Anomalous vortex diffusion in proximity-junction arrays

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The classical motion of a single vortex in a proximity-junction array is investigated with the help of the effective-action formalism. The dynamics of the superconducting order parameter of each superconducting grain is assumed to be of the Langevin type corresponding to local dissipation. The action describing the motion of the vortex in the array is shown to incorporate the propagator, the form of which in terms of the real-time dynamics corresponds to the logarithmical depression of mobility. Since in the array the motion of the vortex takes place in the effective periodic potential we check also that the presence of the periodic potential does not renormalize the anomalous behavior of the mobility in the low frequency limit. The results obtained are in agreement with experimental data on frequency dependence of the vortex contribution to proximity-junction-array response.

I. INTRODUCTION

Application of the inductive method for the investigation of two-dimensional superconducting systems gives the advantage of being able to measure both the resistive and inductive parts of the sample response in a wide range of frequencies.[1,2] Recently this method was applied for the investigation of proximity-junction arrays in small magnetic fields[3] at temperatures well below the temperature of the Beresinskii-Kosterlitz-Thouless transition and has allowed the isolation of the contribution to the response which is proportional to the applied field and therefore to the concentration of the vortices which are induced by this field. The anomalous frequency dependence of the inductive part of the response can be interpreted as evidence for the logarithmic depression of vortex mobility at low frequencies.[3]

Remarkably both the resistive and inductive part of the response turned out to be in agreement with the simple phenomenological theory which was developed by Minnhagen[4] for the properties of the neutral vortex plasma above the Beresinskii-Kosterlitz-Thouless transition. This theory is based on the assumption that at finite frequencies the form of the inductive part of the response function should be the same as at zero frequencies but with the substitution of the static correlation length (determined by the concentration of the free vortices) by the dynamical length l_ω which is defined as a diffusion length for the time ω^{-1}, the resistive part of the response being related to the dissipative part by the Kramers-Kronig relation.

A suggestion was made by Beck[5] that the origin of the anomalous behavior of mobility should be related to the presence of local dissipation on each superconducting grain of the array, which would correspond to applicability of the time-dependent Ginzburg-Landau equation for the description of the dynamics of the superconducting order parameter. However, in the calculation presented in Ref. 5 the anomalous frequency dependence of the vortex mobility appears only as a collective property of the vortex plasma, which is related to the particular form of the structure factor describing the static correlations of the vortices, whereas the diffusion of a single vortex is assumed to be normal (with constant mobility) according to the previous results of Beck and Ariosa.[6]

From the theory of vortex dynamics in bulk superconductors[7,8] it is known that the zero-frequency friction coefficient can be logarithmically large if the ratio of the normal conductance of a superconductor to the dissipative coefficient determining the dynamics of the superconducting order parameter is high enough. In homogenous materials this cannot happen since both dissipation coefficients are determined by the same relaxation time. On the other hand, in an array the local dissipation and normal conductance of the array are not related to each other. Therefore it is possible to consider a model in which only the local dissipation on each grain is taken into account (as was done in Refs. 5 and 6), but in that case the zero-frequency friction coefficient for the vortex should be expected to be logarithmically divergent.

In the present work we reinvestigate the problem of vortex dynamics in the framework of the model with local dissipation and show that the anomalous frequency dependence of mobility indeed appears already as a property of a single vortex motion. To demonstrate this we use the effective-action formalism, which allows one to extract not only the real but also the imaginary part of the vortex mobility (inductive part of the response function). In Sec. III we do it directly for the lattice system by deriving the effective action describing the motion of the vortex between two neighboring cells of a proximity-junction array. In Sec. IV we consider the continuous analog of the same model in which the motion of the vortex on a plane is unrestricted, and verify that for low frequencies it gives the same frequency dependence of the response function. In Sec. V we check what is the influence of the underlying periodic potential on the motion of the particle with the logarithmically depressed mobil-

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ity (since the vortex in the array *should* be considered as moving in the periodic potential[9,10]) and show that the correction to the inverse response function does not renormalize its anomalous part, and so the low frequency asymptotic behavior of the mobility remains the same. The concluding remarks are relegated to Sec. VI.

II. THE MODEL

Let us consider the Josephson junction array, assuming that it can be described by the following Euclidean action:

$$A = \frac{\eta}{4\pi}\sum_j \int dt \int dt' \left[\frac{\varphi_j(t)-\varphi_j(t')}{t-t'}\right]^2 + J\sum_{\langle jj'\rangle}\int dt\,\{1-\cos[\varphi_j(t)-\varphi_{j'}(t)]\}, \tag{1}$$

where φ_j denotes the phase of the superconducting order parameter on the jth superconducting grain.

The form of the first term in Eq. (1) corresponds to having for each of the variables φ_j the traditional Langevin dynamics with viscosity η.[11] This term cannot be associated with the proper Ginzburg-Landau dissipation inside the superconductor since in the array each superconducting grain is a macroscopic object for which the deviations from the Josephson relation

$$\hbar\frac{d\varphi_j}{dt} = 2eV_j \tag{2}$$

(where V_j is the electrostatic potential of the grain) are suppressed.[12] The origin of the dissipative term in Eq. (1) should be related rather to the possibility of normal current flow between each superconducting grain and a substrate which is a normal metal. In that case one can introduce the effective resistance to the ground R which determines the value of η: $\eta = (\hbar/2e)^2R^{-1}$.

The second term in Eq. (1) describes the Josephson coupling between the superconducting grains. In this term the summation is assumed to be performed over the pairs of grains which are connected by the Josephson junctions, the coupling constant for which is denoted by J. In terms of electrical parameters J is proportional to the critical current of the junction, I_c: $J = (\hbar/2e)I_c$.

Since we are interested in vortex diffusion in an overdamped proximity-junction array we do not include into the consideration the mass term describing the charging effects. We also neglect all the screening effects. For magnetic field screening this is fully justified since in the experimental situation the effective penetration depth for the array is usually large.[13] The situation with electrical field screening is less certain and will be discussed in the conclusion.

We are starting from the Euclidean action just to simplify the derivation and that does not imply that we are going to consider any quantum properties of the system. The same action can be applied also for the description of a two-dimensional planar magnet with overdamped spin dynamics.

III. EFFECTIVE ACTION FOR THE MOTION OF THE VORTEX BETWEEN THE NEIGHBORING ARRAY CELLS

In the equilibrium vortex configuration on a square lattice the cell which can be associated with the core of the vortex is surrounded by four bonds, on each of which the phase difference

$$\Phi_{jj'} = \varphi_j - \varphi_{j'} \tag{3}$$

is equal to $\pi/2$. When the vortex moves to the neighboring cell the phase difference $\Phi_{jj'}$ on the bond separating the initial and the final positions of the core goes through the point $\Phi_{jj'} = \pi$ corresponding to the maximum of the energy for the particular junction which is crossed by the core, and ends up in becoming equal to $3\pi/2$ (which is equivalent to $-\pi/2$), the total change of $\Phi_{jj'}$ being equal to π.

For all the other junctions the change of the phase difference $\Phi_{jj'}$ in the considered process is not so large and the maximum of the cosine potential is not achieved, so for each of these junctions it is reasonable to substitute the cosine potential by a parabolic one with suitably chosen positions of the minima:

$$J\{1-\cos[\varphi_j(t)-\varphi_{j'}(t')]\} \Rightarrow \frac{J}{2}[\varphi_j(t)-\varphi_{j'}(t')-2\pi m_{jj'}]^2, \tag{4}$$

where, since we want to consider the configuration which contains a vortex, the integer variables $m_{jj'} \equiv -m_{j'j}$ should be chosen in such a way as to make the sum of $m_{jj'}$ over each contour surrounding the vortex core equal to 1 and for the contours not surrounding the vortex core equal to zero.

After such a transformation we obtain that the action depends quadratically on all the degrees of freedom but one, which is certainly the phase difference $\Phi_{jj'}$ on the particular bond which is crossed by the vortex and which we will denote simply as Φ starting from now. Therefore all the variables except Φ can be eliminated from the modified action by taking its variation with respect to φ_j, solving the system of linear equations with the condition that for the particular bond $\varphi_j(t) - \varphi_{j'}(t) = \Phi(t)$ [where $\Phi(t)$ is in principle an arbitrary function of time], and substituting the solution back into the action. This can be done both at zero and at finite temperature since we are now considering an approximation in which the fluctuations of all the other degrees of freedom but Φ are Gaussian.

Such an approach was introduced by Larkin, Ovchinnikov, and Schmid[14] for the investigation of quantum vortex tunneling in the presence of charging effects in an array without dissipation. It produces a new effective action which depends on a single variable $\Phi(t)$ and can be naturally decomposed into two parts,

$$A_{\rm eff} = \frac{1}{2}\int dt\int dt' g^{-1}(t-t')\Phi(t)\Phi(t') + \int dt\, V[\Phi(t)], \tag{5}$$

the first of which can be associated with the dynamics of the variable Φ and the second with the effective potential in which it moves. In our problem the Fourier transform of the function $g^{-1}(t)$ for the case of a regular square lattice has the form

$$g^{-1}(|\omega|) = 2\left[\int \frac{d^2\mathbf{q}}{(2\pi)^2}\,\frac{k^2}{\eta|\omega| + Jk^2}\right]^{-1} - 2J, \qquad (6)$$

where

$$k^2 \equiv k^2(\mathbf{q}) = 2(1-\cos q_x) + 2(1-\cos q_y)$$

and the integration is performed over the Brillouin zone.

The potential $V(\Phi)$ which we obtain has the double-well form

$$V(\Phi) = \frac{J}{2}(\Phi - \pi)^2 + J(1-\cos\Phi) + \text{const}, \qquad (7)$$

in which the positions of the minima are slightly misplaced with respect to the points $\pi/2$ and $3\pi/2$. This displacement is quite natural since we have treated the different bonds around the vortex core in a different way. The correct positions of the minima will be recovered if we also make a substitution of the form of the potential (from the cosine to the piecewise parabolic potential) for the particular bond which is crossed by the vortex core.

But the particular form of the potential $V(\Phi)$ which is obtained in the calculation of the effective action is of small importance since it is evident that in the framework of a more general description the vortex should experience a periodic rather than a double-well potential.[10] However, if we are interested in vortex motion at temperatures which are larger than the amplitude of this potential its presence can be expected to be not very relevant and the motion of the vortex can be expected to be satisfactory described by some frequency-dependent propagator which in the first approximation does not depend on the potential.

We propose that, since the first (dynamic) part of the action (5) is harmonic in variable Φ, we can associate the motion of the vortex with the change of Φ and assume that it is described by the same propagator divided by the scaling factor π^2 which appears because the change of Φ by π corresponds to the motion of the vortex by one lattice constant. If we furthermore are interested in the real-time dynamics we should make an analytical continuation from the imaginary to the real frequencies. For the propagator (the response function) this procedure reduces to a simple substitution of $|\omega|$ by $-i\omega$ which is more convenient to do directly in Eq. (6) before the integration is performed. Thus the propagator describing the motion of the vortex in the absence of the potential will be given by

$$G(\omega) = \frac{1}{\pi^2}g(-i\omega). \qquad (8)$$

The integration in Eq. (6) gives then that the frequency dependence of the inverse response function $G^{-1}(\omega)$ can be characterized by the following asymptotic behavior:

$$G^{-1}(\omega) \approx \begin{cases} -i\frac{\pi}{2}\eta\omega\ln\frac{Jq_0^2}{\eta|\omega|} + \frac{\pi^2}{4}\eta|\omega| & \text{for } |\omega| \ll J/\eta, \\ \frac{\pi^2}{2}(-i\eta\omega + J) & \text{for } |\omega| \gg J/\eta, \end{cases} \qquad (9)$$

where the value of the effective cutoff for the momentum q_0 can be chosen by equating the area of the Brillouin zone to that of a circle with radius q_0. For the square lattice this gives $q_0^2 = 4\pi$.

An analogous calculation of $G^{-1}(\omega)$ for a triangular lattice produces a result which differs from Eq. (9) by a factor of $2/\sqrt{3}$ in the low frequency limit and by a factor of $4/3$ in the high frequency limit. Note that the low frequency factor $2/\sqrt{3}$ is related not to the structure of the lattice, but simply to the average density of sites which for a triangular lattice is $2/\sqrt{3}$ times larger than for the square lattice with the same lattice constant.

Thus we have obtained that the mobility of the vortex $\mu(\omega) \equiv -i\omega G(\omega)$ at low frequencies does not remain constant but decreases inversely proportional to the logarithm, the real part of the inverse response function being proportional to $|\omega|$ as was observed in experiment.[3] Let us emphasize that the low frequency asymptotics of $G(\omega)$ is determined by integration over small wave vectors, which in terms of the original problem corresponds to distances far away from the vortex core where our approximation (4) becomes correct not only qualitatively but also quantitatively.

In the so-called resistance-shunted model the dissipation is assumed to take place only on the junctions, so the dynamics is of the Langevin form for the variables $\Phi_{jj'}$ and the first term in Eq. (1) should be substituted by

$$\frac{\eta_S}{4\pi}\sum_{\langle jj'\rangle}\int dt\int dt'\left[\frac{\Phi_{jj'}(t) - \Phi_{jj'}(t')}{t-t'}\right]^2,$$

where η_S is inversely proportional to the normal resistance R_S shunting the junction: $\eta_S = (\hbar/2e)^2 R_S^{-1}$. For such a model the analogous calculation gives instead of Eq. (6) an expression which differs from it by the substitution

$$\eta \Rightarrow \eta_S k^2$$

and the dynamics of the vortex is of the normal Langevin type without any anomalies in the low frequency limit [that is, with finite mobility $\mu = 1/(2\pi^2\eta_S)$]. When two types of dissipation are present simultaneously, at low frequencies their contributions to $G^{-1}(\omega)$ are just added to each other:

$$G^{-1}(\omega) = -i\left[\frac{\pi}{2}\eta\ln\frac{Jq_0^2}{\eta|\omega|} + 2\pi^2\eta_S\right]\omega + \frac{\pi^2}{4}\eta|\omega|. \qquad (10)$$

However, two points still remain to be clarified: (i) since our definition of the relevant physical variable describing the motion of the vortex is local we have to check if this motion can still be described by the same propagator when it is not confined to two nearest cells of the array, and (ii) since at low frequencies the diffusion of the vortex is slower than that of a particle with simple

Langevin dynamics we have to check if this does not make the periodic potential in which the vortex moves relevant at any temperature.

IV. CONTINUOUS APPROXIMATION

Let us now consider the continuous analog of the same model. In that case it would be convenient to introduce into the action the auxiliary field $\mathbf{m}(\mathbf{r},t)$ defining the vortex position right from the beginning:

$$A = \int d^2\mathbf{r} \int dt \left\{ \frac{\eta}{4\pi} \int dt' \left[\frac{\varphi(\mathbf{r},t) - \varphi(\mathbf{r},t')}{t-t'} \right]^2 + \frac{J}{2}(\nabla\varphi + 2\pi\mathbf{m})^2 \right\}. \quad (11)$$

Since we want to consider the situation with only one vortex present the constraint for $\mathbf{m}$ will have the form

$$\nabla \times \mathbf{m} = \delta[\mathbf{r} - \mathbf{R}(t)], \quad (12)$$

where $\mathbf{R}(t)$ is the position of the vortex as a function of time.

Such a formulation allows one to explicitly take into account the peculiar properties of differential operators which appear in the presence of the vortex. The absence of $\mathbf{m}$ in the first term of Eq. (11) implies that we are considering some particular gauge for the auxiliary field $\mathbf{m}$. The velocity of the vortex $d\mathbf{R}/dt$ in this gauge is determined only by the time derivatives of $\mathbf{m}$:

$$\frac{\partial \mathbf{m}}{\partial t} = -\hat{\mathbf{z}} \times \frac{d\mathbf{R}}{dt}\delta[\mathbf{r} - \mathbf{R}(t)]. \quad (13)$$

Since the action (11) is quadratic with respect to φ this variable can be eliminated from it in the same way as was done in the previous section for the lattice model. Variation of (11) with respect to φ gives

$$\varphi(\mathbf{q},\omega) = \frac{2\pi J}{\eta|\omega| + Jq^2} i\mathbf{q}\cdot\mathbf{m}(\mathbf{q},\omega), \quad (14)$$

whereas substitution of (14) back into (11) produces an expression for the effective action which depends only on $\nabla \times \mathbf{m}$ and $\partial\mathbf{m}/\partial t$ and not on $\mathbf{m}(\mathbf{r},t)$ itself. With the help of the relations (12) and (13), this expression can be rewritten as

$$A = \frac{1}{2}\int dt \int dt' \left\{ U_1[\mathbf{R}(t) - \mathbf{R}(t'), t-t'] + U_2[\mathbf{R}(t) - \mathbf{R}(t'), t-t'] \frac{d\mathbf{R}(t)}{dt}\frac{d\mathbf{R}(t')}{dt'} \right\}, \quad (15)$$

where the functions $U_{1,2}(\mathbf{R},t)$ are defined by their Fourier transforms

$$U_1(\mathbf{q},|\omega|) = \frac{4\pi^2 J^2}{\eta|\omega| + Jq^2}; \qquad U_2(\mathbf{q},|\omega|) = \frac{4\pi^2 J\eta|\omega|^{-1}}{\eta|\omega| + Jq^2}.$$

An analogous nonlocal expression for the action of the vortex in the absence of dissipation (but in the presence of charging effects) has been obtained by Eckern and Schmid.[10]

If we are interested in the linear response function for the vortex we have to expand Eq. (15) to the second order in $\mathbf{R}$ and make an analytical continuation of the propagator thus obtained to real frequencies. This gives a rather simple expression:

$$G^{-1}(\omega) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left\{ \frac{q^2}{2}[U_1(\mathbf{q},-i\omega) - U_1(\mathbf{q},0)] + \omega^2 U_2(\mathbf{q},-i\omega) \right\} = 2\pi^2 J \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{-i\eta\omega}{-i\eta\omega + Jq^2}, \quad (16)$$

the form of which implies that the integration over momenta should be restricted by some form of cutoff. If the sharp cutoff at $|\mathbf{q}| = q_0$ is assumed the integration in Eq. (16) gives

$$G^{-1}(\omega) = -i\frac{\pi}{2}\eta\omega \ln\left[1 + \frac{\gamma}{-i\omega}\right]; \quad (17)$$

where $\gamma = Jq_0^2/\eta$.

The low frequency behavior of $G^{-1}(\omega)$ is determined by the integration over small momenta and only logarithmically depends on the cutoff. Moreover, for $\omega \ll \gamma$ the expression (17) has the same asymptotic behavior as was obtained for the vortex propagator in the lattice system [Eq. (9)] and corresponds to logarithmic depression of the mobility.

The agreement between the results of the continuous approximation assuming the nonrestricted motion of the vortex and of the lattice calculation in which the motion of the vortex was restricted to a pair of neighboring cells confirms that in a lattice calculation we have made a correct choice of the relevant variable describing the vortex motion and therefore can trust the results of the lattice calculations not only in the limit of small frequencies, but also for higher frequencies. In that case, however, some numerical corrections may be needed since at high frequencies the form of the propagator is determined by all the length scales and in the vicinity of the vortex core the approximation (4) is equivalent to overestimation of the effective coupling.

V. IRRELEVANCE OF THE PERIODIC POTENTIAL

One more point which still remains to be clarified is how the diffusion of the vortex is influenced by the presence of the periodic potential in which this motion takes place. Since the diffusion of the vortex is slower than that of the ordinary particle it is possible that the presence of the periodic potential may be of greater importance and we have to check if it does not lead to a further depression of the diffusion.

To this end we have performed a perturbative calculation of the correction to the mobility of a particle moving in the periodic potential

$$V(X) = V_0 \cos(2\pi X)$$

in the same fashion as was done by Chui and Weeks[15] for the two-dimensional sine-Gordon model. We obtain that the first nonvanishing correction to the inverse of the response function appears in the second order in the amplitude of the periodic potential and is of the form

$$\Sigma(\omega) = -i\omega \frac{(2\pi V_0)^2}{2} \int_0^{+\infty} dt \exp(i\omega t) \times \exp\{-(2\pi)^2[D(t=0) - D(t)]\}, \quad (18)$$

where $D(t)$ is the equilibrium correlation function for the variable X in the absence of the potential:

$$D(t) \equiv \langle X(t+t')X(t')\rangle = \int \frac{d\omega}{2\pi} D(\omega) \exp(-i\omega t), \quad (19)$$

the Fourier transform of which, $D(\omega)$, can be related to the response function $G(\omega)$ with the help of the fluctuation-dissipation theorem:

$$D(\omega) = \frac{T}{i\omega}[G(\omega) - G(-\omega)], \quad (20)$$

T being the temperature of the system.

For the particle which in the absence of a potential has constant mobility $\mu = \eta^{-1}$, calculation of $D(t)$ gives that the correlations diverge in accordance with the diffusion law,

$$D(0) - D(t) \equiv \tfrac{1}{2}\langle [X(t+t') - X(t')]^2\rangle = \frac{Tt}{\eta},$$

so the integral in Eq. (18) can be explicitly calculated, giving a correction to G^{-1} of the form

$$\Sigma(\omega) = \frac{(2\pi V_0)^2}{2T} \frac{-i\omega}{-i\omega + \omega_0}, \quad \omega_0 \equiv \frac{T}{\eta}, \quad (21)$$

which corresponds to effective increase of the viscosity η and appearance of the inductive term in the inverse response function which at low frequencies is proportional to ω^2. The correction to the viscosity is small in comparison with the viscosity itself for $T \gg V_0$ and that defines the region where the corrections of higher order can be neglected.

Now if we take the particle (the vortex) the propagator of which is defined by Eq. (17), calculation of $D(t)$ becomes a more complicated problem. But since the only singularity in Eq. (17) is a cut on the imaginary axis, integration over $d\omega$ in Eq. (19) can be shifted to that cut, allowing one to rewrite the expression for $D(0) - D(t)$ as an integral of a real positive function,

$$D(0) - D(t) = \frac{2T}{\pi\eta} \int_0^{\gamma} d\Omega \frac{1 - \exp(-\Omega|t|)}{\Omega^2 \left(\ln^2 \frac{\gamma - \Omega}{\Omega} + \pi^2\right)}, \quad (22)$$

which is easier to evaluate. The most important contribution to the integral in Eq. (22) comes from the interval of small Ω where $1 - \exp(-\Omega|t|)$ behaves as $\Omega|t|$. Calculation of this contribution for $|t| \gg \gamma^{-1}$ gives then that for the vortex propagator the real-time correlations diverge with logarithmic corrections to the diffusion law,

$$D(0) - D(t) \approx \frac{2T}{\pi\eta} \frac{t}{\ln(\gamma t)}, \quad (23)$$

which is rather natural for a system with logarithmic behavior of the mobility.

Substitution of Eq. (23) into Eq. (18) shows that in the expansion of the expression (18) in powers of $(i\omega)$ all the coefficients remain convergent, as they are for the particle with the constant mobility; therefore the anomalous part of the response function which is singular at zero frequency remains unrenormalized. The condition for the quantitative applicabilty of the lowest order pertrubative calculation remains essentially the same, $V_0 \ll T$, if we disregard the logarithmic corrections. But it can be expected that for low enough frequencies the asymptotic behavior of the mobility will remain the same even if the potential is not small.

VI. CONCLUSION

We have investigated the classical motion of the vortex in a two-dimensional proximity-junction array assuming the dynamics of the superconducting order parameter to be of the Langevin type, and have found the frequency dependence of the complex vortex mobility.

In contrast to the resistance-shunted model in which the dissipation is assumed to take place only on the junctions, the presence of local (on-grain) dissipation makes the dispersion law for the long-wavelength fluctuations gapless, which allows for the appearance of nonanalytical contributions related to the effective volume of the system which is involved in the motion of the vortex at the given frequency. Earlier it was shown for systems without dissipation[10,14] that when the form of the capacitance matrix is dominated by the capacitance to the ground the mass of the vortex also becomes logarithmically dependent on frequency.

Our results allow us to explain the experimental data[3] by the assumption that the vortex contribution to the array response is given by the single vortex response times the concentration of the vortices. In that way we get a frequency dependence which is in qualitative agreement with the experimental results both for the resistive and the inductive part of the proximity-junction array response.[3]

The question of quantitative comparison with the experiment is more subtle. We have used a model in which the sheet resistance r of the substrate is not taken into account, that is, the electrical potential of the substrate is assumed to be uniform. In other words, this can be described as complete neglect of the electric field screening in the array. Such an approximation can be justified only in the case $R_S, R \gg r$, otherwise the frequency range in which the mobility of the vortex has logarithmic dispersion cannot exist. Unfortunately, the direct measurement of all these resistances is impossible. And if we compare the value of R which can be extracted from the data on vortex mobility[3] with the normal state resistance of the array measured at temperatures well above the temperature of the Berezinskii-Kosterlitz-Thouless transition[3] (when the grains are superconducting but incoherent) we find that their ratio is not so large as could be expected. On the other hand the experimental data prove that the vortex mobility has dispersion and therefore the screening of the electric field is not important. The nature of this discrepancy still has to be clarified.

On the triangular lattice the height of the barrier which the vortex has to overcome is very small (about 4% of the coupling constant[9]) and accordingly small is the amplitude of the effective potential in which the vortex can be considered to be moving. That is why a temperature range can exist in which there are no temperature-activated vortex pairs (whose presence will produce additional problems for the motion of field-induced vortices), but the periodic potential is still not important.[3] On the square lattice the height of the barrier is five times higher[9] so the observation of the anomalous diffusion of field-induced vortices can be more problematic, but in principle it may be also possible, since we have shown that the corrections related to the presence of the periodic potential do not renormalize the asymptotic form of the response function.

Calculation of the real-time response function for the vortex is equivalent to derivation of the equation of motion for it which for the continuous system has been undertaken earlier by Beck and Ariosa.[6] These authors have obtained the result that the equation of motion for the vortex is of the Langevin type (with constant mobility) both for the case of local dissipation and for the continuous limit of the resistance-shunted model. However, their calculation involves linearization with respect to variation of the phase due to the motion of the vortex (which in the vicinity of the core may be not so small) and the equation of motion for the vortex is constructed by choosing some combination of equations of motion for the phase which does not appear to be an unambiguous procedure. The effective action formalism which has been used in this paper allows us to bypass both of these obstacles.

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