Possible destruction of the ordered phase in Josephson-junction arrays with positional disorder

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The Josephson-junction array with geometrical irregularities in the presence of the magnetic field with an integer number of flux quanta per plaquette can be described by the XY model with some random distribution of phase shifts on the bonds (so-called positional disorder). We show that the existing theoretical understanding of the properties of this model is based on an incomplete analysis. The form of the higher-order corrections indicates that the phase with quasi-long-range order, in which the vortices are bound in pairs, in the presence of positional disorder is probably no longer stable. The experimental observation of a superconducting transition in Josephson-junction arrays with small disorder may be then ascribed to finite-size effects.

I. INTRODUCTION

Recent developments in the techniques of lithography have given rise to experimental investigation of various models of two-dimensional statistical mechanics with the help of Josephson-junction arrays and superconducting wire networks (see Ref. 1 for the review). Whereas in the absence of the magnetic field a Josephson-junction array can be described by the ordinary two-dimensional (2D) XY model, the presence of the magnetic field penetrating the array requires that a frustrated XY model be used. For the values of the field corresponding to an integer number of flux quanta per unit cell, a Josephson-junction array with geometrical irregularities can be described by the XY model with random phase shifts on the bonds:

$$H = -J \sum_{(j,j')} \cos(\phi_j - \phi_{j'} - A_{jj'}), \tag{1}$$

where the summation is assumed to be performed over pairs of nearest neighbors on some regular lattice.

If fluctuations in the positions of the superconducting grains forming the array are local, there will not be any long-range correlations between quenched variables $A_{jj'}$. In that case these variables can be considered as independent. The particular form of the distribution of $A_{jj'}$ around zero is not relevant as long as it remains symmetric under a change of sign. Such a system is often referred to as a Josephson-junction array with positional disorder.[2]

Model (1) has been studied by Rubinstein, Shraiman, and Nelson[3] who have found that in the presence of disorder the domain of stability of the ordered phase is decreased. According to Ref. 3, for strong disorder the ordered phase is completely wiped out whereas for weaker disorder the phase diagram should possess a reentrant transition to the disordered phase at low temperatures. On the other hand the reentrant transition was not discovered either in numerical simulations[4,5] or in the experimental work on Josephson-junction arrays with built-in disorder.[6]

In the present paper we reinvestigste the problem both in terms of the replica representation (Sec. II) and without it (Sec. III) and show that the analysis of Ref. 3 is insufficient. In the framework of a more general approach it appears very probable that even small positional disorder may always destroy the quasiordered phase in which the vortices are present only in the form of the bound pairs. In Sec. IV we discuss how the observation of the phase transition in computer[4,5] and real[6] experiments can be reconciled with our findings. A preliminary account of the results of this paper has been published previously.[7]

II. REPLICA REPRESENTATION

It is well known that an ordinary XY model is equivalent to a 2D Coulomb gas, the charges of which correspond to the vortices of the XY model. In the vortex representation the presence of the positional disorder amounts to the appearence of the random potential for the vortices $V(\mathbf{R})$, the correlations of which in real space diverge logarithmically with distance:

$$U(\mathbf{R}) \equiv \frac{1}{2}\overline{[V(\mathbf{R}_0+\mathbf{R}) - V(\mathbf{R}_0)]^2} \approx 2\pi\sigma K^2 \ln R. \tag{2}$$

Here σ characterizes the strength of fluctuations of $A_{jj'}$ and $K = J/(k_BT)$. The sign of the random potential is different for positive and negative vortices. We use the overbar to designate the averages over disorder.

Rubinstein, Shraiman, and Nelson[3] had shown that the replicated version of the model (1) after the averaging of the partition function over disorder can be reduced to the replicated Coulomb gas:

$$H = \frac{1}{2}\sum_{\mathbf{R},\mathbf{R}'} m_\alpha(\mathbf{R})G_{\alpha\beta}(\mathbf{R}-\mathbf{R}')m_\beta(\mathbf{R}') + \left(\ln\frac{1}{Y}\right)\sum_{\mathbf{R}} m_\alpha^2(\mathbf{R}) \quad (3)$$

in which the interaction of charges is logarithmic not only for the charges in the same replica, but also for the charges belonging to different replicas:

$$G_{\alpha\beta}(0) - G_{\alpha\beta}(\mathbf{R}) \approx \begin{cases} 2\pi\left(K - \frac{\sigma K^2}{1+n\sigma K}\right)\ln R & \text{for } \alpha=\beta, \\ -2\pi\frac{\sigma K^2}{1+n\sigma K}\ln R & \text{otherwise.} \end{cases} \quad (4)$$

Here and in the following n stands for the number of replicas and Greek indices designate the number of the replica and run from 1 to n. The fugacity of the charges Y can be used as a formal parameter of expansion.

If the interaction of the charges in the same replica is strong enough one would naturally expect only neutral pairs of unit charges belonging to the same replica to be present in the system. Rubinstein *et al.*[3] have studied how the existence of such neutral pairs renormalizes the interaction between the other charges, have found the renormalization group equations, and have used them to construct the phase diagram discussed above.

The renormalization-group equations are really only needed for the investigation of the details of the critical behavior. The general structure of the phase diagram can be understood in the framework of the perturbation expansion by noting that the first correction to $G_{\alpha\beta}(\mathbf{q})$, which is given by the expression

$$-Y^2 G_{\alpha\gamma}(\mathbf{q})G_{\gamma\beta}(\mathbf{q})\int d^2R(1-\cos\mathbf{q}\cdot\mathbf{R}) \times \exp[-G_{\alpha=\beta}(0) + G_{\alpha=\beta}(\mathbf{R})] \quad (5)$$

in the limit of $n \to 0$ is not dangerous at small $\mathbf{q}$ only for

$$2\pi(K - \sigma K^2) > 4. \quad (6)$$

Then for small Y one would expect higher-order corrections also to be negligible, at least if one is not too close to the border of the domain (6). Such reasoning gives approximately the same shape of the domain of stability of the ordered phase as suggested in Ref. 3.

We would like to emphasize that in addition to neutral pairs of charges belonging to the same replica the more complex objects should also be included into analysis. Consider an agglomeration consisting of, say, N positive unit charges, all belonging to different replicas (in what follows we shall call such object N-complex). The charges of the same sign belonging to different replicas attract each other so one can consider such an object as well defined. Then according to Eq. (4) the logarithmic interaction of an N-complex with an analogous complex constructed from negative charges will have a prefactor equal to

$$P_N = 2\pi(KN - \sigma K^2 N^2). \quad (7)$$

Here the number of replicas n has been set to zero.

The positive term in Eq. (7) is linear in N, but the negative term is quadratic, so for any values of K and σ for large enough N the interaction of N-complexes will be repulsive. That means that they will not be bound in pairs but will exist as free particles.

The renormalization group analysis of Ref. 3 is based on the existence of the quasiordered phase in which the charges of the equivalent Coulomb gas are bound in neutral pairs. The line of the transition to the disordered phase has been found as a line at which these bound pairs start to break and free charges appear in the system. Now we have discovered that at any point of the phase diagram the large-enough N-complexes cannot be bound in pairs but should exist in the free state. The finite concentration of these large N-complexes will screen the logarithmic interaction of the smaller complexes and of the single charges and therefore should lead to the appearence of free single charges.

This makes the task of writing down the renormalization-group equations impossible because the dielectric phase in which only the neutral pairs or complexes of charges are thermodynamically stable is completely wiped out. In terms of the Josephson-junction array this may signify that the superconducting phase is no longer stable.

An objection may be put forward that the N-complex of N charges belonging to different replicas is not a well-defined object in the limit of a zero number of replicas. So it seems worthwhile emphasizing that the single charges in our system correspond to N-complexes with $N = 1$. Therefore if we assume that it is important to take into account the existence of bound pairs of single charges belonging to the same replica, we have to conclude that it is no less important to consider the bound pairs of N-complexes with $N > 1$. Nevertheless it would be certainly more convincing to obtain the same results with the help of some other approach (without replicas) and it proves to be possible to do so.

III. DIRECT AVERAGING OF THE PERTURBATION EXPANSION

In the absence of disorder the perturbation expansion for the interaction of charges in the Coulomb gas can be written as

$$G(\mathbf{q}) = G_0(\mathbf{q}) - G_0^2(\mathbf{q})\Sigma(\mathbf{q}), \quad (8)$$

$$\Sigma(\mathbf{q}) = \Sigma_2(\mathbf{q}) + \Sigma_4(\mathbf{q}) + \cdots . \tag{9}$$

The Coulomb gas is assumed to be in the dielectric phase and so only the terms corresponding to the even number of charges are present in the rhs of Eq. (9). In the following we shall use the explicit form of the first two terms in Eq. (9):

$$\Sigma_2(\mathbf{q}) = Y^2 \int d^2R_1 F_{01} F_{01}^* W_{01}, \tag{10}$$

$$\Sigma_4(\mathbf{q}) = \frac{Y^4}{4} \int\int\int d^2R_1 d^2R_2 d^2R_3 F_{0123} F_{0123}^* (W_{0123} - W_{01}W_{23} - W_{03}W_{21}). \tag{11}$$

Here factors

$$F_{01} \equiv \exp(i\mathbf{q}\cdot\mathbf{R}_0) - \exp(i\mathbf{q}\cdot\mathbf{R}_1), \tag{12}$$

$$F_{0123} \equiv \exp(i\mathbf{q}\cdot\mathbf{R}_0) - \exp(i\mathbf{q}\cdot\mathbf{R}_1) + \exp(i\mathbf{q}\cdot\mathbf{R}_2) - \exp(i\mathbf{q}\cdot\mathbf{R}_3) \tag{13}$$

bear only geometrical meaning whereas W_{01} and so on are the statistical weights one should ascribe to this or that particular configuration of charges (positive charge at point $\mathbf{R}_i$ and negative charge at point $\mathbf{R}_j$ in case of W_{ij}, etc.). In the absence of negative counterterms in Eq. (11), $\Sigma_4(\mathbf{q})$ would diverge with the area of the system. Following our assumption we do not include in Eqs. (10) and (11) the statistical weights W corresponding to the odd number of charges. The expansion we are using can be derived directly for the Coulomb gas but also can be obtained from the diagrammatic analysis of the sine-Gordon model.[8]

In the presence of disorder statistical weights W in addition to the interaction betweeen charges also incorporate the random potential due to positional disorder. For example W_{01} acquires the form

$$W_{01} = \exp\{-G_0(0) + G_0(\mathbf{R}_1 - \mathbf{R}_0) - V(\mathbf{R}_1) + V(\mathbf{R}_0)\} \tag{14}$$

and, after averaging over disorder, reduces to

$$\overline{W_{01}} = \exp\{-[G_0(0) - G_0(\mathbf{R}_1 - \mathbf{R}_0)] + U(\mathbf{R}_1 - \mathbf{R}_0)\} \tag{15}$$

so that $\overline{\Sigma_2}$ remains not diverging in domain (6) in complete accordance with the results of the replica analysis.

The influence of the disorder on the higher-order terms is similar. Upon averaging it introduces an additional interaction which is equal to $U(\mathbf{R}) \approx \sigma K G_0(\mathbf{R})$ between all the charges in the corresponding configuration. For the term in $\Sigma_4(\mathbf{q})$ containing W_{0123} this means only overall substitution of $K - \sigma K^2$ for K, so no new divergence appears. But in the two other terms related to counterterms this really happens. It is so because in both of them in absence of disorder each positive charge is interacting only with one negative charge, whereas disorder introduces the additional interaction between all four charges involved. Thus, for example, for $\mathbf{R}_2 \approx \mathbf{R}_0$, $\mathbf{R}_3 \approx \mathbf{R}_1$, and $R \equiv |\mathbf{R}_1 - \mathbf{R}_0| \gg 1$ we obtain

$$\overline{W_{01}W_{23}} \propto R^{-2\pi(2K - 4\sigma K^2)} \tag{16}$$

that leads to the same divergence as induced by N-complexes with $N = 2$ in the replica approach.

Speaking more precisely the form of the contribution to $\overline{\Sigma(\mathbf{q})}$ related to $\overline{W_{01}W_{23}} + \overline{W_{03}W_{21}}$ coincides with the contribution to the diagonal part of the self-energy in the replica representation associated with the bound pairs of N-complexes with $N = 2$. The correspondence between the different counterterms and pairs of N-complexes with different N can be generalized to the higher orders of the perturbation expansion. For example in the $2N$th order of the perturbation expansion the contribution can be isolated which corresponds to the neutral pair of N-complexes and so on. Therefore all the conclusions concerning the renormalization and screening of the vortex-vortex interaction obtained in Sec. II in terms of the replica representation can also be proved without introducing replicas.

IV. DISCUSSION

So with the help of two different methods we have shown that the phase diagram of the two-dimensional XY model with positional disorder based on second-order renormalization-group equations[2,3] is likely to be incorrect. It looks very probable that if the N-complexes with $N > 1$ (or, equivalently, the higher orders of the perturbation expansion) are taken into account the phase with quasi-long-range order in which the vortices are bound into neutral pairs is completely wiped out from the phase diagram. In terms of the Josephson-junction array that may signify the total destruction of superconductivity in a rigorous sense of the word.

Unfortunately we cannot suggest any quantitative description incorporating the dangerous high-order terms. The renormalization-group analysis of Ref. 3 was based on the assumption that no new types of divergencies appear in higher orders of perturbation theory. On the other hand our analysis has revealed that a new type of divergence appears in every order of perturbation expansion. The correction to $G_{\alpha\beta}(\mathbf{q})$ associated with neutral pairs of N-complexes becomes dangerous when $P_N = 4$. Thus we do not envisage any possibility of constructing a closed renormalization procedure since an infinite number of relevant charges should be involved.

The large number of divergences appearing in the problem is certainly related to the non-Gaussian form of the fluctuations of the vortex-vortex interaction $G(\mathbf{q})$ [see Eqs. (8)–(14)]. It may be possible that the approach based on the direct averaging of the expansion for $G(\mathbf{q})$ is an oversimplification and that the problem should be studied in terms of distribution functions for $G(\mathbf{q})$. In any case the phase which is only quantitatively different from the low-temperature phase of the ordinary XY model (without disorder) is likely to no longer be present on phase diagram. Of course the question of whether a transition to some glasslike state can occur still remains open.

If the conclusion about the destruction of the quasiordered phase is really valid the relevant question is why the experiments (both real[6] and numerical[4,5]) still demonstrate the existence of the transition into the phase with the superconducting response (finite value of the helicity modulus). In our opinion this phenomenon may be ascribed to the finite-size effects.

In the case of weak disorder ($\sigma \ll 1$) and not-very-low temperatures ($K \sim 1$) the dangerous corrections to $G(\mathbf{q})$ appear only in the high-order terms of the perturbation expansion (starting from the Lth term with $L \sim 1/\sigma$). So it is very likely that they will not manifest themselves up to exponentially large distances. Thus in the case of finite Josephson-junction arrays just a small broadening of the transition from the high-temperature (disordered) to the quasiordered phase should occur, and the transition should seem to take place as was observed in Refs. 4–6. But the low-temperature (reentrant) transition in that case may be completely wiped out, because it is supposed to occur in the domain where the new divergences appear in all orders of the perturbation expansion.

Thus we can conclude that our results provide at least a tentative explanation for the absence of reentrant transition in Josephson-junction arrays with positional disorder and hope that they will encourage a further investigation of this interesting problem. Comparison with the random field XY model[9] gives a hint that at low temperatures the superconductivity may still really exist if a transition takes place to some glasslike phase in which different almost degenerate disordered states are separated from each other by infinite barriers.

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