Replica symmetry breaking in vortex glasses

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Destruction of the long range order in a vortex crystal by a random potential is expected to be accompanied by the formation of a glasslike phase—the so-called vortex glass. The properties of such a phase are investigated in the framework of the self-consistent harmonic approximation, taking into account the possibility of replica symmetry breaking. The main attention is given to the problem of the uniaxial vortex glass in which the vortices are free to move only in one direction. We obtain the result that in the two-dimensional case, upon lowering the temperature, a phase transition takes place between the phase in which the random potential is irrelevant, to the phase with one-step replica symmetry breaking. For $2 < D < 4$ the random potential is always relevant and the replica symmetry breaking is of the hierarchical type. In both cases, the fluctuations of the displacement in the glassy phase diverge logarithmically. The same conclusions are shown to be valid for the case of a biaxial vortex glass in the absence of dislocations. The results obtained are also applicable to the description of the pinning of charge density waves.

I. INTRODUCTION

Experimental investigation of high-T_c superconducting materials has led to increased interest in the theoretical understanding of the properties of a vortex glass phase which may be formed when a vortex crystal interacts with a random pinning potential.[1–8] An example of such a system is the two-dimensional uniaxial vortex crystal formed by a sequence of vortex lines confined to move in a plane.[3–6] One can think, for example, about a large-area Josephson junction between two bulk pieces of type-I superconductor in the presence of a magnetic field along the plane of the junction. The inhomogeneities in the width of the junction will provide a random potential for the Josephson vortices which then form a two-dimensional uniaxial vortex crystal. In three dimensions the vortex crystal can be assumed to be uniaxial if the magnetic field is applied parallel to the layers of a superconductor with a well-developed layered structure. Then, at least at low temperatures, the large core energy will prevent the vortices from crossing the superconducting planes.[9]

If fluctuations of the intervortex distance are not too large compared to its average value, the uniaxial vortex crystal interacting with a random potential can be described by the Hamiltonian:[3–6]

$$H = \int d^D r \left[\frac{J}{2}(\nabla u)^2 + V_0(\mathbf{r})\nabla_x u + V_1(\mathbf{r})\cos u + V_2(\mathbf{r})\sin u \right], \qquad (1)$$

where $u \equiv u(\mathbf{r})$ is the displacement of the vortices with respect to the equilibrium configuration of the vortex lattice. We assume that on the average the vortex lines are parallel to the y axis and can move only in the x direction. The displacement u is considered to be rescaled in such a way as to make the period of the vortex lattice (in the x direction) equal to 2π.

The first term in Eq. (1) represents the elastic energy of the uniaxial vortex crystal, the elastic moduli of which are assumed to be local. In that case one can always make this term isotropic by the proper choice of the units of length for different directions.

Three other terms describe the interaction of the vortex crystal with the random potential. They contain three different independent random functions $V_i(\mathbf{r})$ which have Gaussian distributions:

$$\overline{V_i(\mathbf{r})} = 0, \qquad \overline{V_0(\mathbf{r})V_0(\mathbf{r}')} = W\delta(\mathbf{r}-\mathbf{r}'),$$
$$\overline{V_i(\mathbf{r})V_j(\mathbf{r}')} = 0 \quad (i \neq j), \qquad (2)$$
$$\overline{V_1(\mathbf{r})V_1(\mathbf{r}')} = \overline{V_2(\mathbf{r})V_2(\mathbf{r}')} = Y\delta(\mathbf{r}-\mathbf{r}').$$

Here and further on the overbar designates the average over different realizations of the random potential and angular brackets—the thermal average.

The second term in Eq. (1) is proportional to the density of vortices and describes the interaction of the vortex crystal with the slowly changing part of the random potential. Since it is linear in u one can always get rid of it by a transformation:

$$u(\mathbf{r}) = \tilde{u}(\mathbf{r}) - \frac{1}{J}\nabla^{-2}\nabla_x V_0, \qquad (3)$$

which also changes the form of the functions $V_1(\mathbf{r})$ and $V_2(\mathbf{r})$ (for a given realization of random potential) but in case of the Gaussian distribution does not change their statistical properties [Eqs. (2)]. Therefore the existence of this term can manifest itself only in the reducible part of the correlation functions, for example,

$$\tilde{\mathcal{G}}(\mathbf{q}) \equiv \overline{\langle u(\mathbf{q})\rangle\langle u^*(\mathbf{q})\rangle} = \overline{\langle \tilde{u}(\mathbf{q})\rangle\langle \tilde{u}^*(\mathbf{q})\rangle} + \frac{Wq_x^2}{(Jq^2)^2}, \qquad (4)$$

whereas the irreducible correlation functions cannot depend on W.

Two last terms in Eq. (1) describe the interaction of the vortex crystal with the short-wavelength components of the random potential which in the x direction have approximately the same period as that of the vortex crystal. This is the most relevant part of the interaction.[4,5]

The same Hamiltonian can be applied to the description of pinning of the uniaxial charge density waves.[4,10,11] In the absence of the second term it also coincides with the Hamiltonian of an XY model with random field in which the possibility of the creation of vortices is neglected. In that context it was investigated earlier by a number of authors.[10–14] For $D = 2$ it has been shown with the help of the renormalization procedure in the replica representation[10–12] that with decrease in temperature a phase transition takes place from the high-temperature phase in which the random potential is irrelevant and the fluctuations of u diverge logarithmically:

$$w(\mathbf{R}) \equiv \overline{\langle [u(\mathbf{r}+\mathbf{R}) - u(\mathbf{r})]^2 \rangle} = \eta \ln R \quad (R \to \infty) \tag{5}$$

to the glassylike low-temperature phase in which the random potential is relevant and the fluctuations of u diverge more rapidly:

$$w(\mathbf{R}) \propto \ln^2 R \tag{6}$$

(Refs. 11 and 13). These conjectures are also supported by the results of dynamical renormalization-group analysis.[14,15]

For $2 < D < 4$ Villain and Fernandez[13] have suggested a real-space renormalization scheme which shows that a random potential is always relevant and that the fluctuations (at least for $T \to 0$) diverge according to Eq. (5) (with $\eta \propto 4-D$ for $D \to 4^-$). On the other hand Nattermann on the basis of a real-space variational calculation[4] suggests that for $2 < D < 4$ Eq. (6) is also valid.

In the present paper we reinvestigate the same model in terms of the replica representation taking into account the possibility of the replica symmetry breaking in the same fashion as was done recently by Shakhnovich and Gutin[16] for the problem of random heteropolymers and by Mézard and Parisi[17] for the problem of the fluctuating manifold in a random potential. We show that for $D = 2$ the glassy state (low-temperature phase) is characterized by a one-step replica symmetry breaking whereas for $2 < D < 4$ a full hierarchical replica symmetry breaking takes place. In both cases the correlation function $w(\mathbf{R})$ diverges in the glassy phase according to Eq. (5) contrary to the results of the replica symmetric renormalization-group analysis[10–13] for $D = 2$.

Earlier an attempt to consider replica symmetry breaking in a pinned-vortex crystal was undertaken by Bouchaud, Mézard, and Yedidia[8] for the case of the ordinary (not uniaxial) vortex crystal. However these authors have not taken into account the periodic nature of the vortex crystal and have come to the conclusion about the algebraical behavior of $w(\mathbf{R})$. The applicability of our results for the case of the biaxial vortex crystal without dislocations is briefly discussed in Sec. VI.

II. SELF-CONSISTENT HARMONIC APPROXIMATION

After introducing n replicas of the Hamiltonian (1) and averaging the partition function over the random potential, the effective Hamiltonian for the replicated variables will acquire the form

$$H_{\rm repl} = \int d^D r \left[\frac{J}{2}\sum_a (\nabla u_a)^2 - \frac{W}{2}\sum_{a,b} (\nabla_x u_a)(\nabla_x u_b) - \frac{Y}{2}\sum_{a\neq b}\cos(u_a - u_b) \right]. \tag{7}$$

Here and throughout, the replica indices a and b run from 1 to n. At the end of the calculation n should be put equal to zero.[18] The factor $\beta \equiv 1/T$ is assumed to be included into the definition of the Hamiltonian, therefore $J \propto 1/T$ and $W, Y \propto 1/T^2$.

We shall calculate the free energy corresponding to the Hamiltonian (7) using the self-consistent harmonic approximation[19] which is equivalent to the variational calculation of free energy:

$$F_{\rm var} = F_0 + \langle H - H_0 \rangle_0 \tag{8}$$

with the help of the harmonic trial Hamiltonian:

$$H_0 = \frac{1}{2}\int \frac{d^D\mathbf{q}}{(2\pi)^D} \sum_{a,b} G_{ab}^{-1}(\mathbf{q}) u_a(\mathbf{q}) u_b^*(\mathbf{q}). \tag{9}$$

In Eq. (8) F_0 stands for the free energy for the trial Hamiltonian and $\langle\cdots\rangle_0$ for the thermal average calculated with the help of H_0. Both terms can be calculated exactly. Such an approach was introduced for analysis of systems with the sine-Gordon structure by Saito[19] and has proved to give a correct qualitative description of both phases. Recently it was used by Mezard and Parisi[17] for the problem of the fluctuating manifold in random media.

Substitution of Eq. (7) and Eq. (9) into Eq. (8) gives the following expression for the free energy of the system formed by n coupled replicas:

$$F_{\rm var} = -\frac{1}{2}\int \frac{d^D\mathbf{q}}{(2\pi)^D} \Big(\ln\{\det[\hat{G}(\mathbf{q})/2\pi]\} + {\rm Sp}\{[\hat{G}^{-1}(\mathbf{q}) - \hat{G}_0^{-1}(\mathbf{q})]\hat{G}(\mathbf{q})\} \Big) - \frac{Y}{2}\sum_{a\neq b} \exp\left(-\frac{B_{ab}}{2}\right), \tag{10}$$

where B_{ab} describes the amplitude of fluctuations of $u_a(\mathbf{r}) - u_b(\mathbf{r})$:

$$B_{ab} \equiv \langle (u_a - u_b)^2 \rangle_0 = \int \frac{d^D\mathbf{q}}{(2\pi)^D} [G_{aa}(\mathbf{q}) + G_{bb}(\mathbf{q}) - G_{ab}(\mathbf{q}) - G_{ba}(\mathbf{q})] \tag{11}$$

and

$$\hat{G}_0^{-1}(\mathbf{q}) = G_0^{-1}(\mathbf{q})\delta_{ab} - D_0(\mathbf{q}) \tag{12}$$

with

$$G_0^{-1}(\mathbf{q}) = Jq^2, \quad D_0(\mathbf{q}) = Wq_x^2$$

is the inverse of the bare Green's function corresponding to the harmonic part of the Hamiltonian (7). After that F_{var} should be minimized with respect to the trial Green's function $G_{ab}(\mathbf{q})$.

Variation of the trial free energy (10) with respect to $G_{ab}(\mathbf{q})$ gives

$$G_{ab}^{-1}(\mathbf{q}) = G_0^{-1}(\mathbf{q})\delta_{ab} - D_0(\mathbf{q}) + \Sigma_{ab} \tag{13}$$

where the self-energy matrix $\Sigma_{ab} \equiv -J\sigma_{ab}$ does not depend on $\mathbf{q}$. Its nondiagonal elements Σ_{ab} (with $a \neq b$) are negative and should be found self-consistently from the equations

$$\sigma_{ab} = \frac{Y}{J}\exp\left(-\frac{B_{ab}}{2}\right), \tag{14}$$

whereas the diagonal elements (with $a = b$) can be determined from the relation

$$\sum_b \sigma_{ab} = 0, \tag{15}$$

which follows from the form of expression (11).

III. REPLICA SYMMETRIC CASE

Let us discuss the simplest case when σ_{ab} (with $a \neq b$) is assumed to be not dependent on replica indices a and b. In that case in the limit of $n \to 0$ the matrix element $\sigma \equiv \sigma_{a\neq b}$ drops out from the expression for $B \equiv B_{ab}$:

$$B = 2\int \frac{d^D\mathbf{q}}{(2\pi)^D}\frac{1}{G_0^{-1}(\mathbf{q}) + nJ\sigma} \tag{16}$$

and therefore there is no need to solve any self-consistent equation. That means that σ is given directly by the expression

$$\sigma = \frac{Y}{J}\exp\left[-\int \frac{d^D\mathbf{q}}{(2\pi)^D}G_0(\mathbf{q})\right] \tag{17}$$

and accordingly is finite for $D > 2$ and zero for $D = 2$.

Thus we have obtained the result that if the possibility of replica symmetry breaking is excluded from the consideration the self-consistent harmonic approximation does not predict any phase transition in the case of the two-dimensional system. Such a conclusion is in contradiction with the results of the renormalization-group analysis.[10–12] There is nothing surprising in that since in the terms of the Coulomb gas representation[12] the self-consistent harmonic approximation corresponds to the Debye-Huckel approximation which can take into account only the influence of free charges whereas according to Ref. 12 in both phases the charges are bound in pairs because their interaction is not renormalized. Therefore the self-consistent harmonic approximation cannot distinguish between these two phases. Below we show that in reality the phase transition in a two-dimensional system is accompanied by the replica symmetry breaking and the appearance of the free charges.

IV. ONE-STEP REPLICA SYMMETRY BREAKING

One can expect that at least at low temperatures the vortex crystal interacting with the random potential may become quenched in one of the many different almost degenerated states separated by infinite barriers.[1] Therefore it is advisable to consider the possibility of replica symmetry breaking. We shall start with considering a case of the one-step replica symmetry breaking which is realized for $D = 2$.

The case of the one-step replica symmetry breaking corresponds to such a form of the self-energy matrix σ_{ab} when its nondiagonal elements can acquire two different values σ_1 and σ_0 depending on whether the two indices a and b belong to the same block of the length m or not. In order to represent this form of the matrix with the help of a simple mathematical notation it would be convenient to split each of the replica indices running from 1 to n into two:

$$a = ma' + a'',$$

where the first index a' (running from 1 to n/m) is the number of the block and the second index a'' (running from 1 to m) is the number of replica inside the block. In that notation σ_{ab} for the case of one-step replica symmetry breaking can be written as

$$\sigma_{ab} = \sigma_0 + (\sigma_1 - \sigma_0)\delta_{a'b'} - [n\sigma_0 + m(\sigma_1 - \sigma_0)]\delta_{ab}, \tag{18}$$

where the value of the coefficient in the last term follows from Eq. (15).

For σ_{ab} of the form (18) inversion of Eq. (13) gives the expression

$$\begin{aligned} G_{ab} = {} & \frac{1}{n}\left[\frac{1}{Jq^2 - nD_0(\mathbf{q})} - \frac{1}{J(q^2 + \Delta_0)}\right] \\ & + \frac{1}{m}\left[\frac{1}{J(q^2 + \Delta_0)} - \frac{1}{J(q^2 + \Delta_1)}\right]\delta_{a'b'} \\ & + \frac{1}{J(q^2 + \Delta_1)}\delta_{ab}, \end{aligned} \tag{19}$$

which contains two different gaps:

$$\Delta_0 = n\sigma_0, \quad \Delta_1 = n\sigma_0 + m(\sigma_1 - \sigma_0).$$

Substitution of Eq. (19) into Eq. (11) shows that Δ_1 can be associated with mutual fluctuations of replicas belonging to the same block whereas Δ_0 is related with mutual fluctuations of the replicas belonging to different blocks.

The general equation (14) for the self-energy matrix σ_{ab} can be then rewritten as

$$\sigma_1 = \frac{Y}{J}\exp[-g(\Delta_1)], \quad (20)$$

$$\sigma_0 = \frac{Y}{J}\exp\left\{-\frac{1}{m}[g(\Delta_0) - g(\Delta_1)] - g(\Delta_1)\right\}, \quad (21)$$

where the function

$$g(\Delta) = \int \frac{d^D\mathbf{q}}{(2\pi)^D}\frac{1}{J(q^2+\Delta)} \quad (22)$$

describes the amplitude of the fluctuations corresponding to the gap Δ.

For $D = 2$ the factor $g(\Delta)$ diverges logarithmically for small Δ, therefore in the limit of $n \to 0$ for which $\Delta_0 \to 0$ we get $\sigma_0 = 0$. In what follows for simplicity we shall use the explicit expression for $g(\Delta)$ of the form corresponding to the sharp cutoff:

$$g(\Delta) \approx K \ln\frac{\Delta_c + \Delta}{\Delta}. \quad (23)$$

Here the prelogarithmical factor $K \equiv 1/4\pi J$ is proportional to the temperature T and $\Delta_c \propto \xi^{-2}$ is determined by the cutoff length ξ which should coincide with the period of the vortex lattice or with the correlation radius of the random potential (if it is larger).

Substitution of Eq. (19) with $\sigma_0 = 0$ into Eq. (10) gives (in the limit of $n \to 0$) the following simple expression for the free energy per replica:

$$\begin{aligned} f_{\text{var}} &\equiv \lim_{n\to 0}\frac{1}{n}[F_{\text{var}}(\Delta) - F_{\text{var}}(\Delta = 0)] \\ &= \frac{1}{2}\left\{\left(\frac{1}{m} - 1\right)\int_0^{\Delta} d\Delta'\,\Delta'\frac{dg(\Delta')}{d\Delta'} + Y(1-m)\exp[-g(\Delta)]\right\} \\ &= \frac{1}{2}\left[\left(1 - \frac{1}{m}\right)K\Delta_c \ln\frac{\Delta_c+\Delta}{\Delta_c} + Y(1-m)\left(\frac{\Delta}{\Delta_c+\Delta}\right)^K\right] \end{aligned} \quad (24)$$

which does not depend on the replica symmetric part of $G_{ab}^{-1}(\mathbf{q})$. Variation of the trial free energy (24) with respect to Δ and m gives a system of two equations which can be transformed to the form

$$\Delta = Ym\left(\frac{\Delta}{\Delta_c + \Delta}\right)^K, \quad (25)$$

$$m = K\frac{\ln(1+\Delta/\Delta_c)}{\Delta/\Delta_c}. \quad (26)$$

Equation (25) resembles very much the analogous equation for the sine-Gordon model.[19]

For high temperatures ($K > 1$) these equations have only one physical solution with $\Delta = 0$ whereas for the lower temperatures ($K < 1$) a solution with the finite value of the gap also appears. The value of the transition temperature coincides with that obtained in the framework of the renormalization-group analysis for the replica symmetric case.[10–12] In the vicinity of the transition Δ is much smaller than Δ_c, so Eqs. (25) and (26) give

$$\Delta \approx YK\left(\frac{YK}{\Delta_c}\right)^{\frac{K}{1-K}}, \quad (27)$$

$$m \approx K, \quad (28)$$

whereas for $T \to 0$ m becomes much smaller than K. It follows from Eq. (26) that the ratio K/m is always smaller than 1.

Thus we have obtained that at $K = 1$ a phase transition takes place in which the irreducible correlation function

$$\mathcal{G}(\mathbf{q}) = \overline{\langle[u(\mathbf{q}) - \langle u(\mathbf{q})\rangle][u^*(\mathbf{q}) - \langle u^*(\mathbf{q})\rangle]\rangle} \quad (29)$$

changes its form from

$$\mathcal{G}(\mathbf{q}) = \frac{1}{Jq^2} \quad (30)$$

in the high-temperature phase to

$$\mathcal{G}(\mathbf{q}) = \frac{\Delta}{m(q^2+\Delta)}\frac{1}{Jq^2} + \frac{1}{J(q^2+\Delta)} \quad (31)$$

in the low-temperature phase. At the same time the reducible part of the correlation function is the same for both phases:

$$\tilde{\mathcal{G}}(\mathbf{q}) = \frac{Wq_x^2}{(Jq^2)^2}. \quad (32)$$

In the framework of the self-consistent harmonic approximation no correction to the reducible part of the correlation function ever appears. It remain always equal to its bare form (32) which corresponds to the logarithmical divergence for $D = 2$ and to the absence of divergence for $D > 2$.

In the high-temperature phase the irreducible correlation function $\tilde{\mathcal{G}}(\mathbf{q})$ has the same form (30) as in the absence of the random potential. This is in agreement with the results of the renormalization-group analysis which shows that the random potential in that phase is irrelevant.[10–12] In the low-temperature phase the expression for $\tilde{\mathcal{G}}(\mathbf{q})$ contains two terms of different origin. According to Ref. 17 the first (gapless) term in Eq. (31)

can be associated with quenching of the system in different almost degenerate states whereas the second term, which has a finite gap, is related with thermal fluctuations in the vicinities of these states. In both phases the correlation function $w(\mathbf{R})$ diverges logarithmically but, in contrast to the high-temperature phase in which the prelogarithmic factor η for the irreductable part of $w(\mathbf{R})$ decreases with decrease in temperature ($\eta = 4K$), in the low-temperature phase $\eta = 4K/m$ increases with decrease in K. At $K = 1$ the derivative of η with respect to the temperature has a discontinuity.

In terms of the Coulomb gas representation[12] the phase transition at the point $K = 1$ corresponds to the appearance of free charges. On the whole there are $n(n-1)/2$ different types of the charges (which can be numbered by *two* replica indices $a < b$) in the system but they do not become free all simultaneously. The fraction of the types of the charges which are free at a given temperature is equal to $1 - m$ and increases continuously from zero to one when K decreases from one to zero.

In the renormalization-group analysis of Cardy and Ostlund[12] the appearance of the free charges in the low-temperature phase was not discovered because it was assumed that all the types of charges behave in the same way (conservation of replica symmetry). In the limit of $n \to 0$ the presence of the replica symmetric self-energy function does not change the interaction of the charges (cf. Sec. III) so in that description they, in both phases, remain bound in pairs. The difference between two phases in the behavior of the full correlation function $w(\mathbf{R})$ in that case is related to its reducible part. For $K > 1$ the correction to D_0 induced by the presence of the bound pairs behaves itself at small $\mathbf{q}$ as $\tilde{K}q^2$, so the reducible part of $w(\mathbf{R})$ diverges also logarithmically. For $K < 1$ the expression for $\tilde{K}$ becomes divergent. The renormalization-group analysis shows that an additional factor proportional to $\ln(1/q)$ appears in $\tilde{\mathcal{G}}(\mathbf{q})$,[12] leading thus to the anomalous behavior of the full correlation function $w(\mathbf{R})$ [Eq. (6)].

In the self-consistent harmonic approximation only the corrections induced by the free charges are taken into account. Formally they correspond to the lower orders of the perturbation expansion than the corrections related to the bound pairs and therefore are more important. But our analysis has revealed that for $0 < K < 1$ only part of the charges becomes free whereas the charges of the other types remain bound in pairs. Since in the replica symmetric case the corrections related to the bound pairs can change the behavior of the correlation function, we must check whether they do not do the same in case of the replica symmetry breaking.

It follows from the form of Eq. (19) that the prelogarithmic factor in the expression for the interaction of those charges that remain bound in pairs for $K < 1$ is equal to $4K/m > 4$, so in contrast to the replica symmetric case the correction to $D_0(\mathbf{q})$ induced by these bound pairs remains of the same analytical form as in the high-temperature phase. That means that the higher-order corrections do not introduce any qualitative changes in the behavior of the correlation functions with respect to the results of the self-consistent harmonic approximation.

V. HIERARCHICAL REPLICA SYMMETRY BREAKING

In a more general case of the hierarchical replica symmetry breaking the form of the self-energy matrix σ_{ab} can be described by a continuous function $\sigma(m)$ with $0 < m < 1$[18]. In that case the trial free energy acquires the form

$$f_{\rm var} = \frac{1}{2}\int_0^1 dm \left(\frac{1}{m^2}\int_0^{\Delta(m)} d\Delta\, \Delta \frac{d}{d\Delta} g(\Delta) + Y \exp\{-B[\Delta(m)]\} \right) \tag{33}$$

in which the dependence on $\sigma(m)$ enters only through the continuously distributed gap function:

$$\Delta(m) = \int_0^m ds\, s \frac{d\sigma(s)}{ds} \tag{34}$$

and the functional

$$B[\Delta(m)] = \frac{1}{m} g[\Delta(m)] - \int_m^1 ds \frac{1}{s^2} g[\Delta(s)] \tag{35}$$

plays the role of B_{ab}. In that notation the case of the one-step replica symmetry breaking considered in Sec. IV corresponds to the steplike function

$$\sigma(m) = \begin{cases} \sigma_0 & \text{for } 0 < m < m_1, \\ \sigma_1 & \text{for } m_1 < m < 1. \end{cases} \tag{36}$$

Substitution of Eq. (36) into Eq. (33) reproduces the form of the trial free energy for the case of the one-step replica symmetry breaking.

Variation of Eq. (33) with respect to $\Delta(m)$ gives the equation

$$\Delta(m) = Y \int_0^m ds\, s \frac{d}{ds} \exp\{-B[\Delta(s)]\}, \tag{37}$$

comparison of which with Eq. (34) shows that

$$\sigma(m) = Y \exp\{-B[\Delta(m)]\}. \tag{38}$$

Our aim consists of finding a solution $\Delta(m)$ of the nonlinear integral equation (37) in which integration is involved also through the definition of B.

For $\frac{d\Delta}{dm} \neq 0$ it turns out to be possible to transform this functional equation to the algebraic form. By differentiating Eq. (38) two times with the help of relations

$$\frac{d\sigma}{dm} = \frac{1}{m}\frac{d\Delta}{dm}, \quad \frac{dg}{dm} = \frac{1}{m}\frac{dB}{dm},$$

which can be obtained by the differentiation of Eqs. (34) and (35), one can get rid of the exponential expression and obtain a simple equation

$$\frac{d}{d\Delta}\left(\frac{dg}{d\Delta}\right)^{-1} = -\frac{1}{m}, \tag{39}$$

which does not contain the operation of integration.

Taking for the case of $D = 2$ the factor $g(\Delta)$ in the form (23), we get

$$\frac{d}{d\Delta}\left(\frac{dg}{d\Delta}\right)^{-1} = -\frac{1}{K}\left(1+\frac{2\Delta}{\Delta_c}\right)$$

so the solution of Eq. (37) is given by

$$\Delta(m) = \frac{\Delta_c}{2}\left(\frac{K}{m}-1\right) \qquad (40)$$

and accordingly

$$\sigma(m) = \text{const} + \frac{\Delta_c K}{m^2}. \qquad (41)$$

These results are in contradiction with the condition $\Delta(0) = 0$ which follows from the form of Eq. (34). Therefore we have to conclude that for $D = 2$ hierarchical replica symmetry breaking cannot be realized and the one-step replica symmetry breaking considered in Sec. IV takes place. It corresponds to the function $\sigma(m)$ of the form (36) for which $\frac{d\Delta}{dm}$ is equal to zero everywhere save one point. We have checked also that the solution found in Sec. IV is stable with respect to the second step of the replica symmetry breaking.

For $2 < D < 4$ the factor $g(\Delta)$ can be approximated at small Δ as

$$g(\Delta) = \int \frac{d^D\mathbf{q}}{(2\pi)^D}\frac{1}{J(q^2+\Delta)} \approx \frac{1}{J}(C - A\Delta^{D/2-1}), \qquad (42)$$

where the dimensionless coefficient

$$A = \frac{S_D}{(2\pi)^D}\frac{\pi}{2\sin\frac{D-2}{2}\pi} \qquad (43)$$

depends only on the dimensionality of the system (S_D is the area of D-dimensional sphere). For $g(\Delta)$ of the form (42)

$$\frac{d}{d\Delta}\left(\frac{dg}{d\Delta}\right)^{-1} = -\frac{J}{A}\frac{4-D}{D-2}\Delta^{1-D/2}$$

and therefore the solution of Eq. (37) should be chosen in the form

$$\Delta(m) = \begin{cases} \left[\frac{(4-D)Jm}{(D-2)A}\right]^{\frac{2}{D-2}} & \text{for } 0 < m < m_c, \\ \left[\frac{(4-D)Jm_c}{(D-2)A}\right]^{\frac{2}{D-2}} & \text{for } m_c < m < 1, \end{cases} \qquad (44)$$

where m_c also depends on C.

As in the case of the one-step replica symmetry breaking the correlation function for the displacement is given by the diagonal element of the matrix $\hat{G}$:[17]

$$\mathcal{G}(\mathbf{q}) = \frac{1}{Jq^2}\left[1+\int_0^1 dm\frac{1}{m^2}\frac{\Delta(m)}{q^2+\Delta(m)}\right]. \qquad (45)$$

Substitution of Eq. (44) into Eq. (45) gives then that the correlation function diverges at large distances logarithmically:

$$w(R) = 2(4-D)\ln R \qquad (46)$$

the prelogarithmical coefficient being not dependent on temperature. The results are in accordance with the results of Villain and Fernandez[13] obtained for $T = 0$ with the help of real-space renormalization procedure.

The character of the replica symmetry breaking in the considered problem (one-step replica symmetry breaking for $D = 2$ and full hierarchical replica symmetry breaking for $2 < D < 4$) turns out not to follow exactly the same pattern as in the problem of a fluctuating manifold in the random potential.[17] But surprisingly the behavior of the correlation function in our case is the same (logarithmical) for any dimension of interest.

VI. BIAXIAL VORTEX GLASS

In the case when the vortices are free to move in two directions the Hamiltonian of the vortex crystal interacting with the random potential can be written in the form

$$H = H_{\text{el}} + H_{\text{imp}}. \qquad (47)$$

Here the first term H_{el} describing the elastic energy of the vortex crystal is assumed to be harmonic and amenable to decomposition into two parts corresponding to longitudal and transversal components of the displacement. The second term describing the interaction of the vortex crystal with the random potential in that case will have the form

$$H_{\text{imp}} = \int dz \sum_{\mathbf{r}_\parallel} V[\mathbf{r}_\parallel + \mathbf{u}(\mathbf{r}_\parallel, z), z], \qquad (48)$$

where $V(\mathbf{r})$ with $\mathbf{r} = (\mathbf{r}_\parallel, z)$ is the random potential and two-dimensional index $\mathbf{r}_\parallel$ numbering the different vortex lines coincides with their positions in the equilibrium configuration of the lattice. For simplicity we have written the explicit form of this expression for $D = 3$. In the case of $D = 2$ the integration over dz should be omitted.

If the possibility of the formation of dislocations in the vortex crystal is excluded from the consideration, the points $\mathbf{r}_\parallel$ can be assumed to form a regular triangular lattice. In that case the summation over $\mathbf{r}_\parallel$ can be substituted by integration with the help of the additional summation over the set of the reciprocal lattice vectors $\mathbf{Q}$ of the triangular lattice:

$$\begin{aligned} H_{\text{imp}} &= \sum_{\mathbf{Q}} \int d^3r V(\mathbf{r}+\mathbf{u}) \exp i\mathbf{Q}\cdot r \\ &= \sum_{\mathbf{Q}} \int d^3R \det\left(\delta^{\alpha\beta} - \frac{\partial u^\alpha}{\partial R^\beta}\right) V(\mathbf{R}) \\ &\quad \times \exp i\mathbf{Q}\cdot[\mathbf{R}-\mathbf{u}(\mathbf{R})]. \end{aligned} \qquad (49)$$

In the last line of Eq. (49) the integration over d^3r is substituted by the integration over d^3R, where $\mathbf{R} = \mathbf{r} + \mathbf{u}(\mathbf{r})$.

Then if one takes the average of the partition function over the random potential $V(\mathbf{r})$ with Gaussian distribution the terms of the form

$$-\frac{1}{2}\overline{H_{\text{imp}}(\mathbf{u}_a)H_{\text{imp}}(\mathbf{u}_b)} = -\frac{\overline{V^2}}{2}\sum_{\mathbf{Q}\mathbf{Q}'}\int d^3\mathbf{R}\,\det\left(\delta^{\alpha\beta}-\frac{\partial u_a^\alpha}{\partial R^\beta}\right)\det\left(\delta^{\alpha\beta}-\frac{\partial u_b^\alpha}{\partial R^\beta}\right)$$
$$\times \exp i(\mathbf{Q}-\mathbf{Q}')\cdot\mathbf{R}\,\exp i[-\mathbf{Q}\cdot\mathbf{u}_a(\mathbf{R})+\mathbf{Q}'\cdot\mathbf{u}_b(\mathbf{R})] \tag{50}$$

will appear in the replica representation. If $\mathbf{u}_a(\mathbf{R})$ are assumed to change with $\mathbf{R}$ much more slowly than $\exp(i\mathbf{Q}\cdot\mathbf{R})$ with $\mathbf{Q}\neq\mathbf{0}$, then only the terms with $\mathbf{Q}=\mathbf{Q}'$ will survive in that sum. In that case the replicated Hamiltonian can be taken in the form

$$H_{\text{repl}} = \sum_a H_{\text{el}}(\mathbf{u}_a) - \frac{\overline{V^2}}{2}\sum_{a,b}\int d^3R\left\{(\nabla_\parallel \mathbf{u}_a)(\nabla_\parallel \mathbf{u}_b)+\sum_{\mathbf{Q}}\cos\mathbf{Q}\cdot[\mathbf{u}_a(\mathbf{R})-\mathbf{u}_b(\mathbf{R})]\right\}, \tag{51}$$

where we have retained only the cross term from the smooth factor related with the determinants.

Keeping in Eq. (51) only the terms corresponding to the smallest wave vectors $\mathbf{Q}$ we get the generalization of Eq. (10) for the case of the vectorial displacements. The same approach as was used earlier for the case of the scalar displacement will also be applicable for the analysis of the Hamiltonian (51). The most essential difference with the scalar case will be that the function $g(\Delta)$ will consist now of two terms (with the same value of the gap Δ) corresponding to longitudal and transversal displacements, respectively. But that will not change qualitatively any conclusions obtained above. Note however that these results can be expected to be valid only at the scales for which the presence of free dislocations (unclosed dislocation lines) can be neglected.

In the analysis of Bouchaud, Mézard, and Yedidia[8] the discrete nature of the lattice was neglected. In our notation that would correspond to taking only the harmonic part of the Hamiltonian (51) plus some anharmonicities related to the determinants which were omitted in Eq. (51). Since these terms contain only the derivatives of the displacement, simple power counting shows that they cannot be expected to induce the appearence of the gap (be it replica symmetric or not). In accordance with that, the authors of Ref. 8 have obtained that in their model the amplitude of the term with broken replica symmetry in the self-energy matrix in the physically correct limit becomes equal to zero. Only in the presence of the additional regularization breaking the symmetry properties of the original system does this term appear. In our description the symmetry of the Hamiltonian with respect to the continuous translation of one replica with respect to the other is broken not due to externally imposed regularization but is related to the nonlinear terms with $\mathbf{Q}\neq\mathbf{0}$ and leads to the logarithmic divergence of the correlation function instead of algebraic divergence. Addition of the higher harmonics of the interaction does not lead to any qualitative changes in the results.

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