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PARITY EFFECT AND CHARGE BINDING TRANSITION IN SUBMICRON JOSEPHSON JUNCTION ARRAYS

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We reconsider the issue of Berezinskii–Kosterlitz–Thouless (BKT) transition into an insulating state in the Coulomb-dominated Josephson junction arrays. We show that previously predicted picture of the Cooper-pair BKT transition at $T = T_2$ is valid only under the condition that T_2 is considerably below the parity-effect temperature $T^* \approx 0.1\Delta$ and even in this case it is not a rigorous phase transition but only a crossover, whereas the real phase transition takes place at $T_1 \approx T_2/4$. Our theory is in agreement with available experimental data on Coulomb-dominated Josephson arrays and also sheds some light on the origin of unusual reentrant temperature dependence of resistivity in the array with nearly-criticial ratio E_C/E_J.

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1. Two-dimensional arrays of micron-scale superconducting islands are extensively studied during last years, both experimentally [1–4] and theoretically [5–7]. It is well-established now that their low-temperature behaviour is determined by the competition between Josephson coupling energy E_J and effective charging energy $E_C = e^2/2C$, where C is some relevant electric capacitance (to be discussed below). Macroscopic superconductive coherence was observed at low temperatures in the arrays with $E_J \gg E_C$, whereas arrays with $E_J \ll E_C$ show insulating behaviour at $T \to 0$. At nearly-critical value of the ratio $x = E_J/E_C \sim x_{cr}$ direct transition between superconductive (SC) and insulating (I) behavior as function of x was observed in zero magnetic field [2]. Moreover, very weak magnetic field $B \leq 1G$ (producing small fractions of flux quantum per unit cell of the array) was shown to switch arrays with $x \approx x_{cr}$ between SC-like and I-like behavior as function of temperature; recently very interesting intermediate region was found [2] where resistance $R(T)$ is basically constant in the temperature range 10 mK$\leq T \leq$ 200 mK, which indicates the existence of a "$2D$ metal" state sandwiched between SC and I phases.

The above-mentioned basic properties of $2D$ arrays are in qualitative agreement with available theories [5, 7] (except for the recently observed $2D$ metal state); however several important features are not understood yet. In particular, resistance of the insulating arrays shows purely activated behaviour $R(T) \propto \exp(E_a/T)$ with constant activation energy E_a through the whole temperature interval studied [2–4], whereas theoretically the charge binding Berezinskii–Kosterlitz–Thouless (BKT) transition [8, 9] from the conducting to insulating phase is expected to occur [5] at the temperature $T_2 \approx E_C/\pi$. Such a transition should occur due to nearly-logarithmic form of Coulomb-interaction between charges in the arrays with self-capacitance of islands C_0 very small compared to the inter-island (junction) capacitance C. In the currently studied arrays the ratio $C/C_0 \sim 100$ (as measured at very low temperatures, about 10 mK, cf. e.g. [2]), which should result in the logarithmic interaction throughout the whole array (effecitve length of interaction Λ should be estimated with the account of 3D nature of electric field, which leads [2, 10] to $\Lambda \sim C/C_0 \sim 100$) and, consequently, to the charge binding BKT transition.

In the case of islands in SC state and under the condition $E_J \ll E_C$ the temperature of this transition was estimated [5] as $T_2 \approx E_C/\pi$, whereas in the case of normal islands (i.e. with superconductivity suppressed by magnetic field) it is expected to be 4 times lower: $T_1 \approx E_C/4\pi$ due to the twofold decrease of an elementary charge available. Nevertheless no indication of such a transition in array of SC islands was found experimentally (except in very recent preprint [11], which is discussed below). Another surprising feature observed in [2] is nonmonotoneous ("reentrant") temperature behaviour of resistance $R(T)$ of the array with a nearly-critical E_J/E_C ratio at $T \leq 200\,\mathrm{mK}$.

In the present Letter we show that the above experimental observations can be naturally understood once the temperature dependence of effective Coulomb-interation between the charges in the array is taken into account properly. In a very broad sense our analysis follows the ideas of Efetov [12] who established the basis for the description of quantum fluctuations in granular superconductors; namely, we consider screening of Cooper-pair Coulomb-interaction by normal quasiparticles existing in each superconductive island at finite temperatures. However we believe that Efetov's treatment of the effect he proposed technically was not quite correct, thus we present here another theoretical approach to the same problem.

Our main qualitative result can be formulated as follows: at the temperatures above the so-called parity-effect [13–16] temperature $T^* \approx \Delta/\ln\mathcal{M} \ll \Delta$ [where $\mathcal{M} = V\nu(0)\sqrt{8\pi T\Delta} \sim 10^4 - 10^5$, V is the volume of the island and $\nu(0)$ is the density of states at the Fermi level in absense of superconductivity] the presence of thermal quasiparticles (with the number $\sim \mathcal{M}\exp(-\Delta/T) \gg 1$) on each island excludes any possibility to observe Cooper-pairs BKT transition at T_2. Since in the most of arrays studied till now the above-defined characterisitic temperature T_2 was in the range 0.3-0.5 K, whereas parity effect temperature $T^* \approx 0.2\mathrm{K} < T_2$, the absense of anything like BKT transition near T_2 is quite natural (measurements below T^* were not possible in these arrays [3, 4] since $R(T)$ becomes immeasurably high ($\geq 10^9\,\mathrm{Ohm}$)). On the other hand single-electron BKT transition is a completely different issue: we do expect such a transition to be observable at approximately the same temperature $T_1 \approx T_2/4$ as in the arrays with islands in the normal state.

2. We proceed now to the derivation of our results, and will follow, with one important modification, Ref. [5]. In the limit when charge tunneling is weak and is important only for the establishment of thermodynamic equilibrium an array of superconducting islands can be described by a classical partition function of a form:

$$Z = \sum_{\{n\}} \exp\left[-\frac{1}{2}\sum_{i,j} G_{ij} n_i n_j - \frac{D}{T}\sum_j \frac{1-(-1)^{n_j}}{2}\right], \qquad G_{ij} = \frac{e^2}{T} C_{ij}^{-1}. \qquad (1)$$

The first term in the exponent in Eq. (1) stands for the electrostatic energy of the array which in the case when only mutual capacitance of nearest islands C is of importance corresponds to the logarithmic interaction of the charges in two-dimensional array:

$$G_{i=j} - G_{ij} \approx \frac{2E_C}{T}\left(\frac{1}{2\pi}\ln R_{ij} + \frac{1}{4}\right), \qquad E_C = \frac{e^2}{2C}, \tag{2}$$

whereas the second term describes the dependence of a free energy of a superconducting island on the parity of the number of electrons n_j on this island [13, 15, 16].

The free energy difference $D(T)$ between the islands with odd and even number of electrons can be expressed as

$$D(T) = -T\ln\tanh(\Omega_{oe}/T). \tag{3}$$

where $\Omega_{oe} = -T\ln(Z_{odd}/Z)$ and Z_{odd} is the "odd grand canonical partition function" introduced in Ref.[13] for the study of parity effect. Accurate expression for the function $\Omega_{oe}(T)$ can be found in [15], but for $T \ll \Delta$ a good approximation is given by

$$\Omega_{oe}(T)/T \approx \mathcal{M}e^{-\Delta/T}, \quad \mathcal{M} = V\nu(0)\sqrt{8\pi T\Delta}, \tag{4}$$

The ratio Ω_{oe}/T in that limit is proportional to the number of thermally exited quasiparticles on one island.

In terms of statistical mechanics partition function (1) defines a lattice Coulomb gas in which the fugacities of odd charges $Y = \exp(-D/T)$ differ from the fugacities of even charges (which are equal to one). Comparison of Eq. (4) with Eq. (3) shows then that for $T \ll T^* = \Delta/\ln\mathcal{M}$ the parity-dependent free energy difference $D(T) \approx \Delta - T\ln\mathcal{M} \gg T$ and $Y \ll 1$, whereas in the opposite limit $T \gg T^*$ the quantity $D(T)$ becomes exponentially small and Y is very close to one.

Previously it has been assumed [5] that in the regime when island charges behave as classical variables the main difference between the array of normal islands and the array of superconducting islands is that in the array of normal islands the charge of each island is quantized in units of e, whereas in the array of superconducting islands the charge is quantized in units of $2e$. The consequence for the array the electrostatic properties of which are dominated by mutual capacitance of nearest neighbours is that the temperature T_2 of the BKT transition in the array of superconducting islands (appearance of free double charges) should be exactly four times higher than the temperature $T_1 \sim E_C/4\pi$ of the BKT transition in the analogous array of normal islands (appearance of free single charges). Comparison with Eq. (1) shows that such description of the array of superconducting islands would be correct only in the limit of $D(T)/T \to \infty$. Since $D(T)$ is always finite this description turns out to be misleading. The behaviour of the array at temperatures close to $T_2 = E_C/\pi$ depends qualitatively on the relation between T_2 and T^*; we consider both cases in turn.

3. At $T_2 \geq T^*$ an array of the superconducting islands is described by practically the same partition function as an array of the normal islands, since $D(T_2) << T_2$ in that case. The phase transition into insulating state in such system can be associated with the binding of the charges ± 1 into neutral pairs; it takes place at the temperature T_1 which is slightly lower than the simple estimate $T_1^{(0)} = E_C/4\pi$ which can be obtained by comparison of the single charge energy with its entropy. The difference between T_1 and $T_1^{(0)}$ is related to the renormalization of charge interaction by bound pairs of charges and decreases with

decrease in fugacities. The appearance of the free single charges induces the screening of the Coulomb-interaction for all types of charges and therefore the double charges also are free at $T > T_1$. Not even a trace of a separate phase transition related to debounding of double charges can be expected to be observed in such a situation, which was realized in the experiments [1-4].

4. In the opposite case $T_2 < T^*$ there is a range of temperatures $T_2 < T < T^*$ where fugacity of single charges Y is much smaller than one. This leads to the increase of phase transition temperature, but it still has to remain smaller than $T_1^{(0)}$. The difference with the case of $Y \approx 1$ is that for $Y \ll 1$ the concentration of free single charges n_1 remains small even at the temperatures considerably higher than T_1. In the region $T_1 < T < T_2$ it can be estimated with the use of the standard Debye-Hückel approximation which gives for n_1 the self-consistent equation:

$$n_1 = 2Y \exp\left\{-\frac{1}{2}\int \frac{d^2\mathrm{q}}{(2\pi)^2}\frac{1}{K[2(1-\cos q_x)+2(1-\cos q_y)]+n_1}\right\} \tag{5}$$

(where $K = T/2E_C$) the solution of which for small n_1 can be expressed as

$$n_1 \approx 2\exp\left\{-\frac{D(T)+aE_C-[\ln(4E_C/T)]E_C/4\pi}{T-T_1^{(0)}}\right\}, \tag{6}$$

where $a = 0.276...$. The main effect of Coulomb-interaction is seen in that it produces singularity at $T = T_1^{(0)}$ in the exponent in Eq.(6). If the shift of the phase transition temperature is taken into acccount $T_1^{(0)}$ should be substituted by T_1. Note that second and third terms in the numenator of the exponent almost canceal each other in the relevant range of parameters.

For $D(T) \gg T$ the screening of the interaction is noticable only on the large scales and the concentration of free double charges also remains small. On the other hand at the temperature $T_2 = 4T_1 \approx E_C/\pi$ the free double charges have to appear even when $Y = 0$ due to mutual influence of pairs of double charges (cf. with Ref.[5]). That means that for $D(T) \gg T$ in the vicinity of T_2 there occurs a crossover characterized by a prolifiration of free double charges.

Close to the transition temperature T_1 the self-consistent approximation is no longer valid and more advanced methods should be used. It is easy to show that when only the single and double charges are taken into account, the Coulomb gas model described by partition function (1) becomes isomorphic (in continuous approximation) to the sine-Gordon model defined by the Hamiltonian:

$$H = \int d^2\mathrm{r}\left[\frac{K}{2}(\nabla\theta)^2 - 2Y\cos\theta - 2\cos 2\theta\right]; \quad K = \frac{T}{2E_C}. \tag{7}$$

The renormalization group equations for the Hamiltonian (7) can be found in Ref.[17]. As can be expected their solution shows that for temperatures lower than T_2 the presence of double charges (in form of neutral pairs) does not introduce any qualitative changes. In the close vicinity of T_1 the temperature dependence of n_1 deviates from the self-consistent result (6) and follows the standard BKT critical behaviour [9] with

$$n_1(T) \propto \exp\left[-\frac{b}{\sqrt{1-T/T_1}}\right], \tag{8}$$

where b is of the order of unity. The array's linear dc resistance should be inversely proportional to the density of free charges n_1. Taking into account Eqs.(6, 8), we get an estimate for this resistance at the temperatures near T_1:

$$\ln \frac{R(T)}{R_1} \approx \min \left[\frac{D(T)}{T - T_1}, \frac{b}{\sqrt{4\pi}} \left(\frac{E_C}{T - T_1} \right)^{1/2} \right], \tag{9}$$

where R_1 is inversely proportional to the probability of tunnelling event which is only weakly dependent on the temperature: $R_1 \sim R_n \mathcal{M}$, where R_n is the normal-state tunnelling resistance (cf. [18]).

5. The representation (7) is also useful for comparison between our results and Efetov's treatment of screening by quasiparticles [12], which can be expressed just as the replacement of the original capacitance matrix C_{ij} by the "effective" one defined (at $T \ll \Delta$) as

$$C_{ij}^{eff} = C_{ij} + \delta_{ij} \cdot V\nu(0)(2e)^2 \sqrt{\frac{2\pi\Delta}{T}} e^{-\Delta/T}. \tag{10}$$

Let us now formally expand the second term in Eq.(7) up to the second order in θ, and neglect the rest of terms. One can easily see that the expression obtained in this way for the effective interaction between $2e$ charges (generated by expansion of the partition function in powers of the $\cos 2\theta$ term) would coinside with the one obtained by inversion of the effective capacitance (10). Physically the above formal operation would mean neglect of the discrete nature of electric charge, which could be reasonable if charge transport between islands could proceed via some "classical" channels able to provide charge in continuous amounts (like charging of macroscopic electric capacitor by external voltage source). It is not the case for submicron arrays with tunnel junctions where charges of islands can change only by $1e$ quantum (which is just reflected by periodic nature of the second term in Eq.(7)). However, basic qualitative feature of Efetov's result-screening of Cooper pair charges by normal exitations - remains valid, in spite of the absense of any simple notion like an effective capacitance matrix.

6. It follows from our results, that in order to observe some growth of effective activation energy (defined as $E_a = d\ln R(T)/d(1/T)$) with T approaching T_2 from above, one needs to use array with the Coulomb-energy E_C few times below than Δ, so that $T_2 \leq T^*$. This conclusion is in complete agreement with recent experimental data [11], where some moderate growth of $E_a(T)$ in the temperature range around 0.2-0.3 K was observed. This experiment differs from few previous ones of the same type [2–4] (where constant E_a was observed) by lower values of E_C and a bit higher reported Δ. On the other hand, we do not agree with the interpretation of that $E_a(T)$ growth as precursor of the BKT transition at T_2, given in [11] for their SC arrays. As follows from our results, no such a transition exists at T_2, which agrees with a rather modest (compared to BKT behaviour) growth of $E_a(T)$ observed in [11] at $T \sim T_2$. Note that the agreeement between the data reported in [11] for normal arrays and the expected genuine BKT transition at T_1 is much better than the above-mentioned comparison for SC arrays.

The above theoretical results point out that any analysis of experimental data on superconductor-insulator transition in artificial arrays of superconducting islands (as well as in the dirty thin films near SC-I transition) should take into account an existence of a characteristic temperature scale of the parity effect T^* (note that T^* is magnetic-field dependent and strongly suppressed by the fields of the order of H_{c2}). In particular, the behavior of I-V characteristics in the intermediate temperature range $T^* < T < \Delta$ cannot be unambigously related to the genuine ground-state properties of the system, as is examplified by non-monotonous $R(T)$ behaviour observed in [2] for a nearly critical ratio E_J/E_C. We interpret this unusual behaviour as follows: at moderately low T screening by quasiparticles is effective and reduces Coulomb-repulsion of Cooper-pairs, leading to the decrease of $R(T)$ behaviour; at still lower T this screening is gone, Coulomb-repulsion increases and effective ratio E_J/E_C^{eff} enters the "insulative" part of the phase diagram, leading to the increase of $R(T)$ at further T decrease.

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