in the approximation of the auxiliary field introduced with the aid of the Stratanovich–Hubbard approximation). This yielded for the phase transition an equation in the form

$$V\left[\frac{m}{4}\left(M+\frac{m}{4}\right)\right]^{1/2}=\text{const}$$

and a conclusion, at variance with our results, that as $M/m\to\infty$ the critical value of the product mV tends to zero. Such an approach, unfortunately, cannot identify a region in which the fluctuations can be disregarded. Our method, on the other hand, becomes more accurate the smaller the ratio m/M and the less important the corrections proportional to higher powers of y.

As already mentioned, at finite temperature the phase-slip probability P becomes different from zero. For a system with effective action (7) and $m_{\rm eff}V\gg\eta_{\rm eff}^2$, the phase-slip probability, i.e., the probability of incoherent tunneling of the variable θ to a neighboring minimum of the periodic potential, depends at low temperatures on the temperature like[22-24]

$$P=\frac{\pi^{1/2}}{4}\frac{\Gamma(\alpha_{eff})}{\Gamma(\alpha_{eff}+1/2)}\frac{\Delta_0^2}{\Omega_0}\left(\frac{\pi T}{\hbar\Omega_0}\right)^{2\alpha_{eff}-1},$$

$$\alpha_{eff}=2\pi\eta_{eff}>1, \qquad \Omega_0^2=\frac{V}{m_{eff}}, \quad (11)$$

which yields, in conjunction with (2), a resistance temperature-dependence coefficient $\lambda=2\pi(mV)^{1/2}-2$. Expression (11) corresponds to a periodic extremal trajectory of the action (5), on which one of the variables θ passes through the potential maximum and then returns.

This approximation is quite adequate in the temperature interval

$$T_0\gg T\gg T_1\sim\hbar(V/m^3)^{1/4},$$

which exists only under the condition $(mV)^{1/2}\gg m_{\rm eff}/m$. $T_0\sim\hbar\Omega_0$ is the temperature of the transition to the thermal-activation regime, for which the classical Arrhenius law is valid: $P\propto\exp(-2E_j/T)$. For $T\lesssim T_l$ it is necessary to consider also configurations corresponding to spatially separated instantons[2]; at $T\ll T_0$ and $T\ll T_1$ this leads to multiplication of (11) by a factor of order T_1/T and correspondingly to a decrease of the exponent λ to

$$\lambda=2\pi(mV)^{1/2}-3. \quad (12)$$

The combination $2\pi(mV)^{1/2}$ in (12) is the pre-logarithmic factor in the nonrenormalized interaction of zero-temperature instantons. In a more rigorous approach Eq. (12) will contain for this factor a renormalized value that assumes on the transition curve a universal value equal to 4. This means that on the transition curve (Fig. 1) the exponent λ becomes equal to unity, and $\lambda>1$ for the entire region in which the zero-temperature coherent state exists.

3. JUNCTION CHAIN WITH LINEAR OHMIC DISSIPATION

Having investigated the influence of the mutual capacitance of neighboring grains on the properties of a regular chain of tunnel junctions, we now take into consideration linear ohmic dissipation, i.e., we consider a system described by the Euclidean action

$$S\{\varphi\}=S_0\{\varphi\}+\sum_{j=-\infty}^{\infty} S_D\{\varphi_j(\tau)-\varphi_{j-1}(\tau)\}, \tag{13}$$

where $S_D\{\theta(\tau)\}$ is of the form (4).

The phase diagram of the model (13) was investigated for the case $M=0$ in Refs. 9 and 10 by using a variational analysis corresponding to replacement of the Josephson term in the energy by a harmonic one (see Fig. 2). This procedure can be used with equal success for a finite ratio M/m, which leads to a shift of the nonuniversal (curved) part of the interphase boundary (Fig. 2). Since, however, in the limiting case $m\to 0$ (corresponding to breakup of the chain into individual junctions) this method leads to patently incorrect results (cf. Refs. 21 and 22), the question of the shape of the phase diagram for finite m requires a more thorough investigation.

The phase diagram of the very same model (including finite M/m) was investigated in Refs. 14 and 15 in the framework of the mean-field theory. We have verified in the preceding section, with the nondissipative case as the example, that this approach is also not reliable enough for one-dimensional systems.

Since in the absence of dissipation, when the partition function of the chain is isomorphous to the partition function of the classical two-dimensional XY model, the junction can be adequately described only by using a recursive allowance for the renormalization of the interaction of topological excitations[5] (and not in the context of the mean-field theory or in the variational approximation for the field variables themselves), we shall eschew even in the presence of ohmic dissipation the concept of an instanton gas. We emphasize that the semiclassical approximation is valid in a wide range. In particular, if the condition $MV\gg 1$ is met it is valid for all values of the parameters m, V, and η.

Just as in the nondissipative case (Sec. 2), the exact form of the extremal trajectory can be obtained only by replacing the cosinusoidal potential by a harmonic one:

$$-V\cos\theta_j\to\frac{V}{2}\theta_j^2,$$

and by a piecewise-parabolic potential for that variable θ_l which undergoes tunneling to the neighboring minimum:

$$-V\cos\theta_l\to\frac{V}{2}\min_{p\in Z_n}(\theta_l-2\pi p)^2.$$

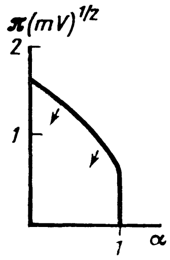

FIG. 2. Phase diagram of chain of tunnel junctions with linear ohmic dissipation (for $M=0$), obtained in Refs. 9 and 10 by a variational analysis. The arrows indicate the direction the displacement of the curved section of the interphase boundary with increase of M.

The extremal trajectory takes in this case the form

$$\theta_j(\tau)=2\pi\nu\int_{-\pi}^{\pi}\frac{dk}{2\pi}e^{ik(j-l)}\left\{-\frac{1}{2}+\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(\tau-\tau_0)}\times\frac{V}{-i\omega[V+g_0(k,\omega)]}\right\}, \tag{14}$$

where l and τ_0 are the coordinates of the instanton center, $\nu=\pm 1$ is its topological charge, and

$$g_0(k,\omega)=\left\{\eta|\omega|+\left[M+\frac{m}{2(1-\cos k)}\right]\omega^2\right\}^{-1} \tag{15}$$

is the propagator for the field θ in the absence of a potential.

Substituting (14) in (13) (with allowance for the change of the form of the potential) we arrive at the logarithmically diverging expression

$$S=\frac{1}{2}\int_{-\pi}^{\pi}\frac{dk}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}G_0(k,\omega),$$

where the Green's function

$$G_0(k,\omega)=\frac{4\pi^2}{\omega^2/V+\omega^2 g_0(k,\omega)}$$

$$=\frac{4\pi^2 mV}{(V+\eta|\omega|+M\omega^2)2(1-\cos k)+m\omega^2}\frac{V}{V+\eta|\omega|+M\omega^2}+\frac{4\pi^2}{|\omega|/(\eta+M|\omega|)+\omega^2/V} \tag{16}$$

determines also the law governing the interaction of widely spaced instantons:

$$\Delta S(R,\tau)=\int_{-\pi}^{\pi}\frac{dk}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}G_0(k,\omega)\exp i(kR-\omega\tau),$$

which is valid (in the leading orders in R and τ) also when the initial cosinusoidal potential is retained (cf. Ref. 25).

The first term in (16) corresponds to a logarithmic space-time instanton interaction that has the same asymptotic form and is characterized by the same constant $2\pi(mV)^{1/2}$ as in the nondissipative case. The second term, on the other hand, determines the dissipation-induced additional instanton interaction which is diagonal in the site number and is logarithmic in time (with a prelogarithmic factor $4\pi\eta$). The chemical activity of the instantons, just as in the nondissipative case, turns out to be exponentially small in $(MV)^{1/2}$ as $M\to\infty$ and has a finite limit as $M\to 0$. Neither constant of the logarithmic interactions depends on M.

The problem has thus been reduced to an investigation of the phase diagram of a two-dimensional Coulomb gas with an additional strongly anisotropic (diagonal in the sites) logarithmic interaction.

4. STABILIZATION OF COHERENT STATE IN THE CASE OF LARGE DISSIPATION

We recall once more that in terms of an instanton gas the coherent state of a chain corresponds to a dielectric phase in which all the instantons are bound into neutral pairs, while the coherent state corresponds to that of a plasma and is characterized by a finite density of the free unbound instantons.[1,2]

As the simplest approximation in which the existence of a phase transition is possible in the system considered, we use the Debye–Hückel approximation, according to which a finite free-instanton density c leads to a Green's-function renormalization that reduces to the appearance of a self-energy part that is independent of k and ω and is equal to c. In turn, c is determined by a renormalized Green's function, so that we can write the self-consistent equation[25]

$$G^{-1}(k,\omega)=G_0^{-1}(k,\omega)+2y\exp[-{}^1/_2G(R=0,\tau=0)]. \quad (17)$$

Solution of (17) shows that for small y there exist two parameter ranges, in one of which Eq. (17) has a unique solution $G(k,\omega)=G_0(k,\omega)$ (corresponding to coherent state of the chain), and in the other this solution turns out to be unstable to the onset of a finite self-energy part. For $G_0(k,\omega)$ in the form (16), the boundary between these regions follows the line

$$\pi(mV)^{1/2}/2+\alpha=1,\ \alpha=2\pi\eta, \quad (18)$$

the position of which at small y does not depend on y.

Undoubtedly, this approximation calls for further refinement, since it takes into account the influence exerted on the instanton-interaction renormalization only by the free instanton, and the effect due to neutral pairs is neglected, so that $G(k,\omega)$ has the same form in the entire region of existence of the dielectric phase. Since the neutral-pair polarization also weakens the interaction between remote instantons, one can expect the true phase separation boundary to be shifted, compared with (18), towards larger values of the parameters.

In contrast to the case considered in Sec. 2, the additional logarithmic instanton interaction which is diagonal in the sites causes a pair of instantons of unlike sign to have a finite action only if both instantons are at the same site, otherwise the action turns out to diverge logarithmically also for the instanton pair. Thus, at sufficiently large η the instantons will be present in the system only in the form of bound one-site pairs. We consider now the influence of such pairs on the renormalization of the interaction of widely spaced instantons.

The correction to $G_0(k,\omega)$ of principal (first) order in the bound-pair density (i.e., of second order in y) takes the form[26]

$$\delta G(k,\omega)=-G_0^2(k,\omega)\Sigma_1(k,\omega),$$

where

$$\Sigma_1(k,\omega)=2y^2\sum_{R=-\infty}^{\infty}\int_{-\infty}^{\infty}d\tau[1-\cos(kR-\omega\tau)]$$
$$\times\exp[-G_0(R=0,\ \tau=0)+G_0(R,\tau)]\equiv\Sigma_1\{G_0\}. \quad (19)$$

Since we are dealing here with one-site pairs, the summation over R can be omitted (terms with $R\neq 0$ vanish all the same), making $\Sigma_1(k,a)$ independent of k: $\Sigma_1(k,\omega)\equiv\Sigma_1(\omega)$.

Summing the infinite series of corrections of degree higher than y^2 we arrive at the self-consistent equation

$$G^{-1}(k,\omega)=G_0^{-1}(k,\omega)+\Sigma_1\{G\}, \quad (20)$$

whose solutions must be investigated by using a renormalization-group analysis.[5] For $G_0(k,\omega)$ in the form (16), the appearance of a self-energy part that depends only on ω can be described as a renormalization of V: $V\rightarrow V(\omega)$, so that Eq. (20) can be simplified to

$$\frac{\omega^2}{V(\omega)}=\frac{\omega^2}{V}+2z^2\int_{\tau_0}^{\omega^{-1}}d\tau\,(1-\cos\omega\tau)\left(\frac{\tau_0}{\tau}\right)^{2\alpha+2\pi[mV(\omega)]^{1/2}}, \quad (21)$$

where $z\propto y$ and τ_0 is the short-time cutoff parameter for the correlator $G_0(R=0,\tau)$.

Breaking up in traditional manner the integral in the right-hand side of (21) into two and changing to an analogous equation with a shifted value of the cutoff parameter,[5] we obtain renormalization-group equations that can be conveniently written in the form

$$\frac{\partial}{\partial\xi}\frac{1}{V}=Z,\quad \frac{\partial}{\partial\xi}Z=[3-2\pi(mV)^{1/2}-2\alpha]Z, \quad (22)$$

where $\xi=\ln(1/\omega\tau_0)$ and $Z=\tau_0^3z^2$.

The system (22) has a continuous set of stable stationary points

$$\pi(mV)^{1/2}+\alpha<{}^3/_2,\ Z=0.$$

Investigating (22) we note easily that as $\xi\rightarrow\infty$ the solutions take on stationary values $V_R\equiv V(\infty)>0$, and $Z(\infty)\equiv 0$ if the parameters of the initial model correspond to the region above the curve

$$\pi(mV)^{1/2}+\alpha={}^3/_2+O(y^2). \quad (23)$$

In this case the self-energy part has for small ω the asymptotic form $\Sigma_1(\omega)\propto\omega^2$.

For parameters corresponding to the region under the curve (23) but to the right of the line $\alpha=1$, $V^{-1}(\xi)$ will increase as $\xi\rightarrow\infty$ like $\exp(3-2\alpha)\xi$, corresponding to a self-energy part in the form

$$\Sigma_1(\omega)\propto\omega^{2\alpha-1}, \quad (24)$$

which leads to screening (at large distances in space-time) of the isotropic logarithmic interaction of the instantons. The interaction diagonal in the sites, however, not only remains long-range in this case, but also keeps the value of the prelogarithmic factor 2α non-renormalized. For $\alpha<1$ and $\frac{3}{2}-\pi(mV)^{1/2}$ the limiting growth law of $V^{-1}(\xi)$ will correspond to a change of the character of the behavior of $G(R=0,\tau=0)-G(R=0,\tau)$ as $\tau\rightarrow\infty$, and Eqs. (22) now cease to be valid in the limit of large ξ.

An instanton-interaction renormalization that leads to the onset of a self-energy part of form (24) occurs also in the case of a single junction with linear ohmic dissipation (for $1<\alpha<\frac{3}{2}$).[27]

The phase-separation boundary [the boundary of the region where a self-consistent solution of Eq. (20) exists] obtained in the present section is shown by the thick line of Fig. 3. We have terminated this line at $\alpha=1/2$ because at smaller α it would turn out to lie lower than the prior line of absolute instability of the coherent state of the chain with respect to spontaneous creation of free instantons. This points out the insufficiency of our analysis and requires, when α is decreased, consideration also of the possibility of

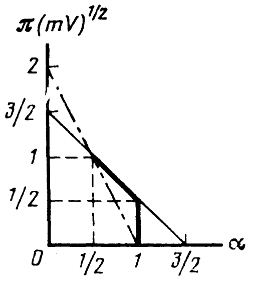

FIG. 3. The dash-dot line is the phase-separation boundary for a chain of tunnel junction with linear ohmic dissipation in the Debye–Hückel approximation. The solid lines are the boundaries of regions where various types of solutions of (20) for $M/m\to\infty$ exists.

formation of spatially separated instanton pairs; this will be done in Sec. 6.

5. CHAIN RESISTANCE IN THE INCOHERENT STATE

It is worth while emphasizing that the position of the vertical section of the phase-separation boundary obtained in the preceding section has turned out to be universal (independent of y). A similar vertical section is present also on the phase diagram obtained in Refs. 9 and 10 with the aid of a variational calculation.

This form of the phase diagram means that for $\alpha>1$ the chain will be in a coherent state at arbitrarily small V. To verify additionally this universal behavior as $V\to 0$, when the semiclassical approximation and with it, naturally, the instanton-gas concept, no longer holds, we can use another method—perturbation theory in V. Just as in the case of a single junction,[21,22] separation of the quadratic part of the action (13) and representation of

$$\exp\Big[\sum_j\int d\tau V\cos\theta_j(\tau)\Big]$$

by a power series allow us, after integrating with respect to θ, to go over to a two-dimensional (1 + 1) logarithmic gas with chemical activity $V/2$ and interaction $g_0(k,\omega)$, which coincides with the correlation function $\langle\theta(k,\omega)\theta^*(k,\omega)\rangle_0$ calculated for $V=0$ [see (15)].

In contrast to the instanton interactions (16), only the component diagonal in the sites diverges logarithmically (with a coefficient $2/\alpha$) in the interaction (15) after a Fourier transformation, while the off-diagonal component decreases like

$$(R^2+2\eta|\tau|/\pi m)^{-1/2}/\pi\eta,\quad R,\ \tau\to\infty.$$

Nonetheless, the divergence of the diagonal part suffices for the corresponding charges to be bound at $\alpha<1$ into neutral pairs. In the representation considered this corresponds to an incoherent state of the chain (in contrast to the instanton-gas representation, for which the opposite is true).

Considering the self-consistent equation having the same structure as (20) for the nonrenormalized interaction function $g(k,\omega)$ (this function coincides with the correlator $\langle\theta(k,\omega)\theta^*(k,\omega)\rangle$), we find that for small V the self-energy part $\sigma(\omega)$ has at low frequencies the form

$$\sigma(\omega)\propto\omega^\gamma,\quad \gamma=\begin{cases}2, & \alpha<2/3\\ 2/\alpha-1, & \alpha>2/3\end{cases},\tag{25}$$

and the region of existence of the self-consistent solution is bounded by the straight line $\alpha=1$ (exactly).

Both the $\sigma(\omega)$ frequency dependence and the position of the transition line turned out to be the same as for a single junction.[27] Recall that for $1<\alpha<3/2-\pi(mV)^{1/2}$ the frequency dependence of the self-energy part $\Sigma(\omega)$ of the instantons has the same form as for a single junction (see Sec. 4). It can thus be stated that the properties of the phase transition on the line $\alpha=1$ are determined by the properties of the individual junctions, and their interaction (via the capacitive interaction with the conducting substrate) turns out to be insignificant.

At zero temperature, the chain resistance (per junction) can be obtained from the relation

$$\frac{1}{R}=\frac{1}{R_{sh}}+\frac{1}{R_Q}\lim_{\omega\to0}\lim_{k\to0}\frac{\sigma(k,\omega)}{|\omega|},\quad R_Q=\frac{\hbar}{4e^2},\tag{26}$$

therefore the presence of a self-energy part in the form (25) does not lead to a difference between R and R_{sh}, i.e., the chain resistance will be equal to the shunting resistance.

Let us show that, for an incoherent state of the chain, $\sigma(k,\omega)$ will have similar properties also in the region where the instanton-gas concept is applicable. In the semiclassical approximation the correlation function $g(k,\omega)$ can be regarded as consisting of two terms,[1,27] of which the first

$$(V+g_0^{-1}(k,\omega))^{-1}\tag{27}$$

is connected with small oscillations near the minima of the potential, and the second

$$\sum_R\int_{-\infty}^{\infty}d\tau\exp\{-ikR+i\omega\tau\}$$
$$\times\sum_{R_1,R_2}\iint_{-\infty}^{\infty}d\tau_1\,d\tau_2\,\Xi(R-R_1,\tau-\tau_1)\,\Xi(R-R_2,\tau-\tau_2)$$
$$\times F(R_1-R_2,\tau_1-\tau_2)\tag{28}$$

with instantons. Here

$$\Xi(R,\tau)=\int_{-\pi}^{\pi}\frac{dk}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\exp\{-ikR+i\omega\tau\}\frac{2\pi}{-i\omega}\frac{V}{V+g_0^{-1}(k,\omega)}\tag{29}$$

is the instanton trajectory (14) displaced to have its symmetry center coincide with the point $R=0$, $\tau=0$, and

$$F(R_1-R_2,\tau_1-\tau_2)=\langle\nu(R_1,\tau_1)\nu(R_2,\tau_2)\rangle$$

is the correlator of the instanton charges ν. If only instanton-pair interaction is taken into account, $F(k,\omega)$ specifies the connection between the unrenormalized $G_0(k,\omega)$ and renormalized $G(k,\omega)$ instanton interaction functions in the form[28,29]

$$G(k,\omega)=G_0(k,\omega)-G_0^2(k,\omega)F(k,\omega),$$

whence

$$F(k,\omega)=\{G_0(k,\omega)+\Sigma^{-1}(k,\omega)\}^{-1}.\tag{30}$$

Substituting (29) and (30) in (28) and adding to (27) we arrive at a form $g(k,\omega)$ that corresponds to

$$\sigma(k,\omega)=\left\{\frac{1}{V}+\frac{4\pi^2}{\omega^2}\Sigma(k,\omega)\right\}^{-1}. \tag{31}$$

For $\Sigma(k,\omega)$, which has as $k\to 0$ and $\omega\to 0$ a finite limit or which decreases like ω^γ ($\gamma<1$), substitution of (31) in (26) against leads to $R=R_{sh}$.

We have thus shown for different limiting cases that in the parameter region corresponding to an incoherent state the chain resistance at zero temperature equals the shunt resistance. The same holds also for a single point contact with linear ohmic dissipation.[21,22]

The result means that for $\alpha=0$ the incoherent state is simultaneously also dielectric. The latter can also be verified for arbitrary values of V by calculating the average fluctuating charge of a chain segment of length L:

$$\left\langle\left[\sum_{j=l+1}^{l+L} n_j\right]^2\right\rangle,$$

which tends as $L\to\infty$ exponentially to a finite limit.

6. ADDITIONAL PHASE TRANSITION AT $\alpha=1/2$

In an investigation of a generalized d-dimensional ($d=1,2,3$) version of the model (13), Zaikin and Panyukov[16] have pointed out that in the presence of dissipation instantons describing tunneling through $\pm 2\pi$ phases of each of the grains (we shall explain later their connection with the instantons introduced by us) will interact logarithmically (with coefficient $4z\alpha$, where $2z$ is the number of nearest neighbors in the lattice). Neglecting logarithmic interaction of such instantons on neighboring sites, they have changed over to an effective single-particle problem isomorphous to that of a single point contact with effective viscosity $2z\eta$. This led them directly to the conclusion that there exists at $\alpha=1/2z$ a phase transition between states in one of which the phase of each grain is fixed (does not execute $\pm 2\pi$ jumps), while in the other it fluctuates strongly, i.e., the correlator

$$\langle[\varphi_j(t)-\varphi_j(t_0)]^2\rangle=\left\langle\left[\int_{t_0} dt'\frac{\partial\varphi_j}{\partial t'}\right]^2\right\rangle$$

diverges as $t-t_0\to\infty$.

The same result was later duplicated in Ref. 10, with the same neglect of the instanton interaction that is not diagonal in the sites. By way of an historic aside, we mention that, from among the models with planar symmetry, a phase transition connected with instanton depairing of this form was investigated in Ref. 30 for the quantum XY model with a nonstandard kinetic term, with the model used to describe the free surface of a quantum crystal. The instanton interaction in this problem was substantially different from that in the model (13).

In the case of a one-dimensional chain, each of the instantons considered by Zaikin the Panukov (tunneling of φ_l by $\pm 2\pi$ while preserving the remaining φ_j with $j\neq l$) will correspond, in terms of our instantons ($\pm 2\pi$ tunneling with $\theta_l=\varphi_l-\varphi_{l-1}$) to a pair of instantons of opposite sign located on neighboring sites. It becomes clear that the phase transition can then be described as the appearance of free non-single-site pairs of instantons[2)] (recall that at high viscosity only single-site instanton can exist in free form), i.e., of screening of an instanton logarithmic interaction that is diagonal in the sites. It is doubtless of interest to investigate this phase transition by methods more adequate than the reduction, used without foundation in Refs. 16 and 10, to a single-particle problem.

The form of expression (29) for $\Sigma_1\{G\}$ shows that when free non-single-site pairs appear in $\Sigma_1(k,\omega)$ it is necessary to add to the term $\Sigma_1\propto\omega^2$, which leads to the renormalization of V [we discuss everywhere in this section the region above the curve (23)], a term $\Sigma_1'(k,\omega)$ having as $k\to 0$ and $\omega\to 0$ the form

$$\Sigma_1'(k,\omega)\approx uk^2. \tag{32}$$

For $mV\gg 1$ it can, in particular, be assumed that

$$\Sigma_1'(k,\omega)\approx 2(1-\cos k)u$$

for any k. Substituting $\Sigma_1(\omega)+\Sigma_1'(k,\omega)$ in (20) and solving this self-consistent equation, we find that $u=0$ for $\alpha>1/2$ and $u\propto y^{2/(1-2\alpha)}$ for $1-2\alpha\ll 1$, i.e., the phase-transition line does indeed coincide with $\alpha=1/2$.

Substitution of the expression used by us for $\Sigma_1(k,\omega)$ in the expression for $G(k,\omega)$ shows that the appearance of a term in the form (32) in the self-energy part leads to screening of the logarithmic instanton interaction which is diagonal in the sites for times $\tau\sim\tau_1\sim 1/\eta u$, and only to a small quantitative change of the constant of the isotropic logarithmic instanton interaction:

$$2\varkappa_R=2\pi(mV_R)^{1/2}\to 2\pi[mV_R/(1+mu)]^{1/2},$$

after which, over time scales exceeding τ_1, the instantons can be regarded as a Coulomb gas with isotropic logarithmic interaction and with renormalized chemical activity:

$$Y=y\exp\left[-\frac{1}{2}\int_{-\pi}^{\pi}\frac{dk}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}4\pi^2\left\{\frac{|\omega|}{\eta+M|\omega|}+\frac{\omega^2}{V_R}+\Sigma_1'(k)\right\}^{-1}\right]\propto yu^\alpha\propto y^{1/(1-2\alpha)}. \tag{33}$$

If the line $\alpha=1/2$ is crossed at $\varkappa_R>2$, this Coulomb gas turns out to be in the dielectric phase. When V is decreased to a value corresponding to $\varkappa_R=2$, the usual Berezinskiĭ–Kosteritz–Thouless transition will take place in it to the plasma phase that corresponds to incoherent states of the chain and is characterized by a finite correlation radius r_c.

On the other hand, if we cross the line $\alpha=1/2$ at $1<\varkappa_R<2$, we find ourselves immediately in an isotropic-Coulomb-gas plasma phase for which r_c has, in the limit as $Y\to 0$, a dependence on the parameters Y and $\varkappa_R$ in the form[5]

$$r_c\propto Y^{-1/(2-\varkappa_R)}. \tag{34}$$

On substitution of (33) in (34) it turns out that when the line $\alpha=1/2$ is crossed in the discussed region a finite correlation radius will occur in a continuous fashion:

$$r_c\propto y^{-2/(2-\varkappa_R)(1-2\alpha)}, \tag{35}$$

but the dependence of r_c on the distance to the transition curve turns out to be substantially different from that obtained by Kosterlitz[5] for a phase transition in an isotropic Coulomb gas.

We can verify additionally that the phase transition due to the appearance of free non-single-site pairs takes place exactly at $\alpha = 1/2$ when approached from the direction of large α. Since the transition is manifested by the appearance of a term that depends strongly on in the self-energy part of $\Sigma(k,\omega)$, while the term $\Sigma_1(k,\omega)$ does not depend on k in this region, the approach to the plane transition should be manifested in the behavior of the $\Sigma(k,y)$ terms of higher order in y^2.

The correction to $G_0(k,\omega)$ of fourth order in y takes in general the form

$$\delta G(k,\omega) = -G_0^2(k,\omega)\Sigma_2(k,\omega),$$

where

$$\Sigma_2(k,\omega) = \frac{1}{4} y^4 \sum_{R_2 R_3 R_4} \iiint d\tau_2\, d\tau_3\, d\tau_4 \exp[-2G_{aa}]$$
$$\times\{\exp[G_{12}+G_{34}-G_{13}-G_{24}+G_{14}+G_{23}]'$$
$$-\exp[G_{12}+G_{34}]-\exp[G_{14}+G_{23}]\}$$
$$\times \sum_{a,b=1}^{4} (-1)^{a+b} \exp i[k(R_a-R_b)-\omega(\tau_a-\tau_b)], \qquad (36)$$

and $G_{a,b} \equiv G(R_a - R_b, \tau_a - \tau_b)$. If $mV \gg 1$ and $G_0(k,\omega)$ is given by (16), the main contribution to the momentum-dependent part of $\Sigma_2(k,\omega)$ is made by charge configurations [whose coordinates enter in (36)] of the type shown in Fig. 4. This leads to

$$\Sigma_2'(k,\omega) \propto y^4 2(1-\cos k)\omega^\gamma, \qquad \gamma = \begin{cases} 2, & \alpha > 3/4 \\ 4\alpha - 1, & \alpha < 3/4 \end{cases}. \qquad (37)$$

Substituting (37) in the self-consistent equation

$$G^{-1}(k,\omega) = G_0^{-1}(k,\omega) + \Sigma_1(\omega) + \Sigma_2(k,\omega),$$

we see that it has a solution only if $\alpha > 1/2$, while on the line $\alpha = 1/2$ it becomes necessary to take into account additional terms in $\Sigma(k,\omega)$, thus confirming the foregoing conclusion concerning the position of the additional-phase-transition line.

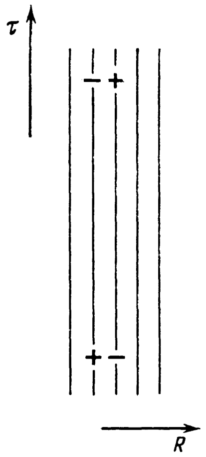

FIG. 4. Typical arrangement of the instantons corresponding to that contribution to $\Sigma_2'(k,\omega)$ which is singular in ω.

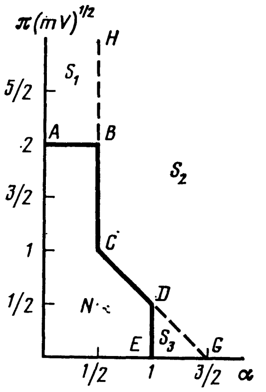

FIG. 5. Phase diagram of a chain of tunnel junctions with linear ohmic dissipation in the limit as $MV \to \infty$. With decrease of M the lines AB and CG shift upwards. The dashed lines are those of the additional phase transitions on which the exponent λ of the temperature dependence of the resistance has discontinuities.

7. GENERAL FORM OF PHASE DIAGRAM. EXPONENT OF RESISTANCE TEMPERATURE DEPENDENCE

The results of Secs. 4–6 are summarized in Fig. 5, which shows the phase diagram of the chain of tunnel junctions with linear ohmic dissipation in the limit as $y \to 0$ ($MV \to \infty$). The boundary between the coherent and incoherent states is the line $ABCDE$, which is subdivided by the singular points into four sections. With increase of y (decrease of M) the lines AB and CG move upwards and remain straight, while the segments BC and DE lie, as before, on the straight lines $\alpha = 1/2$ and $\alpha = 1$.

It follows from a computer simulation of the classical XY models[31] that when y reaches a value $y_{\max}$ corresponding to $M = 0$ the relative displacement of the point A is of the order of 75%. The displacement of the line CG is due to the same cause as that of the line AB, viz., instanton-interaction renormalization due to neutral pairs. One can thus expect it to be of the same order or even smaller, since we are dealing in this case with single-site pairs. The fact that the points B and C correspond to different values of V_R leads to preservation of the straight-line section BS on the phase transition curve also in the limit as $M = 0$. The phase diagram obtained by variational calculation will not have such a segment, regardless of the ratio M/m.

Zaikin and Panyukov[16] have advanced the hypothesis that the phase transition that takes place in the upper part of the phase diagram at $\alpha = 1/2$ is in fact a transition from a coherent to an incoherent state. Analysis has shown that this assumption is valid only for the lower part of the HC line. We shall be able to propose a lucid physical meaning of the phase transition that takes place on the upper part of this line only after we discuss the behavior of the chain resistance at constant temperature.

Investigating the phase diagram of the model (13), we have shown that, in the region to the right of the HCG line and designated S_2 in Fig. 5, contributions to the partition function are made only by single-site bound pairs of instantons, the presence of which leads only to a small quantitative renormalization of one of the constants contained in the instanton-interaction function. This region is therefore most favorable for the approximation used in Sec. 2, wherein the phase-slip is described with the aid of the effective single-particle action (7). In the case of the model (13) the propagator in (7) takes the form

$$G_{eff}^{-1}(\omega)=\eta|\omega|+\left\{\frac{m}{4}\left[V+\eta|\omega|+\left(M+\frac{m}{4}\right)\omega^2\right]\right\}^{1/2}$$
$$\times|\omega|+\left(M+\frac{m}{4}\right)\omega^2,$$

which corresponds to an effective viscosity $\eta_{\rm eff}=\eta+\frac{1}{2}(mV)^{1/2}$ and to the same effective mass (9) as in the absence of dissipation.

For a particle with parameters $\eta_{\rm eff}$ and $m_{\rm eff}$, located in a sinusoidal potential of amplitude V, the phase-slip probability for $m_{\rm eff}V\gg\eta_{\rm eff}^2$ is determined for $T\ll\hbar(V/m_{\rm eff})^{1/2}$ by expression (1) (Refs. 23 and 24), and in the opposite limiting case it takes, at temperatures up to the point of transition to the thermal-activity regime, the form[27,32]

$$P=\frac{(2\eta_{eff}^7)^{1/2}}{Vm_{eff}^2}e^{-2\alpha_{eff}}\left(\frac{\alpha_{eff}T}{\hbar V}\right)^{2\alpha_{eff}-1}$$

In both cases the temperature dependence of P corresponds to the exponent $\lambda=2\pi(mV)^{1/2}+4\pi\eta-2$ of the temperature dependence of the resistance.

These results of the reduction to an effective single-particle problem are rigorously valid in the limit as $M/m\to\infty$. For finite (or zero) M it is necessary to replace V in them by the renormalized value V_R (which, however differs less from V the larger the exponent λ itself).

In the remaining regions of the phase diagram the temperature dependence of the resistance is not so clearly defined, and account must be taken both of the change of the Green's-function structure upon normalization, and (in the S_1 region) of the contribution from the non-one-site instanton pairs.

To find the limiting value of λ at $T\to 0$ we can substitute in the expression for the phase-slip probability,[2,26]

$$P=y^2\sum_{R=-\infty}^{\infty}\int_{-i\infty+\beta/2}^{i\infty+\beta/2}d\tau$$
$$\times\exp\left\{-\int_{-\pi}^{\pi}\frac{dk}{2\pi}\frac{T}{\hbar}\sum_{s=-\infty}^{\infty}[1-\cos(kR-\omega_s\tau)]G(k,\omega)\right\},$$

$$\omega_s\equiv\frac{2\pi T}{\hbar}s,\qquad \beta=\omega_1^{-1}$$

the Green's functions obtained for $T=0$. Doing so for various regions of the phase diagram we find that

$$\lambda=2\pi(mV_R)^{1/2}-3\quad\text{in the region of } S_1, \tag{38}$$

$$\lambda=2\pi(mV_R)^{1/2}+2\alpha-2\quad\text{in the region of } S_2. \tag{39}$$

$$\lambda=2\alpha-2\quad\text{in the region of } S_3. \tag{40}$$

As the phase-transition line is approached from the superconducting side, the exponent λ tends, independently of the M/m ratio, to a universal value l in the case of segments AB and CD, to a universal value 0 for the DE segment, while $\lim\lambda$ (as $1/2+0$) varies along the segment BC smoothly from 1 (at point B) to 3 (at the point C). When the DG line is approached from the regions S_1, the exponent λ tends to the same universal value $\lambda=1$ as on the segment CD of which DT is the continuation. On tending to the universal value $\lambda=1$, the exponent λ will have a square-root singularity, $\lambda-1\propto\delta^{1/2}$ (δ is the distance to the phase-transition line), the same as in the nondissipative case.[2] It must be borne in mind, however, that the critical-behavior region of λ is narrower the smaller the ratio m/M. We emphasize that in the region S_3 the value of λ is not renormalized and is independent of m, M, and V.

For $\alpha=1/2$ (on the HB line) the exponent λ has a jump discontinuity of universal value $\Delta\lambda=2$, and on the DG line the jump has a nonuniversal value that reaches a maximum equal to 1 at the point D. Although strictly speaking the exponent λ is discontinuous, the temperature interval $0<T\ll T_*$ in which Eqs. (38) and (40) are valid narrows down without limit ($T_*\to 0$) as HB and DG are approached from the regions S_1 and S_2, respectively. In this case the $R(T)$ dependence is described at $T\gtrsim T_*$ by the exponent (39).

Concluding the list of the results, we recall also that the critical properties of the phase transition on the DE line turn out to be the same as for a single junction with ohmic dissipation, while the critical behavior of the correlation radius on crossing the BC line is described by Eq. (35). In the region of stability of the incoherence zero-temperature state (designated N in Fig. 5) we have $R\to R_{\rm sh}$ as $T\to 0$.

In summary, our method of changing to the instanton-gas representation has disclosed a much more elaborate structure of the phase diagram of a chain of junctions with linear ohmic dissipation than is possible by other methods (Refs. 9–11 and 14–16). Furthermore, its use to investigate the temperature dependence of the chain resistance in the low-temperature limit has made it possible not only to reveal universal properties of the various segments inot which the singular points break up the phase boundary, but also to elucidate clearly the physical meaning of two additional phase transitions that take place in the system and that turn out to be connected with the jumps of the exponent of the temperature dependence of the resistance.

[1)]The parameter can be changed by applying a magnetic field.
[2)]From here on the term "instanton" will be used only in the one sense corresponding to our definition.

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