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Phase diagram of a Josephson junction array with positional disorder

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Abstract

A Josephson junction array with geometrical irregularities in presence of perpendicular magnetic field can be described by a two-dimensional XY-model with random phase shift. If the magnitude of the field corresponds to having on the average the integer number of flux quanta per plaquette, the problem can be discussed in terms of a two-dimensional Coulomb gas interacting with a random potential the correlations of which diverge logarithmically. In the present work we discuss the phase diagram of such system and show that the low-temperature reentrant transition into disordered phase is not possible. Our conclusions are in agreement with the results of both real and computer experiments.

1. Introduction

A Josephson junction array in a perpendicular magnetic field can be described by the Hamiltonian

$$H = -J\sum_{jj'}\cos(\varphi_j - \varphi_{j'} - A_{jj'}), \tag{1}$$

where φ_j is the order parameter phase on jth superconducting island and summation is performed over all the junctions which connect the islands. The phase shift on each junction $A_{jj'}$ is determined by the vector potential of the magnetic field and in a typical experimental situation can be ascribed entirely to the external field [1].

If the magnetic field is absent or its value corresponds to the integer number of flux quanta per plaquette, all variables $A_{jj'}$ can be put equal to zero. In that case the Hamiltonian (1) reduces to the Hamiltonian of the ordinary two-dimensional XY-model which demonstrates a phase transition between a quasi-ordered low-temperature phase in which all vortices are bound in pairs and a disordered high-temperature phase in which vortex pairs are debound [2–4].

But if the array has geometrical irregularities (the random displacements in the positions of the junctions) it becomes impossible to get rid of the phase shifts $A_{jj'}$ even if the average flux per plaquette is integer. In such case one has to treat $A_{jj'}$ as quenched random variables. In the following we assume that the random variables $A_{jj'}$ are independent from each other and obey the Gaussian statistics with

$$\overline{A_{jj'}} = 0, \qquad \overline{A_{jj'}^2} = \sigma. \tag{2}$$

Here and further on the average over disorder is denoted by an overbar.

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Model (1) has been introduced by Rubinstein et al. [5] for the description of a planar magnet with random Dzyaloshinskii–Moriya interaction and only afterwards it has been suggested that the same model can be applied for the description of the Josephson junction array with positional disorder in presence of perpendicular magnetic field [6]. The statistical mechanics of vortices in model (1) can be described with the help of a Coulomb gas Hamiltonian [5]:

$$H_{CG} = \frac{1}{2}\sum_{r,r'} s_r G_0(r - r')\, s_{r'} - \sum_r V(r)\, s_r, \tag{3}$$

where the integer variables s_r (describing the topological charges of the vortices) are defined on the sites r of a dual lattice. The presence of random variables $A_{jj'}$ manifests itself in the appearance of a random potential $V(r)$ which is linearly related to $A_{jj'}$. Therefore, if the distribution of $A_{jj'}$ is Gaussian then the distribution of $V(r)$ is also Gaussian.

In terms of the Coulomb gas charges each of the random variables $A_{jj'}$ corresponds to a quenched dipole. Since a potential of a dipole in two dimensions decays as $1/R$, the fluctuations of the potential created by the random dipoles uniformly distributed over whole plane have to diverge logarithmically [5]:

$$d(r_{12}) \equiv \overline{[V(r_1) - V(r_2)]^2} \approx 4\pi J^2 \sigma \ln |r_{12}|,$$

$$r_{12} = r_1 - r_2. \tag{4}$$

The expression $V(r_1) - V(r_2)$ which enters Eq. (4) can be also interpreted as a random contribution to the energy of a neutral vortex pair (with one vortex at point r_1 and the other at point r_2) and therefore Eq. (4) defines the width of the distribution of this quantity.

The function $G_0(r)$ describing the interaction of the vortices remains the same as in absence of disorder. In contrast to the energy of a single vortex which is logarithmically divergent the energy of a neutral vortex pair $g(r_{12})$ is finite and for the large separation between the vortices depends logarithmically on the separation

$$g(r_{12}) \equiv G_0(r = 0) - G_0(r_{12}) \approx 2\pi J \ln |r_{12}|. \tag{5}$$

In Eqs. (4) and (5) and further on we assume that the lattice constant is equal to one.

The analysis of model (1) developed by Rubinstein et al. [5] is based on the consideration of the renormalization of vortex interaction and random potential distribution (in the lowest order in vortex fugacity). The main conclusions of Ref. [5] are that for large disorder the ordered phase is always destroyed, whereas for small disorder it should exist but with the decrease in temperature the reentrant transition to the disordered phase should always take place. Only the first of these conclusions has been confirmed (both in the experiments on Josephson junction arrays [7] and in numerical simulations [7–9]) whereas the reentrant transition to the disordered phase has never been observed.

Recently, it has been shown [10] that the consideration of the higher-order corrections to the calculation of Rubinstein et al. [5] reveals the appearance of a new divergence in each order of the expansion and leads to the further suppression of the domain of stability of the ordered phase until it completely disappears. This can mean that the ordered phase (in which all the vortices are supposed to be bound in pairs) is always destroyed or that the correct description of the system should be based on some other approach [10]. On the other hand, Ozeki and Nishimori [11] have proved with the help of the formal gauge transformation that if the ordered phase of the XY-model with random phase shift does exist then the phase diagram cannot include a reentrant transition into the disordered phase. Unfortunately, this approach also does not allow to make any conclusions about the existence of the ordered phase.

In the present work we briefly describe two different approaches which demonstrate that if the disorder is sufficiently weak the ordered phase should be stable both at zero and at low enough temperatures. One of them consists in a simple comparison of the proper (elastic) energy of a vortex with a maximal energy gain it can acquire due to presence of disorder, whereas the other is based on the systematic calculation of the corrections to

vortex interaction and random potential distribution due to presence of bound vortex pairs. The main improvement with respect to the analysis of Rubinstein et al. [5] is that instead of discussing the lowest-order corrections in vortex fugacity, we consider the lowest-order corrections in vortex pair concentration (in a disordered system they do not concide). Such approach seems to be equivalent to the effective resummation of the infinite set of divergencies one encounters in the frame-work of the fugacity expansion [10] and leads to the drastic consequences for the shape of phase diagram from which the reentrant transition to the disordered phase is eliminated. Some other arguments in favour of the same conclusions can be found in an earlier publication [12]. A more detailed presentation of the approach developed in Section 3 can be found in Ref. [13].

2. A single vortex consideration

A phase transition in a regular XY-model can be associated with an appearence of free vortices due to thermal fluctuations. We start our discussion of the disordered system by considering if the presence of logarithmically divergent random potential can induce the spontaneous creation of vortices even at zero temperature.

In the case of the finite size array $(L \times L)$ the regular part of the vortex energy can be estimated as

$$E_0(L) \approx \pi J \ln L, \tag{6}$$

whereas the disorder-induced contribution V to the energy of the single vortex is characterized by a Gaussian distribution with a width $\varepsilon(L)$ given by

$$\varepsilon(L) \approx J\sqrt{2\pi\sigma \ln L}. \tag{7}$$

Comparison of Eq. (7) with Eq. (6) shows that in the limit of $L \to \infty$ the typical value of V becomes negligible in comparison with $E_0(L)$.

But this does not mean that the random potential cannot induce the spontaneous creation of vortices. In order to check if the creation of the vortex is energetically favourable it is necessary to compare $E_0(L)$ not with the typical value of $V(\boldsymbol{r})$ for the given L but with the maximal value of $V(\boldsymbol{r})$ for the given potential realization (that is with the maximal energy gain V_m the vortex can acquire by finding the suitable position in the array).

An estimate for this quantity can be constructed by neglecting the correlations between the values of a random potential on different sites. This corresponds to considering $N = L^2$ independent random variables $V(\boldsymbol{r})$ all of which are characterized by the same Gaussian distribution

$$p(V) = \frac{1}{\sqrt{2\pi}\varepsilon} \exp\left[-\frac{V^2}{2\varepsilon^2}\right] \tag{8}$$

the width of which is given by Eq. (7) and therefore depends on the number of these variables.

The distribution of V_m is then given by

$$P(V_m) = Np(V_m)\left[\int_{-\infty}^{V_m} \mathrm{d}V\, p(V)\right]^{N-1} \equiv \frac{\mathrm{d}}{\mathrm{d}V_m}\left[\int_{-\infty}^{V_m} \mathrm{d}V\, p(V)\right]^N. \tag{9}$$

For $N \gg 1$ the most convenient way to describe the form of $P(V_m)$ consists in looking at

$$A(V_m) \equiv -\ln\frac{P(V_m)}{p(0)} = -(N-1) \times \ln\left[\int_{-\infty}^{V_m} \mathrm{d}V\, p(V)\right] - \ln\frac{p(V_m)}{p(0)} - \ln N. \tag{10}$$

The minimum of $A(V_m)$ [and therefore the maximum of $P(V_m)$] is achieved when

$$\frac{\mathrm{d}A}{\mathrm{d}V_m} = 0. \tag{11}$$

Since for $V_m \gg \varepsilon$

$$\frac{\mathrm{d}A}{\mathrm{d}V_m} \approx -(N-1)p(V_m) + \frac{V_m}{\varepsilon^2}, \tag{12}$$

the solution of Eq. (11) is given by

$$[V_m^{(0)}]^2 \approx 2\varepsilon^2\left[\ln(N-1) - \ln\frac{\sqrt{2\pi}\,V_m^{(0)}}{\varepsilon}\right] \approx 2\varepsilon^2 \ln N. \tag{13}$$

On the other hand, the width of the distribution of V_m can be estimated by calculating the second

derivative of A:

$$\left.\frac{d^2A}{dV_m^2}\right|_{V_m=V_m^{(0)}} \approx (N-1)\frac{V_m^{(0)}}{\varepsilon^2}p(V_m^{(0)}) + \frac{1}{\varepsilon^2} \approx \frac{2\ln N}{\varepsilon^2}. \tag{14}$$

Comparison of Eq. (14) with Eq. (13) shows that for $N \to \infty$ the distribution of V_m is rather narrow:

$$\frac{[\overline{(V_m - V_m^{(0)})^2}]^{1/2}}{V_m^{(0)}} \sim \frac{1}{2\ln N}, \tag{15}$$

and therefore the average value of V_m should be very close to $V_m^{(0)}$:

$$\overline{V_m} \approx \sqrt{2\ln N}\varepsilon. \tag{16}$$

By calculating the next derivative of A it is possible to check that the distribution of V_m is not Gaussian.

Substitution of Eq. (7) into Eq. (16) shows that in the considered approximation the average value of the maximal energy gain diverges with the size of the system logarithmically:

$$\overline{V_m} \approx J\sqrt{8\pi\sigma}\ln L, \tag{17}$$

and therefore the average energy of the vortex created at the most favourable position in the array is given by

$$E_m(L) \equiv E_0(L) - \overline{V_m}(L) \approx \pi J\left(1 - \sqrt{\frac{\sigma}{\sigma_*}}\right)\ln L, \tag{18}$$

where $\sigma_* = \pi/8$.

For $\sigma < \sigma_*$ the regular contribution to $E_m(L)$ dominates and therefore the creation of the vortex requires (in the limit of $L \to \infty$) the infinite energy. On the other hand, for $\sigma > \sigma_*$ the addition of the vortex to the system can decrease its energy and therefore the spontaneous creation of the vortices should take place.

When the correlations of random potential on different sites are taken into account they tend to decrease the mutual scattering of the random variables $V(r)$ and therefore the value of $\overline{V_m}(L)$ can only be decreased. Therefore, the previous calculation gives an upper border for $\overline{V_m}$ and a lower border for E_m and suggests that at least for $\sigma < \sigma_*$ the model is stable with respect to spontaneous creation of vortices by disorder.

If the real dependence of $\overline{V_m}$ on L is indeed logarithmical but with the smaller prefactor than in Eq. (17) this would correspond simply to the shift of the critical value of the disorder from $\pi/8$ to the larger value. On the other hand, if the real dependence of $\overline{V_m}$ on L is slower than logarithmic the conclusion of a single vortex consideration should be that the arbitrarily strong random potential cannot induce the creation of vortices. To show that this is not the case it is necessary to construct the lower border for $\overline{V_m}$ which also demonstrates the logarithmic dependence on L.

A possible way to do it consists in separating the contributions to $V(r)$ from different scales. To this end, let us divide our system of $N = L^2$ sites into $M \gg 1$ equal parts and discuss the potential which is created inside of each of these subsystems by all the other subsystems. To estimate this potential we can substitute each of the other $M - 1$ subsystems by a random dipole. The width of the distribution of these random dipoles will be $\sqrt{N/M}$ times larger than the distribution width $\sigma^{1/2}$ for the original random dipoles – the random variables $A_{jj'}$. But since the linear size of each subsystem is larger by the same factor (in comparison with the original lattice unit) and the potential of a dipole in two dimensions decays as $1/R$ the two factors $\sqrt{N/M}$ in the expression for the potential canceal each other. This brings us back to the original problem in which the number of sites N is substituted by the number of subsystems M.

So if we now choose the subsystem with the largest value of the potential (created inside of it by he other subsystems) the average value of this potential will be given by the same function $V_m(l)$ (with $l = \sqrt{M}$) we are trying to estimate. But in the current approach we have kept the possibility to continue the search of the favourable position in array (for the vortex creation) by repeating the same procedure again and again. This means that at the next step we have to divide our chosen subsystem into M parts (sub-subsystems) and then choose the one for which the value of the potential created by the other subsystems is the largest. This adds another term $V_m(l)$ to the value of the potential. On the whole, such procedure can be repeated $\ln N/\ln M = \ln L/\ln l$ times and therefore the

average value of the potential at the site which is chosen in the end is given by

$$V_m^{(1)}(L) = \frac{\ln L}{\ln l} V_m(l). \tag{19}$$

Thus, we have proposed the explicit algorithm for the search of the site with large enough potential and have found that the average value of the potential on this site is logarithmical in the size of the system L. Since our algorithm does not allow to find the site with the largest value of $V(\underline{r})$ the expression (19) provides the lower border for $\overline{V_m}(L)$. This lower border is logarithmical in L and therefore the critical value of disorder σ_* has to be finite.

All arguing above has been based on comparison of different contributions to vortex energy and therefore is directly applicable in the case of zero temperature. As is well known the temperature of the phase transition in the regular system can be estimated [3] by comparing the energy of the single vortex given by Eq. (6) with its entropy $S(L)$ which in absence of the disorder is equal simply to the logarithm of the number of possible states (that is of the number of lattice sites). The free energy of a single vortex is then given by

$$F(L) = E_0(L) - TS(L) \approx (\pi J - 2T) \ln L, \tag{20}$$

and vanishes at $T = (\pi/2)J$ which signifies that at higher temperatures the thermal fluctuations make possible the spontaneous creation of vortices.

In the case of disordered system the entropy of a single vortex is given by

$$S(L) = \ln\left\{\sum_{\underline{r}} \exp\left[-\frac{V_m - V(\underline{r})}{T}\right]\right\}, \tag{21}$$

and since the difference between the values of a random potential in two different points grows with the increase of the distance between them, the expression (21) can be expected to be dominated by some finite vicinity of the point at which $V(\underline{r})$ is maximal (that is equal to V_m) and therefore $S(L)$ cannot grow with the increase in L as fast as $\ln L$. This means that at small temperatures the critical value of the disorder should remain the same as at zero temperature.

3. Vortex pairs unbinding and phase diagram

A more systematic approach to a phase diagram of a Coulomb gas should take into account that the presence of bound pairs (or other neutral complexes of charges) can induce the renormalization of the interaction of charges and of their energy [3, 4]. This can lead to the shift of the phase transition line or to the complete destruction of the ordered phase.

The renormalized interaction of the charges $G(\boldsymbol{R}_1, \boldsymbol{R}_2)$ can be defined as a response of the system to the introduction of two infinitely small test charges e_1 and e_2 at points $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively. The function $G(\boldsymbol{R}_1, \boldsymbol{R}_2)$ can be shown to have a form

$$G(\boldsymbol{R}_1, \boldsymbol{R}_2) = G_0(\boldsymbol{R}_1, \boldsymbol{R}_2) - \sum_{\underline{r}_1, \underline{r}_2} G_0(\boldsymbol{R}_1 - \underline{r}_1) \times \Sigma(\underline{r}_1, \underline{r}_2) G_0(\underline{r}_2 - \boldsymbol{R}_2), \tag{22}$$

where $\Sigma(\underline{r}_1, \underline{r}_2)$ is given by the second derivative of the Coulomb gas free energy with respect to the potential

$$\Sigma(\underline{r}_1, \underline{r}_2) = -\frac{\partial^2 F}{\partial V(\underline{r}_1) \partial V(\underline{r}_2)}. \tag{23}$$

In the lowest order approximation in pair concentration the contributions to the second term in Eq. (22) from different vortex pairs are independent from each other. Therefore, when calculating this correction the interaction between the charges belonging to different pairs can be neglected. To this end the partition function of the Coulomb gas can be rewritten in the form

$$Z = \prod_{(\underline{r}_1, \underline{r}_2)} (1 + w_{\underline{r}_1 \underline{r}_2} + w_{\underline{r}_2 \underline{r}_1}), \tag{24}$$

where

$$w_{\underline{r}_1 \underline{r}_2} = \exp\left[-\frac{g(\underline{r}_1 - \underline{r}_2) - V(\underline{r}_1) + V(\underline{r}_2)}{T}\right] \tag{25}$$

and the product is taken over all pairs of different sites on a dual lattice.

The form of Eq. (24) implies that for each pair of sites $(\underline{r}_1, \underline{r}_2)$ the three possibilities are considered: the absence of any vortex pair, the presence of vortex pair with the topological charge $+1$ at

point $\boldsymbol{r}_1$ and -1 at point $\boldsymbol{r}_2$ and the presence of the vortex pair with opposite orientation. The partition function (24) takes into account the interaction of all the vortices with a random potential and the mutual interaction of the vortices belonging to the same pair and the only contribution which is neglected is the interaction between the vortices belonging to different pairs.

Substitution of the free energy corresponding to the partition function (24) into Eq. (23) allows then to rewrite the expression for the disorder-averaged Fourier transform of $\Sigma(\boldsymbol{r}_1, \boldsymbol{r}_2)$ as

$$\overline{\Sigma}_2(\boldsymbol{q}) = \sum_{\boldsymbol{R}} (1 - \cos \boldsymbol{q}\boldsymbol{R}) \overline{W(\boldsymbol{R})}, \tag{26}$$

the potential-dependent weight factor $W(\boldsymbol{R}, V)$ being of the form

$$W(\boldsymbol{R}, V) = \frac{\partial}{\partial V}\left[\frac{w(\boldsymbol{R}, V) - w(\boldsymbol{R}, -V)}{1 + w(\boldsymbol{R}, V) + w(\boldsymbol{R}, -V)}\right], \tag{27}$$

where $w(\boldsymbol{R}, V)$ is defined by Eq. (25) with $\boldsymbol{r}_1 - \boldsymbol{r}_2 = \boldsymbol{R}$ and $V(\boldsymbol{r}_1) - V(\boldsymbol{r}_2) = V$. The width of the Gaussian distribution for V is given by Eq. (4) and also depends on $\boldsymbol{R}$.

When only the large-distance behaviour of the vortex interaction is important the correlation to the interaction can be expressed as a correction to the coupling constant J:

$$\Delta J = -\pi^2 J^2 \sum_{\boldsymbol{R}} R^2 \overline{W(\boldsymbol{R})}. \tag{28}$$

In the same approximation of diluted pairs all the corrections to the distribution of random potential (4) can be absorbed into the renormalization of J, whereas the parameter σ describing the strength of the disorder remains unrenormalized.

For $R \to \infty$ the result of the averaging $W(\boldsymbol{R}, V)$ over disorder is characterized by an algebraic behaviour:

$$\overline{W(\boldsymbol{R})} \approx B(T) R^{-K(T)} \tag{29}$$

but the temperature dependence of the parameters in Eq. (29) is essentially different for $T > T_*(J, \sigma) \equiv 2J\sigma$ when the results of Rubinstein et al. [5] are reproduced and for $T < T_*(J, \sigma)$:

$$B(T) = \begin{cases} \frac{\pi T/T_*}{\sin(\pi T/T_*)} \frac{1}{\pi J \sqrt{2\sigma \ln R}} & \text{for } T < T_*, \\ \frac{2}{T} & \text{for } T > T_*, \end{cases}$$

$$K(T) = \begin{cases} \frac{\pi}{2\sigma} & \text{for } T < T_*, \\ 2\pi\left(\frac{J}{T} - \frac{J^2\sigma}{T^2}\right) & \text{for } T > T_*. \end{cases} \tag{30}$$

The more attentive consideration shows that $\overline{W(R)}$ always decreases with a decrease in temperature and therefore there are no reasons to expect a reentrant transition.

Substitution of Eqs (29) and (30) into Eq. (28) shows that for $T > T_*(J, \sigma)$ the lowest-order correction to the coupling constant becomes divergent on the line

$$T = T_+(J, \sigma) \equiv 2J\sigma_*(1 + \sqrt{1 - \sigma/\sigma_*}) \tag{31}$$

(the line CD in Fig. 1), whereas at low temperatures $[T < T_*(J, \sigma)]$ the domain of convergence is restricted by the line $\sigma = \sigma_* = \pi/8$ (the line AC in Fig. 1) which is parallel to the temperature axis.

That defines the domain of stability of the ordered phase in the limit of zero fugacity (infinite core energy). For finite or zero core energy the renormalization effects should be taken into account and the domain of the ordered phase will shrink. But since only the coupling constant J is renormalized and the disorder strength σ remains the same the position of the low-temperature part of the phase transition line ($\sigma = \sigma_*$) will not be changed, whereas the other part of the phase transition line (CD) will be shifted to the lower temperatures (BE).

Remarkably, the single vortex consideration of the previous section also leads to the conclusion that at low temperature the critical value of σ should not depend on temperature. Moreover, the critical values of σ predicted by two methods coincide with each other. Since Ozeki and Nishimori [11] have proved that the phase transition line should be parallel to the temperature axis for $T < J\sigma$, we have shown in Fig. 1 that the singular point C is shifted by the renormalization effects to the position B where the line $\sigma = \sigma_*$ crosses the line $T = J\sigma$.

Thus, we have studied the stability of the low-temperature phase of a Josephson junction array

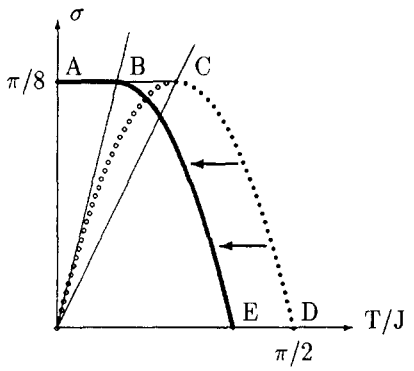

Fig.1. In the limit of zero fugacity the ordered phase is stable below the line ABCD. The renormalization effects shift the curved part of the phase transition line to the lower temperatures.

with positional disorder with respect to spontaneous creation of vortices and unbinding of vortex pairs. Our main conclusion is that in the situation when the system can be described in terms of a Coulomb gas with random potential the reentrant transition to the disordered phase cannot take place since the correctly calculated average of the correction to vortex–vortex interaction always decreases with the decrease in temperature and therefore cannot become more importnat. This conclusion is in agreement with the results of experiments on Josephson junction arrays [7] and of numerical simulations [7–9]. The validity of the additional suggestion that at low temperatures the critical value of disorder does not change with temperature may depend on the distribution function of disorder [11]. Unfortunately, insofar neither real nor computer experiments were accurate enough to check the existence of such a property.

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