Destruction of Superconductivity in Josephson Junction Arrays by Positional Disorder

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Abstract. For the integer number of flux quanta per plaquette the Josephson junction array with geometrical irregularities can be described by the XY-model with random frustration on the bonds (so called positional disorder). We show that the phase with quasi-long-range order in which the vortices are bound in neutral pairs in presence of such a disorder is no longer stable.

In absence of the magnetic field the Josephson junction array (JJA) can be described by the ordinary 2D XY-model. The presence of the magnetic field penetrating the array introduces many new features into the system. For the values of the field corresponding to integer number of flux quanta per unit cell JJA with small geometrical irregularities will be described by the XY-model with random frustration on the bonds:

$$H = -J\sum_{(\mathbf{jj}')}\cos(\phi_{\mathbf{j}} - \phi_{\mathbf{j}'} - A_{\mathbf{jj}'}) \tag{1}$$

where $A_{\mathbf{jj}'}$ are quenched variables with symmetrical distribution around zero. If there is a true long-range order in the positions of the superconducting grains forming the array, the correlations between variables $A_{\mathbf{jj}'}$ will be short-range. Such situation is often being refered to as JJA with positional disorder [1]. According to Rubinstein *et al* [2] in presence of such a disorder the domain of stability of the ordered (superconducting) phase is decreased and a reentrant transition to the disordered phase at low temperatures appears on the phase diagram.

We have reinvestigated this problem and have shown the analysis of Ref.2 to be insufficient. Our main conclusion is that positional disorder always destroys a superconducting phase in which all vortcies are bound in pairs.

An ordinary XY-model is known to be isomorphic to a 2D Coulomb gas the charges of which correspond to vorticies of the XY-model. Rubinstein *et al* [2] had shown that the replicated version of (1) is equivalent to the replicated Coulomb gas described by the Hamiltonian:

$$H = \frac{1}{2}\sum_{\mathbf{R},\mathbf{R}'} m_\alpha(\mathbf{R})G_{\alpha\beta}(\mathbf{R}-\mathbf{R}')m_\beta(\mathbf{R}') + \ln\frac{1}{Y}\sum_{\mathbf{R}} m_\alpha^2(\mathbf{R}) \tag{2}$$

with

$$G_{\alpha\beta}(0) - G_{\alpha\beta}(\mathbf{R}) \approx \begin{cases} 2\pi\left(K - \frac{\sigma K^2}{1+n\sigma K}\right)\ln R & \text{for } \alpha=\beta \\ -2\pi\frac{\sigma K^2}{1+n\sigma K}\ln R & \text{otherwise,}\end{cases} \tag{3}$$

n being the number of replicas and greek indices running from 1 to n. Here σ characterizes the strength of fluctuations of $A_{\mathbf{jj}'}$ and $K \equiv J/(k_B T)$.

In the case of strong interaction between the charges in the same replica only neutral pairs of unit charges belonging to the same replica are likely to be present in the system. In Ref. 2 the renormalization of charge-charge interaction by such neutral pairs was studied, renormalization group equations were found and with their help the phase diagram discussed above was constructed.

In terms of the pertrubation expansion for the replicated Coulomb gas the general structure of the phase diagram can be understood by seeing that the first correction to $G_{\alpha\beta}(\mathbf{q})$ which is given by the expression:

$$-Y^2 G_{\alpha\gamma}(\mathbf{q}) G_{\gamma\beta}(\mathbf{q}) \int d^2\mathbf{R}(1-\cos\mathbf{qR})\exp[-G_{11}(0)+G_{11}(\mathbf{R})] \tag{4}$$

for $2\pi(K-\sigma K^2) > 2$ will be convergent. Then for small Y one would expect next order corrections to be negligible, at least for $2\pi(K-\sigma K^2) \gg 2$.

We would like to emphasize that not only neutral pairs of charges but also some other objects should be considered. Let us call a complex consisting of N positive unit charges belonging to different replicas a N-complex. As positive charges belonging to different replicas attract each other it turns out to be a well defined object. Then according to Eq.(3) the logarithmical interaction of N-complex with its negative counterpart will have

$$P_N = 2\pi(NK - N^2\sigma K^2)$$

as a prefactor. Here n has been already put down to zero. So for any values of K and σ for large enough N the interaction of N-complexes will become repulsive. That means that they will not be bound in pairs but will exist as free particles screening therefore all logarithmical interactions in the system. It makes the task of writing down the renormalization equations impossible because the phase where only the neutral pairs or complexes of charges are thermodinamically stable is completely wiped out. In terms of JJA this means that the superconducting phase will no longer be stable.

The same results can be also obtained in the framework of another approach (without replicas). In vortex representation the positional disorder manifests itself as a random potential for the vorticies the correlations of which diverge logarithmically. If we now take the average over disorder of the pertrubation expansion:

$$G(\mathbf{q}) = G_0(\mathbf{q}) - G_0^2(\mathbf{q})\Sigma(\mathbf{q})$$
$$\Sigma(\mathbf{q}) = \Sigma_2(\mathbf{q}) + \Sigma_4(\mathbf{q}) + \ldots$$

for the vortex-vortex interaction $G(\mathbf{q})$, the most dangerous contribution to $\langle\Sigma_{2N}(\mathbf{q})\rangle$ will be of the same form as the term induced by N-complexes in replica approach.

For the case of $\sigma \ll 1$, $K \sim 1$ one can argue that because only the high-order terms with N larger than $N_c \sim 1/\sigma$ are dangerous, the unbinding of vortex pairs will not manifest itself up to exponentially large distances, so in the experiments on finite arrays just a small broadening of the transition from the high-temperature (disordered) to the quasi-ordered phase will be seen. But the low temperature (reentrant) transition is likely to be completly wiped out. These conclusions are in complete accordance with the results of computer simulations [3]. Of course there still remains a possibility of a transition to a glass-like state.

References

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[3] Forrester M.G., Benz S.P. and Lobb C.J., *Phys. Rev.*, **B41** (1990) 8749.